**PROFESSOR:** Hi everyone. Spring has regressed. So, we have-- we're going to have a guest at the end of lecture today, which should kind of entertaining. Just as a warning, if you see someone come in. So questions, before we get started? No questions about anything? At all? Math? Nothing? Yeah?

**AUDIENCE:** Can you explain the physical significance of the crystal momentum?

**PROFESSOR:** Yeah. OK. Let me go over that. That's a good question. So the question is what again is the significance of the crystal momentum? So let me answer that in a slightly backward way. So this is a form of the explanation I haven't given you. It's going to be a slightly different one.

Let's step back and think about the momentum, and ask what the momentum is. Now you guys showed on a problem set, the following fact. That if you have a wave function, sine of x, such that, the expectation value in the state SI of x in the state SI is equal to x naught, and the expectation value of p in the state SI is p naught. Hat, hat. Then if you want to change the momentum, increase momentum by h bar k, the way to do that is to take SI and build a new wave function, SI tilda, is equal to e to the i, k x, SI of x. And then the expectation value of x is the same, SI tilda, still equal to x naught, because this phase goes away from the two complex, from the wave function is complex content we give the inner product. But the expectation value, the momentum is shifted in state SI tilda, is shifted by each h bar k, p naught plus h bar k.

So all the intuition you have about momentum, you can translate into intuition about the spatial variation of the phase of the wave function. Yeah?

**AUDIENCE:** [INAUDIBLE]

**PROFESSOR:** OK, good. OK, good, So we have a sneaky [INAUDIBLE]. So, the information about the momentum can be encoded in these spatial variation of the phase of the wave function. So another way to answer the question of what is momentum, apart from it's the thing that-- so what are ways to answer the question, what is momentum, you could ask well what is momentum? It's the thing that commutes with p or with x by i h bar.

That's one way to answer it. Another way to answer is to say that translations by l can be expressed in terms of momentum as e to the minus i upon h bar p l. So these are both ways of describing what the momentum is. But another way of talking about the momentum is the momentum p governance the spatial variation, the x dependence of the phase of the wave function. So these are always talking about what the momentum is.

So now let's turn this around, and let's ask about the crystal momentum. Oh, and one last thing, a last defining property of the momentum, a central property from the Schrodinger equation is at the time variation d dt of p is equal to the expectation value of minus d the potential of x d x. Also known as the force. So this is the Ehrenfest Theorem Statement that the classical equation of motion, p dot, is equal to the minus d v d x is equal to the force, Ehrenfest's

Theorem tells us that the classical equations of motion are realized as expectation values. And equivantly, if there's no potential, the potential is constant, this tells us that the momentum expectation value is time independent. Right? A familiar fact. So these are all true lovely and things about the momentum.

So let's turn all these facts around into the crystal momentum. So let's talk about crystal momentum. Which was the question, what is the crystal momentum? So the crystal momentum is defined from beginning, from the following property. If we have a potential v of x, which is invariant under shifting, by one lattice spacing, by some l, v of x, then this tells us that the energy operator is invariant if we shift by l. If we translate by l equals zero. And from this fact, we deduced via block or a la block, that the wave functions are really the energy eigenfunctions, can be written in the

form e cubed is equal to e to the i q x, u of x, where u, we're going to take to be a periodic function.

So what is this parameter q doing? Q is governing the spatial variation of the phase of the wave function. Cool? So in precisely this sense, the momentum difference is space of the wave function. Here, in the case of a periodic potential, the crystal momentum q is governing the spatial variation of the phase of the wave function. So q is the thing the governs the phase as a function of x. Well what about-- another fact about the crystal momentum which you show in your problems set, is that if you impose an external force d q d t, and really d h bar q. d t is equal to-- d dt of the expectation value of h bar q, is equal to the expectation value of the force. I'll just write-- OK?

So again, this is a quantity, and this was assuming that we had a sharply peaked wave packet. So this is for a wave packet sharply peaked at q naught. And so let me just write this as h bar q naught. So the central value of your wave packet-- so this is what you've shown on the problem set that the central value of your wave packet, the peak of your wave packet varies in time according to the external force. And so in particular, if the force is zero, we turn no external driving force, your wave packet maintains its crystal momentum. It's time independent. So the crystal momentum is something that time independent, unless an external force is applied, just like the momentum. And it's something that governs the phase of the wave function just like the momentum. However, it's different in a crucial way.

It is not the eigenvalue p on five sub e q is not equal to a constant p naught times 5 sub e q. Because when we take-- when we active p or we active the derivative, you pick up a term from here, which gives us a constant, but we also have this overall periodic piece. And its spatial variation is generically non-zero. And if the potential is nontrivial, it's always non constant. So when the momentum operator hits this guy, it will generically not give us zero. It'll get two terms and we will not get an eigenvalue equation. So q is not the eigenvalue h bar q is not the eigenvalue of p. And what's the last important property of q that's different from the momentum?

It comes from the commutator, which tells us that the thing that's conserved is the expectation value of p l is really the precise statement. And in particular, what this tells us is that the eigenfunction, or the eigenvalue of our wave function, under translations by l, is a quantity that can be determined simultaneously with knowing the energy. However, the eigenvalue of t sub l, on this state, is equal to e to the i q l. Which means that q is only defined for determining the eigenvalue up to 2 pi over l. If you have q, which is 0, and you increase it to pi over l, that value, pi over l, is effectively the same as the value minus pi over l. Because at least they're the same eigenvalue. But that's really strange because that means that q itself, it's not strictly conserved. It's conserved mod 2 pi over l. When you have momentum conservation, momentum is strictly conserved if there's no force. And even if there is a force, it's increasing control by the force as you turn on the force, it just constantly increases. For the crystal momentum, that's not the case. You turn on a force, it increases according to the conservation law.

But it's not increasing constantly. It's periodic. It's periodically defined. So it increases then it ends up at a smaller value. It increases and ends up at a smaller value. OK? So it carries many of the same properties. It governs the phase. It's time independent unless there's an external force applied. It's the eigenvalue. Controls the eigenvalue of an operator that commutes with the energy when you have a periodic potential, in the same way that the momentum commutes with the energy when you have no external force, when you have a constant potential.

Does that help? Good. OK. So developing an intuition for the crystal momentum, I think, is best done by just playing with examples. And you'll do that more in the course on solids, which I encourage you all to take. Because it's really beautiful stuff. But for our purposes, this is going to be the full set of ideas we'll need for 8.04. Yeah?

**AUDIENCE:**      [INAUDIBLE]

**PROFESSOR:**      Ah. So good. So thank you. So this involves a slight subtlety, which I've been glossing over in the entire story here. Which of the following. So, is u of x a real

function? Well, so when we started out asking what are the eigenfunctions of the transit by l operator, all we showed was that, and I'm going to do this on a separate board just to make it clearer. Tell me if this turns off, because it kept bumping. OK.

