

PROFESSOR: So let's do an example where we can calculate from the beginning to the end everything. Now, you have to get accustomed to the idea of even though you can calculate everything, your formulas that you get sometimes are a little big. And you look at them and they may not tell you too much unless you plot them with a computer.

So we push the calculations to some degree, and then at some point, we decide to plot things using a computer and get some insight on what's happening. So here's the example. We have a potential up to distance, a , to 0. The wall is always there, and this number is minus v naught.

So it's a well, a potential well. And we are producing energy. Eigenstates are coming here. And the question now is to really calculate the solution so that we can really calculate the phase shift. We know how the solutions should read, but unless you do a real calculation, you cannot get the phase shift. So that's what we want to do.

So for that, we have to solve the Schrodinger equation. Ψ of x is equal to what? Well, there's a discontinuity. So we probably have to write an answer in which we'll have a solution in one piece and a solution in the other piece.

But then we say, oh, we wrote the solution in the outside piece already. It is known. It's always the same. It's universal. I don't have to think. I just write this. E to the i delta. I don't know what delta is, but that's the answer, E to the i delta sine kx plus delta should be the solution for x greater than a .

You know if you were not using that answer, it has all the relevant information for the problem, time delays, everything, you would simply write some superposition of E to the i kx and E to the minus i kx with two coefficients. On the other hand, here, we will have, again, a wave.

Now, it could be maybe an E to the i kx or E to the minus i kx . Neither one is very good because the wave function must vanish at x equals 0. And in fact, the k that represents the kinetic energy here, k is always related to E by the standard quantity, k squared equal to mE over h squared or E equal the famous formula.

On the other hand, there is a different k here because you have different kinetic energy. There must be a k prime here, which is $2m E$ plus v naught. That's a total kinetic energy over h squared. And yes, the solutions could be E to the ik prime x equal to minus ik prime x minus ik prime x , but since they must vanish at 0, should be a sine function.

So the only thing we can have here is a sine of $k'x$ for x less than a and a coefficient. We didn't put the additional normalization here. We don't want to put that, but then we must put the number here, so I'll put it here. That's the answer, and that's k and k' .

Now we have boundary conditions that x equals a . So ψ continues at x equals a . What does it give you? It gives you a sine of ka is equal to E to the i delta sine of $ka + \delta$. And ψ' continues at x equals a will give me ka cosine ka equal-- I have primes missing; I'm sorry, primes-- equals $k E$ to the i delta cosine $ka + \delta$.

What do we care for? Basically we care for δ . That's what we want to find out because δ tells us all about the physics of the scattering. It tells us about the scattering amplitude, sine squared δ . It tells us about the time delay, and let's calculate it.

Well, one way to calculate it is to take a ratio of these two equations so that you get rid of the a constant. So from that side of the equation, you get $k \cotangent\ of\ ka + \delta$ is equal to $k' \cotangent\ of\ k' a$. Or $\cotangent\ of\ ka + \delta$ is k' over k . We'll erase this.

And now you can do two things. You can display some trigonometric wizardry, or you say, OK, δ is arc cotangent of this minus ka . That is OK, but it's not ideal. It's better to do a little bit of trigonometric identities. And the identity that is relevant is the identity for cotangent of $a + b$ is $\cot a \cot b - 1$ over $\cot a + \cot b$.

So from here, you have that this expression is $\cot ka \cot \delta - 1$ over $\cot ka + \cot \delta$. And now, equating left-hand side to this right-hand side, you can solve for cotangent of δ . So cotangent of δ can be solved for-- and here is the answer. $\cot \delta$ is equal to $\tan ka + k'$ over $k \cot k' a$ over $1 - k'$ over $k \cot k' a \tan ka$.

Now, who would box such a complicated equation? Well, it can't be simplified any more. Sorry. That's the best we can do.