PROFESSOR: Last time, we talked about the Broglie wavelength. And our conclusion was, at the end of the day, that we could write the plane wave that corresponded to a matter particle, with some momentum, p, and some energy, E. So that was our main result last time, the final form for the wave.

So we had psi of x and t that was e to the i k x minus i omega t. And that was the matter wave with the relations that p is equal to h bar k. So this represents a particle with momentum, p, where p is h bar times this number that appears here, the wave number, and with energy, E, equal to h bar omega, where omega is that number that appears in the [? term ?] exponential.

Nevertheless, we were talking, or we could talk, about non-relativistic particles. And this is our focus of attention. And in this case, E is equal to p squared over 2m. That formula that expresses the kinetic energy in terms of the momentum, mv.

So this is the wave function for a free particle. And the task that we have today is to try to use this insight, this wave function, to figure out what is the equation that governs general wave functions. So, you see, we've been led to this wave function by postulates of the Broglie and experiments of Davisson, and Germer, and others, that prove that particles like electrons have wave properties.

But to put this on a solid footing you need to obtain this from some equation, that will say, OK, if you have a free particle, what are the solutions. And you should find this solution. Perhaps you will find more solutions. And you will understand the problem better.

And finally, if you understand the problem of free particle, there is a good chance you can generalize this and write the equation for a particle that moves under the influence of potentials. So basically, what I'm going to do by trying to figure out how this wave emerges from an equation, is motivate and eventually give you, by the middle of this lecture, the Schrodinger equation. So that's what we're going to try to do.

And the first thing is to try to understand what kind of equation this wave function satisfies. So you want to think of differential equations like wave equations. Maybe it's some kind of wave equation. We'll see it's kind of a variant of that. But one thing we could say, is that you have this wave function here. And you wish to know, for example, what is the momentum. Well you should look at k, the number that multiplies the x here, and multiply by h bar. And that would give you the momentum.

But another way of doing it would be to do the following. To say, well, h bar over i d dx of psi of x and t, calculate this thing. Now, if I differentiate with respect to x, I get here, i times k going down. The i cancels this i, and I get h bar k. So, I get h bar k times the exponential. And that is equal to the value of the momentum times the wave.

So here is this wave actually satisfies a funny equation, not quite the differential equation we're looking for yet, but you can act with a differential operator. A derivative is something of a differential operator. It operates in functions, and takes the derivative. And when it acts on this wave function, it gives you the momentum times the wave function. And this momentum here is a number. Here you have an operator. An operator just means something that acts on functions, and gives you functions. So taking a derivative of a function is still a function. So that's an operator.

So we are left here to think of this operator as the operator that reveals for you the momentum of the free particle, because acting on the wave function, it gives you the momentum times the wave function. Now it couldn't be that acting on the wave function just gives you the momentum, because the exponential doesn't disappear after the differential operator acts. So it's actually the operator acting on the wave function gives you a number times the wave function. And that number is the momentum.

So we will call this operator, given that it gives us the momentum, the momentum operator, so momentum operator. And to distinguish it from p, we'll put a hat, is defined to be h bar over i d dx. And therefore, for our free particle, you can write what we've just derived in a brief way, writing p hat acting on psi, where this means the operator acting on psi, gives you the momentum of this state times psi of x and t. And that's a number. So this is an operator state, number state.

So we say a few things, this language that we're going to be using all the time. We call this wave function, this psi, if this is true, this holds, then we say the psi of x and t is an eigenstate of the momentum operator. And that language comes from matrix algebra, linear algebra, in which you have a matrix and a vector. And when the matrix on a vector gives you a number times the same vector, we say that that vector is an eigenvector of the matrix. Here, we call it an eigenstate. Probably, nobody would complain if you called it an eigenvector, but eigenstate would be more appropriate. So it's an eigenstate of p.

So, in general, if you have an operator, A, under a function, phi, such that A acting on phi is alpha phi, we say that phi is an eigenstate of the operator, and in fact eigenvalue alpha. So, here is an eigenstate of p with eigenvalue of p, the number p, because acting on the wave function gives you the number p times that wave function. Not every wave function will be an eigenstate. Just like, when you have a matrix acting on most vectors, a matrix will rotate the vector and move it into something else. But sometimes, a matrix acting in a vector will give you the same vector up to a constant, and then you've got an eigenvector. And here, we have an eigenstate.