So when we started with translate by l, and we constructed it's eigenfunctions, we said that translate by l q Phi sub cubed is equal to some phase, and this is unitary, so we're talking about they must be an actual phase in the i alpha of Phi sub q of x. And let's just suppose that this is true. Then this tells us that Phi sub q times e to the minus i q l equals u. So, I'm just going to use this to define a new function, u sub q.

Or just u. I'll use sub q. Fine. of x. So this defines a new function, sub q. I take an eigenfunction, I multiply it by some phase. Sorry, minus i q x. If we choose q l to be equal to alpha, then acting on u sub q, by translate by l, on u sub q, of x, is equal to-- well, if we act on Phi sub q with translate by l, what happens to Phi sub q we pick up a phase e d i alpha. What happens to e to the minus i q x? x goes to x plus l.

We pick up a phase e to the minus i q l. So if q l is equal to alpha, those two phases cancel, and we just get u back. u sub q of x. But translate by l, if u sub q, by definition, is equal to u sub q of x plus l. So we've determined is that if we take q l is equal to alpha, then Phi sub q if eigenvalue label by its eigenvalue, q, can be written in the form e to the i q x u sub q of x, where this is periodic. Everybody agree with that? OK. So that's step one.

Step two is to say well look, since the eigenvalue of this guy, under t sub l, e d i alpha is equal to e to the i q l. Since this is periodic under shifts of q, by 2 pi upon l, I can just choose to define q up to 2 pi over l. So 2 q, I will take to be equivalent to q plus 2 pi over l. And the reason I'm going to do that is because it gives the same eigenvalue, and if I want to label things by eigenvalues, it's sort of redundant to give multiple values to the same eigenvalue.

Now there's a subtlety, here though. And this little thing here is this. Suppose we have a free particle. Does a free particle respect translation by l? So if we have a free particle, the potential is zero. That constant function is also periodic under shifts by l. Right? Because it's just zero. So it's stupidly periodic, but it's periodic

nonetheless. So now I'm going to ask the following question. What are the common eigenfunctions of the energy and translate by l for the free particle?

We did this last time. So the common eigenfunctions of translate by l and the energy are the wave functions Phi sub q, comma e, are equal to e to the i q x times some function u of x, on general grounds. But we know what these eigenfunctions are. They're just e to the i k x. Where k squared upon 2 m is e. [INAUDIBLE] So we know that these are the correct eigenfunctions, but we're writing them in the form e v i q x u.

Now you say that's fine. There's nothing wrong with this. We just say u is constant and q is equal to k. These functions are of this form, but they're of this form with e v i q x being e d i k x and u of x being constant. Right? There's nothing wrong with that. Everyone agree? Perfectly consistent.

However, I thought we said that q is periodic by 2 pi? If q is periodic by 2 pi, then that would seem to imply that k is periodic by 2 pi, and we know that's not true because any k is allowed for a free particle. So if we want to think about q is periodic by 2 pi upon l, then we cannot require that u is real. Because it must be the phase that makes this up. It must be, so I can always write this as e to the i q x where q is less than 2 pi upon l.

I'm sorry, where q is between 2 pi or pi upon l and minus pi upon l. So that it's defined only after this periodically thing. But times some additional phase, e to the i k minus q x This is trivially equal to e to the i k x. But now u is not a real function. On the other hand, if we hadn't imposed the requirement that q is periodic, we wouldn't have needed to make u real. We could just taken q to be equal to k, for any value k, and then u would be constant. u would be real. So this is important for answering the excellent question that our fearless restation instructor provoked me to answer.

Which is that so what-- we'll come back to the question in just a second. But what I want to emphasize this, that if we're going to take q to be not periodic, Sorry. If we're going to take q to be defined only up to shifts by 2 pi over l, it's important that we allow u to be not real. It must be able to be an overall phase. But if we want u to

be always real, we can do that. We just can't impose this periodicity. Different values of q mean different wave functions.

And this is really what's going on when you see those plots, sometimes you see the plots as parabolas. The bands are represented by parabolas with wiggles, and sometimes they're folded up. And that's the difference. The difference is that when you fold them up, you're imposing this periodicity and you're labeling the eigenfunctions by q, and the overall amount of the number effectively of k phases that you're subtracting off. Yeah?

**AUDIENCE:**     So is this an arbitrary choice? [INAUDIBLE]

**PROFESSOR:**     Yeah. I mean, how to say? It's exactly akin to a choice of variables. In describing the position of this particle, should we use Cartesian coordinates, or should we use Spherical coordinates? Well it can't possibly matter. And so you'd better make sure in any description of your system, that changing your coordinates doesn't change your results. And here, that's exactly what's going on. Do we want to define our variable to be periodic by 2 pi upon l?

Well, OK then. But u can't be real. Or we could take q to be not periodic by 2 pi l and impose that u is real. It's just a choice of variables. But it can't possibly give different answers. The point is, this is a subtle little distinction it we gloss over, and is glossed over into my knowledge every book on intro to quantum mechanics that even covers periodic potentials. It can be very confusing. Anyway, the reason that I had to go through all this, is that in order to answer the very, very good question professor Evans posed, I'm going to need to deal with this fact.

So for the moment, let me deal with-- let's work with u real. And q, q an unconstrained, real number. OK. So not periodic. Are we cool with that for the moment? So if we do that, then notice the falling nice property of our wave function. Our wave function, Phi sub q, is equal e to the i q x times u of q, or u of x. Which is real.

So when we can construct the current-- remember that j boils down to the imaginary

part, h bar over 2 m i. Well, h bar over m times the imaginary part of SI complex conjugate derivative, with respect to x, which is the current, in the x direction of SI. And we need this to be imaginary, or we will get no current. You show this in a problem set, if you have a pure, real wave function, for example. A single real exponential, that's decaying, as on the wrong side of a barrier. Then you get no current. Nothing flows. And that make sense. It's exponentially decaying. Nothing gets across.

So we need the wave function to be real. So if q were zero we would get zero. And what you can immediately do from this, compute from this, is that while the derivative, if the derivative doesn't hit e to the i q x, if it hits u, than the phase e to the i q x cancels. And so the contribution from that term vanishes. So the only term that's going to contribute in here is when the derivative hits the e to the i q x. But then this is going to be equal to h bar q. And we want the imaginary parts, that's going to be e to the I over m. And then we're left with u squared of x. So this is the current, but we have to do it-- we had take advantage in order for this to be sort of clean, we had to take advantage of u being real. Everybody cool with that?

Now there's one last twist on this, which is that if I have k-- if I have q. So this is a side note. Going back up here, to this logic. If I have q, and I want, I can always write it as some q naught plus n pi over l. And so now what I want to do is I want to take sort of a hybrid of these two pictures. And I want to say Phi sub q is going to be equal to e to the i q naught x. Where this is the value that's periodic by 2 pi. e to the I n pi over l x u. And so now really what's going to happen, what I'm doing here is I'm labeling q, not by a single number. I'm labeling my wave function not by single number q, but by q naught and an integer n. Comma n. So q naught and n. So now q naught is periodic. It's defined up to shifts by 2 pi. n is an additional integer, and what it's telling you is how many times did you have to shift back to get into that fundamental zone between pi and minus pi.

