PROFESSOR: Time evolution of a free particle wave packet. So, suppose you know psi of x and 0. Suppose you know psi of x and 0.

So what do you do next, if you want to calculate psi of x and t? Well, the first step, step one, is calculate phi of k. So you have phi of k is equal 1 over square root of 2 pi integral dx psi of x, 0 e to the minus ikx. So you must do this integral.

Step two-- step two-- with this, now rewrite and say that psi of x, 0 is 1 over square root of 2 pi dk e to the-- no, I'm sorry-- phi of k, e to the ikx. So that has achieved our rewriting of psi of x and 0, which was an arbitrary function as a superposition of plane waves.

Step three is the most fun step of all. Step three-- you look at this, and then you say, well, I know now what psi of x and t is. Evolving this is as easy as doing nothing. What I must do here is 1 over square root of 2 pi-- just copy this-- dk, phi of k, e to the ikx. And I put here minus omega of k, t.

And I remind you that h bar omega of k is the energy, and it's equal to h squared k squared over 2m. This is our free particle. And I claim that, just by writing this, I've solved the Schrodinger equation and I've time-evolved everything. The answer is there-- I didn't have to solve the differential equation, or-- that's it. That's the answer.

Claim this is the answer. And the reason is important. If you come equipped with a Schrodinger equation, what should you check, that ih bar d psi dt is equal to h psi-- which is minus h-- squared over 2m, d second, dx squared psi. Well, you can add with ih d dt on this thing. And you remember all that happens is that they all concentrate on this thing.

And it solves this, because it's a plane wave. So this thing, this psi of x and t, solves the Schrodinger equation. It's a superposition of plane waves, each of which solves the free Schrodinger equation.

So, we also mention that since the Schrodinger equation is first ordered in time, if you know the wave function at one time, and you solve it, you get the wave function at any time. So here is a solution that is a solution of the Schrodinger equation. But at time equals 0-- this is 0-- and we reduce this to psi of x and 0.

So it has the right condition. Not only solve the Schrodinger equation, but it reduces to the right thing. So it is the answer. And we could say-- we could say that there is a step four, which is-- step four would be do the k integral.

And sometimes it's possible. You see, in here, once you have this phi of k, maybe you can just look at it and say, oh, yeah, I can do this k integral and get psi of x and 0, recover what I know. I know how to do-- this integral is a little harder, because k appears a little more complicated. But it has the whole answer to the problem.

I think one should definitely focus on this and appreciate that, with zero effort and Fourier's theorem, you're managing to solve the propagation of any initial wave function for all times. So there will be an exercise in the homework, which is called evolving the free Gaussian-- Gaussian.

So you take a psi a of x and time equals 0 to be e to the minus x squared over 4a squared over 2 pi to the 1/4-- that's for normalization-- square root of a. And so what is this? This is a psi-- this is a Gaussian-- and the uncertainty's roughly a-- is that right? Delta x is about a, because that controls the width of the Gaussian.

And now, you have a Gaussian that you have to evolve. And what's going to happen with it? This Gaussian, as written, doesn't represent a moving Gaussian. To be a moving Gaussian, you would like to see maybe things of [? the ?] from e to the ipx that represent waves with momentum.

So I don't see anything like that in this wave function. So this must be a Gaussian that is just sitting here. And what is it going to do in time?

Well, it's presumably going to spread out. So the width is going to change in time. There's going to be a time in which the shape changes. Will it be similar to what you have here? Yes. The time will be related.

So time for changes. So there will be some relevant time in this problem for which the width starts to change. And it will be related to ma squared over h bar.

In fact, you will find that with a 2, the formulas look very, very neat. And that's the relevant time for the formation of the Gaussian.

So you will do those four steps. They're all doable for Gaussians. And you'll find the Fourier transform, which is another Gaussian. Then you will put the right things and then try to do the integral back. The answer is a bit messy for psi, but not messy for psi squared, which is what we typically ask you to find.