So another way of expressing this, is we say that psi of x and t, this psi of x and t, is a state of definite momentum. It's important terminology, definite momentum means that if you measured it, you would find the momentum p. And the momentum-- there would be no uncertainty on this measurement. You measure, and you always get p. And that's what, intuitively, we have, because we decided that this was the wave function for a free particle with momentum, p. So as long as we just have that, we have that psi is a state of definite momentum.

This is an interesting statement that will apply for many things as we go in the course. But now let's consider another aspect of this equation. So we succeeded with that. And we can ask if there is a similar thing that we can do to figure out the energy of the particle.

And indeed we can do the following. We can do i h bar d dt of psi. And if we have that, we'll take the derivative. Now, this time, we'll have i h bar. And when we differentiate that wave function with respect to time, we get minus i omega times the wave function. So i times minus i is 1. And you get h bar omega psi. Success, that was the energy of the particle times psi.

And this looks quite interesting already. This is a number, again. And this is a time derivative of the wave function. But we can put more physics into this, because in a sense, well, this differential equation tells you how a wave function with energy, E, what the time dependence of that wave function is.

But that wave function already, in our case, is a wave function of definite momentum. So somehow, the information that is missing there, is that the energy is p squared over 2m. So we have that the energy is p squared over 2m. So let's try to think of the energy as an operator. And look, you could say the energy, well, this is the energy operator acting on the function gives you the energy. That this true, but it's too general, not interesting enough at this point.

What is really interesting is that the energy has a formula. And that's the physics of the particle, the formula for the energy depends on the momentum. So we want to capture that.

So let's look what we're going to do. We're going to do a relatively simple thing, which we are going to walk back this. So I'm going to start with E psi. And I'm going to invent an operator acting on psi that gives you this energy. So I'm going to invent an O.

So how do we do that? Well, E is equal to p squared over 2m times psi. It's a number times psi. But then you say, oh, p, but I remember p. I could write it as an operator. So if I have p times psi, I could write it as p over 2m h bar over i d dx of psi.

Now please, listen with lots of attention. I'm going to do a simple thing, but it's very easy to get confused with the notation. If I make a little typo in what I'm writing it can confuse you for a long time. So, so far these are numbers. Number, this is a number times psi. But this p times psi is p hat psi which is that operator, there. So I wrote it this way.

I want to make one more-- yes?

AUDIENCE: Should that say E psi?

PROFESSOR: Oh yes, thank you very much. Thank you. Now, the question is, can I move this p close to the psi. Opinions? Yes?

AUDIENCE: Are you asking if it's just a constant?

PROFESSOR: Correct, p is a constant. p hat is not a constant. Derivatives are not. But p at this moment is a number. So it doesn't care about the derivatives. And it goes in. So I'll write it as 1 over 2m h/i d dx, and here, output p psi, where is that number. But now, p psi, I can write it as whatever it is, which is h/i d dx, and p psi is again, h/i d dx psi.

So here we go. We have obtained, and let me write the equation in slightly reversed form. Minus, because of the two i's, 1 over 2m, two partials derivatives is a second order partial derivative on psi, h bar squared over 2m d second dx psi. That's the whole right-hand side, is equal to E psi.

So the number E times psi is this. So we could call this thing the energy operator. And this is the energy operator. And it has the property that the energy operator acting on this wave function is, in fact, equal to the energy times the wave function.

So this state again is an energy eigenstate. Energy operator on the state is the energy times the same state. So psi is an energy eigenstate, or a state of definite energy, or an energy eigenstate with energy, E. I can make it clear for you that, in fact, this energy operator, as you've noticed, the only thing that it is is minus h squared over 2m d second dx squared.

But where it came from, it's clear that it's nothing else but 1 over 2m p hat squared, because p hat is indeed h/i d dx. So if you do this computation. How much is this? This is A p hat times p hat, that's p hat squared. And that's h/i d dx h/i d dx. X And that gives you the answer. So the energy operator is p hat squared over 2m.

All right, so actually, at this moment, we do have a Schrodinger equation, for the first time. If we combine the top line over there. I h bar d dt of psi is equal to E psi, but E psi I will write it as minus h squared over 2m d second dx squared psi.