And this fits nicely into this story, because now all we're going to get here is q, which is q naught plus n pi. So the current depends on both the part defined mod 2 pi over l, and the integer, which tells you how many factors of 2 pi over l did you have to

subtract off to get into that fundamental domain. So let's think back to our band structure. So what is this n quantity? Let's think back to our band structure. In our band structure, we had something that looks like this.

And here's the value of q. But am I plotting q? No. I'm plotting here q naught. I'm plotting the part that's periodically defined up to 2 pi over l. So this is pi over l. This is minus 2 pi over l. Or minus pi over l. OK. And what we see is that there isn't a single energy.

Because this is the energy the vertical direction for the band pictures. There isn't a single energy for a given value of q. In fact, the set of energy eigenvalue-- or the set of allowed states or energy eigenvalues for an allowed value of q would say this particular value of q naught, how many of them are there. Well, there are as many as there are integers. One, two, three, four, count. So to specify a state, I don't just have to specify q NAUGHT, I also have to specify N.

Which one of these guys I'm hitting. And when you unfold this into the parabola picture, remember where these came from. These came from these curves. Came from shifting over. And the higher up you go, the more you had to shift over. And that's exactly the integer piece in n pi over l. And so we can write the current now, in terms of h bar q naught upon m, u squared-- I'm sorry. h bar q naught upon m plus n pi h bar upon m u squared of x. So we get a contribution from the crystal momentum and from which we're in. OK? So sort of an elaborate story to answer the phase question. Yeah?

**AUDIENCE:**      [INAUDIBLE]

**PROFESSOR:**      Good. So here we had SI-- so SI-- I'm sorry. I should have done this for Phi. But I meant this wave function, right. This is Phi, this is Phi q. So from here we're going to get the imaginary part. So we get the imaginary part of this wave function which is u to the minus i q x u of x derivative of e to the i q x u of x. Now the term that contributes is when the derivative hits the e to the i q. x pulls down a factor of i q, and the two phases cancel from these guys, leaving us with a u of x here, and a u of x here.

**AUDIENCE:** [INAUDIBLE]

**PROFESSOR:** Oh sorry. This is a potential. Good. That's the point. So this is the potential. So in this statement that what we have this translation by x. So this is just some function. It has nothing to the potential. It's defined in terms of the wave function. The eigenfunction of translate by l. So the logic here goes, if we know we have a function of translate by l, then I construct a new function u. Nothing to do with the potential, just a new function.

Which is e to the minus i q x times it. You can't stop me. You hand me a function, I will hand you a different function. And then we pick q felicitously, to show that u is periodic. So u is just some periodic function which is contained which is defined from the wave function. From the energy. From eigenfunction of t l. Did that answer your question? OK. So here, it just came from the fact that u is Phi is the e to the i q x u, x and then a factor of u for each of these. Other questions. Yeah.

**AUDIENCE:** [INAUDIBLE]

**PROFESSOR:** So this picture, when it's unfolded, first off, you know what it is for a free particle. So we want the energy as a function of q. So what is it for a free particle? Parabola. Yeah, exactly. And now let's add in-- let's make this a function of q, not q naught, but so here's pi over l. Here's 2 pi over l. Here's 3 pi over l. And I need to do this carefully, because it's incredibly difficult to get the straight. OK.

My artistic skills are not exactly the thing of legend. OK. So here's the parabola that would have been, if we had not turned on a periodic potential. As we turn on the periodic potential, we know that the energies change. And so in the first band it's easy to see, because for minus pi over l, it's pi over l. We don't have to do anything. So it look exactly the same as the lowest band over here. So in particular-- OK? So what about this second band?

Well what I want to know what's the allowed, the other allowed energy that's say, plus pi over l. Plus pi over l, it's going to be something greater than this value. But plus pi over l, we already know the answer from that diagram, because plus pi over l

is the same as minus pi over l, so what's the value over here? Well, the value over there for the second band is slightly above, and then it increases and decreases. So slightly above, and then it increases.

Shift by pi over l. Whoops. Did I shift by pi over l for this guy? That's one, two. Yes. I did. Good. And it goes the other way. So just noting that it goes away from the top. I have a hard time drawing these things. So for every value of q, there's an allowed energy. But it's different than it would have been for the free particle. And then we do the same thing for the next state. And it looks like this. So now imagine what happens when we take this, and we it over one two. What we get is a band the looks-- that should look like this. That's what the second band should look like. And indeed, when we put it in the fundamental domain, this is what we get. This is what the first band and the second band together look like. And then the third band, we'll move this over once, and then twice, it's going to look like this. And this guy, move it over once, twice, looks like whoops. Yeah?

**AUDIENCE:** If we wanted to plot u with respect to k instead, would that just be a parabola dotted line? If so, why do we not have really--

**PROFESSOR:** If we just wanted-- sorry. Say it again?

**AUDIENCE:** E as a function of k instead of q.

**PROFESSOR:** Oh. Yeah. E as a function of k is always going to look like that. But k is not a well-- so what is k? K is just defined as h bar squared, k squared upon 2 m is equal to e. So this doesn't tell you anything. Right. Because any allowed k. Sure any allowed k is some valid value of e. But this didn't tell you which values of e are allowed. Only some values of e are allowed, right? There are no values of e-- there are no energy eigenstates with energy in between here and here, right? And so that tells you they're no allowed k's because k is just defined, it's just completely defined by e. So this doesn't tell you anything about which states you're at. It just that given an e, there's some quantity that could define k. This is a definition of k, in terms of e. What this diagram is telling you is which e's are allowed.

**AUDIENCE:**    [INAUDIBLE]

**PROFESSOR:**    Yes. Yes. There should be. Let's see. What's

**AUDIENCE:**    [INAUDIBLE]

**PROFESSOR:**    Oh, here. Yes. Yes, you're absolutely right. Over out. Thank you. Excellent. That's exactly right. Yeah. Oh man. I made a dimensional mistake. Thank you. Jesus. OK. Good. Yeah.

**AUDIENCE:**    Could you like re-explain how imperfections and a lattice leads to actual conduction?

**PROFESSOR:**    Yeah. I'm going to do that. So that's an excellent question. The question is could you explain again how imperfections and a lattice leads to actual conduction. As we talked about last time, when you have a perfect lattice, there is actually no current flowing in response to an applied electromagnetic field. If you put on a capacitor, played across your perfect lattice, you don't get any current. So the particle, the charged particle in your lattice, just oscillates back and forth in a block oscillation, running up the band, and down the band, and up the band, and down the band. So, let me slightly change your question, and turn it into two other questions.

