PROFESSOR: Let's look at the magnitude squared of those waves that we've already defined here. We have two solutions, one for no potential and one for a real potential. Both are for some finite range potential.

We have phi of x squared is equal to sine squared of kx. And psi of x squared is equal to sine squared of kx plus delta. Even from this information you can get something.

Think of x plotted here. Here's x equals 0. And there's this sine squared. This wave for phi of x squared.

Suppose you're looking at some feature-- a maximum, a minimum-- of this function. Suppose the feature happens when the argument, kx, is equal to some number, a0. Whatever feature-- this number a0 could be 0, in which case you're looking at a minimum, it could be pi over 2, in which case you're looking at a maximum-- some feature of sine squared.

Well the same feature will appear in this case when the whole argument is equal to a0. So while this one happens at x equals a0 over k, here it happens at x still equals a0 over k minus delta over k. If this is the probability density associated to the solution for no potential and it has a maximum here, the maximum of the true solution-- say, here-- would appear at a distance equal to delta over k. Earlier-- so this is like the x, and this is like the x tilde-- that feature would appear, delta over k in that direction.

So this is psi. This is psi squared. So we conclude, for example, that when delta is greater than 0, the wave is pulled. Delta equals 0, the two shapes are on top of each other. For delta different from 0, the wave function is pulled in. So delta greater than 0, psi is pulled in.

What could we think of this? The potential is attractive. It's pulling in the wave function. Attractive.

Delta less than 0, the wave is pushed out. It would be in the other direction, and the psi is pushed out. Potential is repulsive. So a little bit of information even from the signs of this thing.

We want to define one last thing, and then we'll stop. It's the concept of the scattered wave. What should we call the scattered wave? We will define the scattered wave psi s as the extra piece in the solution-- the psi solution-- that would vanish without potential.

So we say, you have a psi, but if you didn't have a potential, what part of this psi would survive? Think of writing the psi of x as the solution without the potential plus the extra part, the scattered wave, psi x. So this is the definition.

The full scattering solution, the full solution when you have a potential, can be written in a solution without the potential and this scattered thing. Now, you may remember-- we just did it a second ago-- that this original solution and the psi solution have the same incoming wave. The incoming wave up there is the same for the psi solution as for this one.

So the incoming waves are the same. So only the outgoing waves are different. And this represents how much more of an outgoing wave you get than from what you would have gotten with psi. So this must be an outgoing wave.

We'll just plug in the formula here. psi s is equal to psi minus phi. And it's equal to 1 over 2i e to the ikx plus 2i delta minus e to the minus ikx minus 1 over 2i-- the phi-- into the ikx minus e to the minus ikx. So the incoming waves wee the same. Indeed, they cancelled. But the outgoing waves are not. You can factor an e to the ikx, and you get e to the 2i delta minus 1. Which is equal to e to the ikx e to the i delta times sine delta.

There we go. We have the answer for the scattered wave. It's proportional to sine delta, which, again, makes sense. If delta is equal to 0, there is no scattering. It's an outgoing wave and all is good. So I'll write it like this. psi s is equal to As e to the ikx, with As equal to e to the i delta sine delta.