PROFESSOR: That brings us to claim number four, which is perhaps the most important one. I may have said it already.b The eigenfunctions of Q form a set of basis functions, and then any reasonable psi can be written as a superposition of Q eigenfunctions.

OK, so let's just make sense of this. Because not only, I think we understand what this means, but let's write it out mathematically. So the statement is any psi of x, or this physical state, can be written as a superposition of all these eigenfunctions So there are numbers, alpha 1 psi 1 of x plus alpha 2 psi 2 of x. Those are the expansion coefficients with alphas. And in summary, we say from sum over i, alpha i psi i of x.

So the idea is that those alpha i's exist and you can write them. So any wave function that you have, you can write it in a superposition of those eigenfunctions of the Hermitian operator. And there are two things to say here. One is that, how would you calculate those alpha i's?

Well, actually, if you assume this equation, the calculation of alpha i's is simple, because of this property. You're supposed to know the eigenfunctions. You must have done the work to calculate the eigenfunctions. So here is what you can do. You can do the following integral. You can do this one, psi i psi.

Let's calculate this thing. Remember what this is. This is an integral, dx, of psi i star. That's psi. And psi is the sum over j of alpha j psi j. You can use any letter. I used i for the sum, but since I put that psi i, I would make a great confusion if I used another i. So I should use j there. And what is this? Well, you're integrating the part of this. That's a sum. So the sum can go out. It's the sum over j alpha j integral of psi i star psi j d.

And what is this delta ij? That is our nice orthonormality. So this is sum over j alpha j, delta i j. Now, this is kind of a simple sum. You can always be done. You should just think a second. You're summing over j, and i is fixed. The only case when this gives something is when j, and you're summing over, is equal to i, which is a fixed number. Therefore, the only thing that survives is j equals to i, so this is 1. And therefore, this is alpha i.

So we did succeed in calculating this, and in fact, alpha i is equal to this integral of psi i with psi. So how do you compute it now for i? You must do an integral. Of what? Of psi i star times your wave function. So in this common interval. So the alpha i's are given by these numbers. This would prove.

The other thing that you can check is if the wave function squared dx is equal to 1. What does it imply for the alpha i's? You see, the wave function is normalized, but it's not a function of alpha 1, alpha 2, alpha 3, alpha 4, all these things. So I must calculate this. And now let's do it, quickly, but do it.

Sum over i, alpha i, psi i star, sum over j, alpha j, psi j. See, that's the integral of these things squared dx. I'm sorry. I went wrong here. The star is there. The first psi, starred, the second psi. Now I got it right. Now, I take out the sums i, sum over j, alpha i star alpha j, integral dx psi i star psi j. This is delta i j, therefore j becomes equal to i, and you get sum over i of alpha i star alpha i, which is the sum over i of, then alpha i squared. OK.

So that's what it says. Look. This is something that should be internalized as well. The sum over i of the alpha i squared is equal to 1. Whenever you have a superposition of wave functions, and the whole thing is normalized, and your wave functions are orthonormal, then it's very simple. The normalization is computed by doing the sums of squares of each coefficient. The mixings don't exist because there's no mixes here.

So everything is separate. Everything is unmixed. Everything is nice. So there you go. This is how you expand any state in the collection of eigenfunctions of any Hermitian operator that you are looking at.

OK. So finally, we get it. We've done all the work necessary to state the measurement possibility. How do we find what we measure? So here it is. Measurement Postulate.

So here's the issue. We want to measure. I'm going to say these things in words. You want to measure the operator, q, of your state. The operator might be the momentum, might be the energy, might be the angular momentum, could be kinetic energy, could be potential energy. Any Hermitian operator. You want to measure it in your state.

The first thing that the postulate will say is that you will, in general, obtain just one number each time you do a measurement, but that number is one of the eigenvalues of this operator. So the set of possible measurements, possible outcomes, better say, is the set of eigenvalues of the operator. Those are the only numbers you can get.

But you can get them with different probabilities. And for that, you must use this plane. And you must, in a sense, rewrite your state as a superposition of the eigenfunctions, those alphas.

And the probability to measure q1 is the probability that you end up of this part of the superposition, and it will be given by alpha 1 squared, [INAUDIBLE]. The probability to measure q will be given by alpha 2 squared and all of these numbers.

So, and finally, that after the measurement, another funny thing happens. The state that was this whole sum collapses to that state that you obtained. So if you obtained q1, well, the whole thing collapses to psi 1. After you've done the measurement, the state of the system becomes psi 1.

So this is the spirit of what happens. Let me write it out. If we measure Q in the state psi, the possible values obtained are q1, q2. The probability, p i, to measure q i is p i equals alpha i squared. And remember what this alpha i we calculated it. This overlap of psi i with psi squared. And finally, after finding-- after, let's write it, the outcome, q i, the state of the system becomes psi of x is equal to psi i of x. And this is a collapse of the wave function. And it also means that after you've done the measurement and you did obtain the value of q i, you stay with psi i, if you measure it again, you would keep obtaining q i.

Why did it all become possible? It all became possible because Hermitian operators are rich enough to allow you to write any state as a superposition. And therefore, if you want to measure momentum, you must find all the eigenfunctions of momentum and rewrite your state as a superposition of momentum. You want to do energy? Well, you must rewrite your state as a superposition of energy eigenstates, and then you can measure. Want to measure angular momentum? Find the eigenstates of angular momentum, use the theorem to rewrite your whole state in different ways.

And this is something we said in the first lecture of this course, that any vector in a vector space can be written in infinitely many ways as different superpositions of vectors. We wrote the arrow and said, this vector is the sum of this and this, and this plus this plus this, and this plus this plus this. And yes, you need all that flexibility. For any measurement, you rewrite the vector as the sum of the eigenvectors, and then you can tell what are your predictions. You need that flexibility that any vector in a vector space can be written in infinitely many ways as different linear superpositions.

So there's a couple of things we can do to add intuition to this. I'll do, first, a consistency check, and maybe I'll do an example as well. And then we have to define uncertainties, those of that phase. So any question about this measurement postulate? Is there something unclear about it?

It's a very strange postulate. You see, it divides quantum mechanics into two realms. There's the realm of the Schrodinger equation, your wave function evolves in time. And then there's a realm of measurement. The Schroedinger equation doesn't tell you what you're supposed to do with measurement. But consistency with a Schroedinger equations doesn't allow you many things. And this is apparently the only thing we can do. And then we do a measurement, but somehow, this psi of x collapses and becomes one of the results of your measurement.

People have wondered, if the Schroedinger equation is all there is in the world, why doesn't the result of the measurement come out of the Schroedinger equation? Well, people think very hard about it, and they come up with all kinds of interesting things.

Nevertheless, nothing that comes out is sufficiently clear and sufficiently useful to merit a discussion at this moment. It's very interesting, and it's subject of research, but nobody has found a flaw with this way of stating things. And it's the simplest way of stating things. And therefore, the measurement is an extra assumption, an extra postulate. That's how a measurement works. And after you measure, you leave the system, the Schroedinger equation takes over and keeps evolving. You measure again, something happens, there's some answer that gets realized. Some answers are not realized, and it so continues.