The first question is given that that's obviously not what happens in real materials, why don't we just give up on quantum mechanics and say it totally failed? And so this is a totally reasonable question, and I want to emphasize something important to you. Which is the following. That model led to a prediction, which is that if you put a capacitor plate across a perfect crystal, then you would get no current flowing across, you would just see that the electron wave packets oscillate. Or block oscillations as we discussed last time. And that is manifestly what happens with copper. But the experimentalist comes back to you and says look dude. That is a ridiculous model because the copper isn't in fact perfect, it's messy.

So how do you test the model? Well there are two ways to test-- to deal with the situation. One is you improve the model to incorporate properties that copper actually has. And see if you can actually get the same conductivity that you see. But

the other is you could improve the material, instead of improving the theory. So let's make up what-- can we actually build a perfect crystal? This is actually something that I'm doing research on right now.

Not on the building side, but on the theory side, because I'm a theorist and you should not let me in a lab. But I collaborate with experimentalists, so they're nice people. They're very good physicists. So here's something you can do. You can build a system that has exactly a periodic potential. It turns out it's very difficult to do this with quantum systems. But what you can do is you can do it with lattices not of atoms, but lattices of dielectric.

So the equation. Here's a cool fact, the equation for light going through a dielectric, where the dielectric has different constants, like wave guides. You've got glass, you got air. You've got glass, you got air. That equation can be put in exactly the same form as the Schrodinger equation for the time evolution of a wave function. They're both waves. And so it's not so surprising these two wave equations are related to each other a nice way. Meanwhile, the index of the dielectric turns into the potential for the quantum mechanical problem.

So if you have a periodic potential, what do you want? You want a periodic dielectric constant. Yeah. And so you can build a system which incredibly, cleanly, has a periodic dielectric constant and no disorder. And then you can put light into the system, and you can ask what happens to this system. So here's the idea,

I take a system which is a periodic-- I'm going to draw the potential here. So I'm going to draw the dielectric constant. So small, large, small, large, small, large, small, large, et cetera. But instead of having it be a one dimensional lattice, I'm going to make it a two dimensional lattice. So now, basically, I've got a set of wave guides. Let me draw this differently.

So does everyone get the picture here? So literally what you have, is you have glass, glass with a different index, glass, glass with a different-- if you can think of those as a line of glass fibers. Optical fibers. And you shine your light that's reasonably well localized, in both position, and in phase variation, or crystal

momentum. Because you can control the phase of the light. So you send this wave packet in and you ask what happens.

Well not a whole lot happens. It's a wave packet. It's going through a wave guide, but we haven't implemented an electric field. To handle an electric field, you need the potential to be constantly varying. Uh huh. So it's at a linear ramp into the potential. Instead of making it just perfectly periodic, let's make the index ramp just a little bit.

And this experiment has been done. In this experiment, so as the wave packet moves along, what's discovered is that the position-- if I draw the x as a function of t, so now the role of t is being played by the distance it's moved along the wave guide, what you find is that it does this. It exhibits beautiful block oscillations.

And this has been proved in a very small number of real honest quantum mechanical systems. The most elegant experiment that I know of was done by Wolfgang Ketterle, who's here at MIT. And he got three data points because it was preposterously difficult and declared victory.

So I talked to him about this in the hallway one day. And he said yes, this was ridiculous, but we got three data points. We got small, we got large. Victory. We declared victory. But it really needs to be done well. So one of the interesting questions in this part of the field right now is we know that it's true. But we want to see it. We want to feel it, so various people around the world are working on making a truly beautiful demonstration of this bit of physics. Yeah.

**AUDIENCE:**     [INAUDIBLE]

**PROFESSOR:**     It's totally impractical, because any interference is just going to kill you. Unfortunately. So, you have to work ridiculously hard to make systems clean. So the question is really a question about quantum computation, which we'll come to next week. But, the basic question is how robust is this. And the answer is it's not robust at all. But which you can tell because everything in the real world has enough impurity that it conducts. Or as an insulator. Yeah.

**AUDIENCE:** What place sort of like the larger role in sort of like the perfection of a lattice like temperature or impurities.

**PROFESSOR:** That's a very good question. So the question is what's the most important property? What's most important disordering property that leads to conduction? And there's temperature fluctuations, there are impurities in the lattice. There are decohereing effects which is a more complicated story. And that's actually, it depends on the situation, it depends on the system. And exactly how it depends is something that is an active area of research.

Now there are many, many ways to probe this physics. So we know that these block oscillations are true. We see them in all sorts of different systems that are analogous. So there's lots of [INAUDIBLE], it's not like this is an ambiguous bit of physics. But it's one that turns out to be surprisingly difficult to tease apart. The reason I bring all this up is to emphasize the following, our model made a prediction that disagreed explicitly with the connectivity property of copper and other materials. So don't throw away the model. Observe that you've modeled the wrong system. If you find a system that fits your-- that is-- that shares the assumptions of your model, that's when you ask did it work. And it worked like a champ. OK.

So now let's talk about real materials. This is going to close up our discussion bands and solids. And this is actually what I wanted to get to at the beginning of the lecture. But that's OK. There are lots of questions and they were good questions. So this is an extremely brief. But I want to ask you the following question.

What happens in the following three systems? So first, imagine we take why don't we take a system with built out of single wells, which have some set of energy eigenstates, and then we build the periodic array out of them. What do we expect? And let me draw this bigger. What do we expect to see when we build a lattice? We expect that this is going to-- that these states are going to spread out into bands a funny way

Yeah and let's just talk about the 1 d potential. So what we'll find is that this band turns into-- I'm sorry. This state, this single state turns into a band of allowed energy

eigenstates. There's now a plot of the energy. And similarly, this state is going to lead to another band with some width. And this state is going to lead to another band, which is even wider. Everyone cool with that? Quick question? In 1 d, do these bands ever overlap? No. By the node theorem. Right?

OK. Now let's take a single electron, and let's put in-- let's take a single electron, and let's put it in the system. What will happen? Well if we put it in the system, what state will this single electron fall into? Yeah one event. But which state?

**AUDIENCE:** [INAUDIBLE]

**PROFESSOR:** Yeah, if you kick the system around, you let it relax a little bit. It's going to fall down to the ground state. You have to couple to something else like hydrogen has to be coupled with an electromagnetic field to decay. But couple it, kick it, and let it decay. It'll settle down to its ground state. So you get an electron down here in the ground state, and looking back at that band, we know that the band for that ground state looks like this.

So, here it is. There's our electron. It's sitting in the lowest energy eigenstate. Is it moving? Well, it's in a stationary state. Is the expectation value of the position changing in time? No. The expectation values don't change in time, in the stationary state. That's part of what it is to be a stationery state, to be an energy eigenstate. OK. Great. it's not moving.

