PROFESSOR: It's a statement about the time dependence of the expectation values. It's a pretty fundamental theorem. So here it goes.

You have d dt of the expectation value of Q. This is what we want to evaluate. We Now this would be d dt of integral psi star of x and t, Q psi of x and t. And the d dt acts on the two of them. So it gives you integral partial psi star dt Q psi of x and t plus psi star Q partial psi dt. And this is the integral over the x.

You've seen that kind of stuff. And what is it? Well, integral dx, this is this Schrodinger equation, d psi star dt is i over h bar, H psi star. From the Schrodinger equation. Then you have the Q psi of x and t. On this term, you will have a very similar thing. Minus i over H bar this time, psi star QH psi of x and t.

So we use the Schrodinger equation in the form, i d psi dt-- i H bar d psi dt-- equal H psi. I used it twice. So then, it's actually convenient to multiply here by i H bar d dt of Q. So I multiplied by i H bar, and I will cancel the i and the H bar in this term, minus them this term. So we'll have d cube x psi star Q H hat psi minus H hat psi star cube psi.

OK. Things have simplified very nicely. And there's just one more thing we can do. Look, this is the product of Q and H. But by hermiticity, H in here can be brought to the other side to act on this wave function. So this is actually equal to the integral d cube dx psi star QH hat psi minus-- the H can go to the other side-- psi star H cube psi.

But then, what do we see there? We recognize a commutator. This commutator is just like we did for x and p, and we started practicing how to compute them. They show up here. And this is maybe one of the reasons commutators are so important in quantum mechanics. So what do we have here? i H bar d dt of the expectation value of Q is equal to the integral dx of psi star, QH minus HQ psi. This is all of x and t.

Well, this is nothing else but the commutator of Q and H. So our final result is that iH bar d dt of the expectation value of Q is equal to-- look. It's the expectation value of the commutator. Remember, expectation value for an operator-- the operator is the thing here-- so this is nothing else than the expectation value of Q with H.

This is actually a pretty important result. It has all the dynamics of the physics in the observables. Look, the wave functions used to change in time. Due to their change in time, the expectation values of the operators change in time. Because this integral can't depend on time.

But here what you have succeeded is to represent the change in time of the expectation value-- the change in time of the position that you expect you find your particle-- in terms of the expectation value of a commutator with a Hamiltonian.

So if some quantity commutes with a Hamiltonian, its expectation value will not change in time. If you have a Hamiltonian, say with a free particle, well, the momentum commutes with this. Therefore the expected value of the momentum, you already know, since the momentum commutes with H. This is 0. The expected value of this is 0. And the expected value of the momentum will not change, will be conserved.

So conservation laws in quantum mechanics have to do with things that commute with the Hamiltonian. And it's an idea we're going to develop on and on.