Now, in order to make it move, what do you have to do? What kind of state corresponds to the position changing in time? Yes. Superposition. Right? From the superpositions we'll get interference terms. So if we put in a superposition of say, this state, and this state, which corresponds to different energies.

If we put it in a superposition of these guys, then it's meaningfully moving. It has some meaningful, well defined time variation of its position expectation value. So in order to induce a current, in order to induce a current of this system where the electron wave packet carries a little bit of momentum is changing in time it's position, what do I have to do to the electron in the ground state? I have to excite it,

so that it's in a superposition of the grounds state and some excited state.

Or more generally, into a superposition of other states. Yes? In order to induce the current, I must put the electron into a higher energy state and in a particular superposition of higher energy states. Everyone down with that? Here's why this is so important.

Imagine each one of these wells is actually not some square well, but it's an atom. And let's say the atom is hydrogen, just for-- this doesn't actually happen, but just imagine-- in particular what it means is it has the ion, the nucleus is charge plus 1. And so in order for the system to be neutral, I must have one electron for every well.

So if I have n wells, I must have n electrons in the system. Everybody agree with that? In order to be neutral. Otherwise, the thing's charged and all sorts of terrible things-- electrons will get ripped off from nearby cad. So we must have an electron per well. How many states are in this band? For n wells? n. Right? OK. So if I put in the n electrons I need to neutralize a system, where do those n electrons go? Yeah, they fill up the first band. And if we let the system relax with lowest energy configuration, every state in this lowest band will be filled, and none of these states will be filled. Everyone down with that?

So here's my question. When I've got that ground state configuration of this lattice of atoms with one electron per well, in these distributed wave functions, filling out these bands, is anything moving? Wow, you guys are so quiet today. Is anything moving? This system is in an energy eigenstate. In particular, it's in a completely antisymmetrized configuration, because they're identical fermions.

So, nothing is moving. If we want to induce a current, what do we have to do? Yeah. We have put them in a superposition. But where's the next allowed energy eigenstate? Next band. So it's in the next band. The next allowed energy eigenstate. So the configuration we have now is that these guys are all filled, these guys are all empty, but in order to take an electron from here and put it into this excited state, we have to put in a minimum amount of energy, which is the gap between those two bands.

Right? So now think about it this way. Suppose I take light and I send my light at this crystal. In order for the light to scatter off the crystal, you must have electrons in superposition states so that they can have a dipole and absorb and radiate that energy. Yeah.

But in order for that to happen, the light has to excite an electron across the gap. It has to give it this macroscopic amount of energy. Well, it's not macroscopic. it's large. It's not infinitesimally small. That means that there's a minimum amount of energy that that incident light must have in order to excite the electron in the first place. So very long wavelength light will never do that. Light along wavelength will not have enough energy to excite an electron across this gap into the next band to allow there to be a current, which could oppose the electric field. So the only for light to scatter off of this crystal, is if the energy, h bar omega, of the light is greater than or equal to, let's say greater than approximately, the band gap delta e. That cool?

We've just discovered something. Crystals are transparent unless you look at sufficiently high frequencies. That's cool. Right? A crystal is transparent unless you look at sufficiently high frequencies.

If you look at low frequencies, your crystal should be transparent. Well that's really interesting. In particular, we immediately learn something cool about two different materials. Consider diamond and copper. These are both crystals.

They're solids made out of a regular array, perhaps not perfect, but extraordinarily good, regular array of atoms of the same time. Array in a particular structure. Diamond, anything and I think face inner cubic. I don't remember. I really should know that. Anyway, copper. It's a lattice. That's embarrassing. I really should know that. So we have these two materials which one has the larger band gap? Diamond, because it's transparent.

At in the visible. So the band gap, delta e of diamond is much larger than the band gap for copper. But in fact, this is a little more subtle, because copper in fact, doesn't even have a band gap. We made an important assumption here.

So I want to think about-- we're going to come back to copper in a second, but I want to point out the nice thing here. Which is that diamond has to have a band gap. It's transparent. It must have band gap. It must be such that when you fill up all the electrons you need for it to be neutral, there is a gap to the next energy states. And that gap must be larger than visible wavelengths of light.

Yeah. That's cool. And that must be true of all the transparent crystals that you see. Otherwise, they wouldn't be transparent. They would respond by having free electrons that could respond like a metal. Yeah.

**AUDIENCE:**    So, [INAUDIBLE]

**PROFESSOR:**    Yeah. Are diamonds good conductors? No. They're terrible conductors. In fact, there preposterously-- if you compare the number of-- I'll get into this later. But yes, they're terrible.

**AUDIENCE:**    [INAUDIBLE]

**PROFESSOR:**    Uh, that's a slightly more complicated story, which let me come back to. Hold on to that, and if I don't answer today, ask me after in office hours because it's a little more-- what? Really? Wow. Well, MIT. It's all about the intellect. And everything else has to-- OK.

So, this is pretty good, but here's the thing. In one dimensional crystals, the only thing that can happen is, look if you have each band come from allowed energy state and each energy state, each well comes with one electron, or two electrons, or three electrons, you will always have filled bands, and then a gap and filled bands and then a gap. Does everybody agree with that? You can't have a partially filled band if each band comes from a bouncy, in a single well, and each well comes with an integer number of electrons. You just-- you're stuck. Yeah.

**AUDIENCE:**    [INAUDIBLE]

**PROFESSOR:**    Oh. I'm lying about spin. But spin in one dimension is little-- I'm lying about spin. But do you really want me to get in spin? Man. OK. So if we include spin, and we have

splitting, then it becomes a more subtle story. If we include spin, then there are two states for every allowed energy eigenstate of the potential.

However, there are generically going to be interactions between the-- there are generically going to be magnetic interactions which split the energy of those two spin states. Electrons spin up, and electrons spin down, will generically have different energies.

Now in 3D, this isn't such a big deal, because those splittings are tiny, and so the states can sort of overlap. But in 1D they can't. So I mean, that's also not exactly true, but it depends on exactly the details. It depends on the details of the system, is what I wanted to get to. Curse you. So let me talk about the same phenomena in an easier context, where we don't have to worry about spin, which we haven't discussed in detail, in the class. Which is in three dimensions. Where the story changes in a dramatic way. Yeah.

**AUDIENCE:**    [INAUDIBLE]

**PROFESSOR:**    Oh. It's not. Generically, no, it's not. It will depend.

**AUDIENCE:**    [INAUDIBLE]

**PROFESSOR:**    You say it happens to be the salient one. Yeah, exactly. That is exactly right. There the gaps are not the same. That they do not remain constant. OK. So let's talk more about this system, but let's talk about it in three dimensions. So in three dimensions, you guys did an interesting thing, when you studied, you didn't know this was about the structure of solids, but it really was. When you studied the rigid rotor.

And when you studied the rigid rotor, you found that you had energy eigenstates and they were degenerate with degeneracy 2 l plus 1. The various different l z eigenstates. Yeah. And then we turned on an interaction which was the energy costs, the energy penalty for having angle momentum in z direction. Which added an l z term to the energy.

And what you found is that as a function of the coefficient, which I think we called

epsilon, of that perturbation of the energies of the energy was equal to l squared over 2 i plus epsilon l z. What you found is that these guys split. So this remained constant. And this split into, so this is the [INAUDIBLE] l equals 1. So l equals zero. So this is one, this is three, this is five. So the l equals zero state, nothing happens. l equals 1.

There's one that changes, one that doesn't. And then this guy has five. One, two, three, four, five. OK. And what we found here is that these guys could cross. States from different multiplates, with different values of l, had energies that could cross as a function of the strength of the deformation of your system. Right? The deformation is where you have a sphere and you stick out your arm. So it's no longer symmetric top.

So here we can have states crossing. There's no nodes here in three dimensions. So as a consequence, when you have a three dimensional material built out of atoms. So here's my sort of pictorial description of three dimensional system built out of atoms. You have a potential well, potential, potential well. Now, if the energy in one particular potential well, is like this, and like this, and like this, then when we add in a lattice we get bands again.

The structure's a little more intricate because it depends on the momentum. But these bands now can overlap. OK. Everybody see that? Because there's nothing preventing states from different-- in different multiplates from having the same energy in three dimensions. There's no nodes here that tells you have to keep the ordering constant as you turn on the potential. Now we turn on the multiple particle potential, and they can interact, they can overlap.

As a consequence, when we fill up, let's say we two electrons per potential, or per well, when we filled those first two bands, well, there is the first-- so the first band is now filled. The second band and part of the first-- part of the third band and most of the second band are going to be filled. But part of the second band is now available, and much of the third band is now available. We filled in 2n electrons, but we haven't filled up this band, because it's really two bands jammed together. Or really

bands from two different orbitals jammed together. They happened to overlap.

So as a consequence here, if this is the length of the energy of the last electron that you put in, how much energy do you have to give the system, do you have to add the system, to excite the energy-- or to excite the electrons into excited states, in particular into superpositions so that the electrons can move?

**AUDIENCE:**     [INAUDIBLE]

**PROFESSOR:**     Yeah. Preposterously small amount. An amount that goes like one over the number of particles. So in the continuum limit, it's zero. There's an arbitrarily nearby energy. So how much energy does it take to excite an electron and cause a current that opposes the induced electric field? Nothing.

Any electric field that you send in will be opposed by an induced current. So this behaves like a classical conductor. You turn on an electric field, and the charges will flow to oppose that externally imposed electric field. You get charges then building up on the walls of your capacitor plates. So, this is where we have a conductor.

Because there's an unfilled band. And back here , we had an insulator because we had filled bands separated by gap. The gap between the filled band and the next available band. This is actually called a band insulator. Because there are other ways of being an insulator.

So from this so far, just from the basic quantum mechanics of a particle and a periodic potential, we now understand why some crystals are transparent. Why some materials conduct. Why the materials that are transparency are also insulators. And the things that conduct are not transparent, generally. Yeah.

**AUDIENCE:**     [INAUDIBLE]

**PROFESSOR:**     Excellent. Excellent question. So what's so special about diamond and differ from copper? And so the answer goes like this. So what determined the exact band structure in for a 1D periodic potential? Two properties. One was l, the periodicity.

And that came in the q l and k l. And the second is the detailed shape of the

potential. Now in three dimensions, the story's going be a little more complicated. In three dimensions, the things that are going to determine the potential are not just the distance between atoms, but you have a three dimensional lattice.

And the three dimensional lattice could have different shapes. It could be cubic, it could be hexagonal, could be complicating in all sorts of different ways. Right? It could be bent, it could be rhomboidal, and it could have all sorts of different crystallographic structures. So that's going to go into it, in the same way that I went into it, which is the only parameter in one dimension. In the same way that I goes into it. So the crystal structure, the shape of the lattice, is going to determine it.

Secondly, the structure of the orbitals is different. Different atoms are different wells, so they'll give you different band structure. So different materials for example, diamond versus copper, are going to give you different bands allowed energies, because the potential is different.

It has different shape. And so when you solve the problem for the energy eigenvalues is a function of now the three different components of the crystal momentum, you'll just get a different set of equations. And working those out is not terribly hard. But it's a computation that must be done, and it is not trivial. And so one of the sort of, I don't know if I'd say exciting, but one of the things that one does when one takes a course in solids, is you go through a bunch of materials. And you understand the relationship between the potential, at the atomic orbital structure of the individual atom, the crystal structure, and the resulting band structure. And there's some sort of nice mnemonics, and there are calculations you do to get the answer.

**AUDIENCE:**     [INAUDIBLE]

**PROFESSOR:**     You will almost always find overlapping bands in three dimensions in sufficiently high energy. I can't off the top of my head give you a theorem about that, but yeah, it's generic. Yeah.

**AUDIENCE:**     --analog to conductor in one dimension? You have these like, non-zero band

depths?

**PROFESSOR:** Yeah. And this is why Matt was barfing at me. So the answer to that is yeah. There aren't [INAUDIBLE]. But what would we need?

What we need is one of two things. We need either the band gap coincidentally is ridiculously small. What's a good example of that? A free particle. In the case of a free particle, these band gaps go to 0. Right? And so that's a conductor. Just an electron. Right. It conducts, right? OK.

So that can certainly happen. But that's sort of stupid. I mean, it's not totally stupid. But it's sort of stupid. But a better answer would be, well, can you have a system where there are bands but you didn't have one electron per potential well? And yeah. You could orchestrate that in lots of ways.

Now it involves orchestration. So it's not the generic system that we were talking about here. But you can't orchestrate it. So spin is a useful thing that gives you an extra handle. If you have twice as many states per well then you can have half a band filled. So that's one way to do it.

Then it becomes dependent on details of the system, which is what I didn't want to get into. But yeah, you can orchestrate it. It's just not a generic thing from what we've done. And it's really not for spin-less systems.

On the other hand, accidental small gaps. Easy. That happens. That certainly happens.

So that brings me to the last thing I wanted to talk about before getting to entanglement, which is accidental small gaps. So what happens to a system which is-- so there are some systems that are neither conductors nor insulators. They are reasonably good conductors and reasonably bad insulators. But they're not perfect.

And these materials are called semiconductors. I want to talk about why they're called semiconductors and what that means. So this is going to be very brief. Then

I'm going to give you-- we're going to get into entanglement.

So consider a system exactly using the same logic we've used so far which has the following property. We have two bands. And the bottom band is filled because we've got just the right number of charged particles. Bottom band is filled. And this guy is empty, but the gap is tiny. OK. Delta e is very small.

Now delta e has dimensions. It has units, right? So when I say small, that doesn't mean anything. I need to tell you small compared to what. So what's a salient thing that controls an energy scale for a real material? Well the temperature. If you have a hot piece of copper, then the lattice is wiggling around. And every once in a while, an ion can hit one of the electrons and excite it, give it some momentum. And so there's an available reservoir of energy for exciting individual electrons.

You have it really hot, what happens is every once in a while an electron will get nailed by a little thermal fluctuation in the system and get excited above the gap. And now it's in a super-- and generically, it's going to be in a superposition state of one of these excited states. So it's in general going to be moving. It can radiate. It will eventually fall back down.

But you're constantly being buffeted. The sea of electrons is constantly being buffeted by this thermal fluctuation. And as a result, you constantly have electrons being excited up, cruising around, falling back down. So you end up with some population of electrons. And they can ask-- and both when asked, although not quite in this language, how likely are you to get an electron up here? How likely is an electron to be excited up thermally? And those of you taking 8.04 will know the answer to this.

The probability goes as e to the minus delta e over kt. So let's think of this where this is the Boltzmann constant. So what does this mean? At very low temperatures, if the gap isn't 0, then this is 0. It doesn't happen. But at large temperatures, the denominator here is large. If the temperature is large compared to the width of the gap, then this is a small number. And e to the minus of a small number is close to 1.

So at high temperature, you're very likely to excite electrons up here. And now if you have electrons up here, you have a bunch of available states down here-- also known as holes-- and you have a bunch of available electrons up here with lots of available states. So at a high temperature, a material with a small gap-- or at least at temperatures high compared to the size of the gap-- it's basically a conductor. And at low temperatures, it's basically an insulator.

This is called a semiconductor. And there are notes on the Stellar web page that discuss in a little more detail what I just went through and show you how you build a transistor out of a semiconductor. And the important bit of physics is just this. OK.

So that finishes us up for the band gap systems for periodic potentials. We've done something kind of cool. We've explained why diamonds are transparent. We've explained why they don't conduct. We've explained why copper does and it's opaque. And that's pretty good for 15 minutes of work. It's not bad.

But along the way, we also talked about the analogous system of what are called photonic crystals. Systems of periodic arrays of dielectrics. Like wave guides. And those have the same structure. They have bands of allowed energy and gaps of disallowed energies where no waves propagate through.

So you might think that's a little bit of a ridiculous example. So just to close this off, you've all seen a good example of a photonic crystal flying past you. You know that highly reflective at very specific frequency structure on the surface of a butterfly wing that makes it shiny and blue? It looks metallic. It looks like it's a crystal reflecting in a specific frequency. At some sharp blue.

And the reason is, it's a photonic crystal. It is exactly this form. If you look at it under a microscope, you see little rays of protein which have different dielectric than air. And they form exact crystals-- or not exact, but very good crystals-- that reflect at very specific wavelengths. And as a consequence, they have a metallic sheen.

So why would a butterfly put a photonic crystal on its surface? Well it's extremely light. It's fairly rigid. It looks shiny and metallic without actually being shiny and

metallic. And it's not a pigment, so it doesn't absorb light and decay over time. It's like the best thing you could ever do if you wanted to be a shiny, fluttery, flying thing.

Anyway. So there's an incredible amount of physics in this story of the band gaps. And consider this an introduction to the topic. OK. So that's it for band gaps. And I want to move on to the remainder, the last topic of our course. Which is going to be entanglement and quantum computation. And here I need to give you one quick observation and then move on to the punchline of today.

The one quick observation is this. We've talked about identical particles before. And we've talked about identical particles in funny states. So for example, imagine I have two particles described by a wave function where the first particle could be in the state a and the second particles in the state b.

And I can build a wave function for the first particle being in state a and the second particle in state b in the following way. So let's say position a and position b. I could take a single particle wave function, chi of a, and a single particle wave functions phi of b. And we've talked about what this tells us. And you've studied this on your problem set.

What this tells you is that the probability of finding the particle at point A is given by chi a squared. And this is normalized, so when we integrate against it, we get 1. And similarly, the probability that we find the second particle at b is this thing norm squared. And it's independent of what a is.

But we also studied-- and so this was called the distinguishable. We also studied the symmetric configuration, which was equal to 1 over root phi, root 2. Chi of a. Phi of b. Symmetric, plus chi of b phi of a. And this tells us something totally awesome. What's the probability that I find the first particle at a? It's the norm squared of chi of a phi of b, right?

If we integrate over all phi b, this is the norm squared integrates to 1. So it's fine. So there's a factor of one half. We either find it at chi of a or chi of b.

However if I tell you that I've measured the first particle and I find it in the state chi,

what can you say about the second particle? It's in the state phi. If you know the first particle's in the state chi, the second part is in the state phi. Because we measured it and it's not in the state-- the first particle's not in the state phi.

So measuring one particle tells you something about the second particle. And this is deeply disconcerting, because I could've taken these particles, put them in this entangled state, and sent one particle off to a distant planet and the second particle to my sister in DC. And my sister measures this second particle and determines what state it's in and is immediately determined what state the first particle is in over in this distant planet Zorg, right?

So that's deeply disconcerting. And to those of us who have studied quantum mechanics up to this point-- which we all in this room have-- to those of us who have studied quantum mechanics to this level of development and understand that it is a correct description of many experiments, this should be yet another moment of serious discomfort. We've run into a bunch of these over the semester. But this one should be troubling to you.

Because look. How can something here dramatically change the state, the configuration, the initial configuration, of a particle arbitrarily far away? Isn't that deeply concerning? And if you think about relativity, this should be all the more deeply disconcerting. Because how does relativistic causality fit into this? So there was a person that roughly this time, a little earlier, who was troubled by this problem. And his name was Einstein.

And so one of the things that's kind of amazing is that he created a thought experiment which we're going to study in detail next week called the EPR experiment. And there's a beautiful historical story about the setting and the meaning and the particular person. And unfortunately, I'm not a historian so I can't tell you that story. It sure would be nice if we had someone who wrote a biography of Einstein to tell you a little bit about that story. Oh look, it's Tom Levenson who wrote a biography about Einstein. So Tom is--

**TOM LEVENSON:** Oh, I need a microphone. Those of who have taken courses in [INAUDIBLE]-- and

I'm sure that's all of you because of the GIRs-- know this is larger than the usual [INAUDIBLE] class. So I'm very used to microphones, but not in this context. OK.

Is this-- yeah, it's on. Can you hear me? All right. So there are lots of ways to slice the story of Einstein by the time he reaches the EPR experiment, which is Einstein, Podolsky, and Rosen for the three people who actually wrote the paper.

Just to dot the I's and cross the T's on the paper itself, Rosen is apparently for person who first came to Einstein. Podolsky and Rosen were two young physicists in Princeton after Einstein moved to Princeton. Einstein moved to Princeton in 1933. About three weeks before-- I'm sorry, he moved to Princeton '33.

He left Germany in 1932 December, about three weeks before Hitler took power. And he did so with decisiveness and dispatch and a head of almost all of his-- in fact, I think all of his German-Jewish physicist colleagues and those German physicists for whom the Hitler regime was unacceptable. Which shows that Einstein really was smarter than most of his peers. That's one of many different ways you can ascertain that.

And so he came to Princeton in '33. He actually went to Caltech before we went to Princeton. As part of an ongoing visitor-ship he had there. Came back to Europe, hung with the queen of Belgium who was a friend of his. Went to England. And then headed across the Atlantic and took up residency in Princeton at the Institute for Advanced Studies where he stayed for the rest of his life.

And over the course of the-- that was '33, he died in '55, I think. I should know that, but I think that's right. 22 years. He worked with a lot of different, mostly younger physicists. And Podolsky and Rosen were early members of that chain. So Rosen was talking with him some day and starts to frame this experiment. Einstein develops it. The three of them talk about it. They write the paper and they put it out.

And I want to share with you, actually, a really lovely description of the way the problem was represented in a way by-- this is from a book that I recommend to all of you. It's actually really hard to find. It's really sweet. Jeremy Bernstein, who is a

physicist. He's sort of been around. A physicist and writer. He's in his eighties now. He lives in Aspen. He worked with CERN for a number of years. He's always been independent. He wrote for the *New Yorker*.

Anyway. So you've all heard of the physicist Bell, I assume? Bell's inequality? OK. So Bell had a lovely way to describe-- I'm trying to find. I had this marked and then I lost my piece of paper. I have already lost it. That's terrible.

So Bell has this wonderful way of describing this problem of entanglement. And it's based on his description of an actual person. I was going to read you his actual quote. Now I'm just going to paraphrase it for you. He had a friend named, I think, Bartelstein. Or at least someone known to him. Who had two quirks. An unusual color sense and a taste for mismatched socks.

And so Bell used to say, if you saw Bartelstein and you could only see one leg and that sock was pink, you knew to a certainty that the other sock was not pink. He comes up I think-- I'm trying to remember who this is originally attributed to.

Same thing. If you have a coin and you cut it in half down the-- so you've got two coin shape disks. You cut the disk in half, not-- and you have one side that's the head and the other side that's the tail. And they're separated. They get handed to two different gamblers. And one gambler tries to cheat the gambling establishment by tossing in his half coin. And you see the head that you know somebody-- somebody at some other casino is cheating by tossing in the half coin that only has a tail on it.

So there are lots of ways to represent this. And many physicists being very witty indeed have come up with different metaphors for it. So Allan just described for you the basic claim in EPR. Its weirdness. That you have two particles that are entangled in some way and then go their separate ways. And thus you have-- if you have knowledge of what's the state of one, you have certain knowledge of the state of the other, violating relativistic ideas of locality. And just kind of making you queasy if you're sort of approaching it naively.

What Einstein, Podolsky, and Rosen argued was actually something a little bit-- in fact, the paper comes to an end on that note of queasiness. But what they argue is a little bit more subtle. Because what they said is, OK. You perform this thought experiment. You send the two particles off. You measure position of one, you know absolutely the position of the other.

You've conferred-- and the paper turns on a discussion of the connection between a measurement-- a physical measurement-- and a property of physical reality. And they have definition for what reality is. And that is something whose-- if you can perform a measurement, you know that quantity absolutely. I don't have the mathematics to express that properly. But that'll do for this hand waving.

You can then do another experiment and measure a complementary property. And you know that piece of reality. But you can't do the-- so on the one hand, quantum mechanics says you can't know physical reality to this level of precision. And on the other hand, the fact that you can do that measurement violates the relativistic picture of reality. So you have what they claimed was a paradox.

And this paper was published. And it received a range of reactions from indifference by younger physicists who said, we don't care that it's weird. We're going to keep on doing quantum mechanics and performing experiments and making measurements. And just see where this leads us. Remember, this is happening in the mid '30s. 1935. One of these three books will tell me precisely in a moment.

And the quantum theory, as it turned into quantum mechanics, developed in its first period between '23 and '27. And by '35, you have enormous numbers of productive results and unexpected things and the prediction of the positron and then its observation. And I mean, the theory is enormously, dramatically, excitingly productive. So those who are really heads down doing the work are, for the most part, saying, this is fine. We'll get back to it when we're old and retired and bored.

But that wasn't the uniform case. And most notably Niels Bohr found this paper really troubling. And spent about six weeks, apparently, discussing this and trying to come up with a response to it. And what he responded was essentially that-- in

some ways, it was the same reaction as his younger colleagues. Get over it.

But more precisely, it was he said, there's no description of reality that excludes the measuring apparatus anymore. You can't make statements about physical reality unless you include a description of the measuring apparatus. And you've said that we can measure this one quantity with precision and know the other thing. And then we can subsequently, in a separate observation, measure a complimentary quality and know the other one. You still can't know much them at the same time.

It's still true that the complementarity in essence means that once you know one part of the picture, you know some other part the picture. And that's just the nature of the quantum world. Einstein had argued that the EPR paradox suggested that quantum mechanics was incomplete. And Bohr essentially responded in effect that Einstein's description of quantum mechanical explanation was inadequate.

The important thing to remember-- and I want to just spend a couple minutes going back into the pre-history of all this, and then a couple minutes speculating on why Einstein reached the position he did. And what that might tell you about the practice of science as a lived experience as opposed to one reflected in your textbooks.

But the thing to remember is that there's nothing logically wrong with the EPR paper. Right? You know. It does what it says it does and there's no overt error in it. And there's nothing wrong with Bohr's response. And in fact, when the experiments were-- Bell formalized the-- what Bell's inequality really does is it formalized the two arguments. It says, if Bohr is right, you will observe this in the experiment. And if Einstein is right, you would observe something different.

The experiments were done, and I imagine are still being done, as sort of demonstrations. And they showed that Bohr's interpretation was correct and that yes, quantum mechanics produces results that are non-local just as Allan described to you. And that the world really is as strange as people first glimpsed in 1925, '26, and '27.

And the question of whether or not that strangeness is adequately explained without

the explanations that you're going to learn in this class and subsequent ones are quote "complete" or not. And completeness is a very funny, very, very tricky concept. But the question whether or not the framework of quantum mechanics is somehow unsatisfactory in any kind of formal a technical sense is one that's at least partly dependent on your scientific temperament, I think.

So that's the cartoon version of what happened in '35. Einstein with his two young colleagues proposes-- really you should understand the EPR paper as a description in detail of a consequence of quantum theory as it was then expressed with the conclusion-- and I just want to read you this thing-- no reasonable definition of reality could be expected to permit such a result. In fact, it's called a paradox, but it isn't. It's a complaint. You know, it's a memo to the Flying Spaghetti Monster that the universe shouldn't be this way if, in fact, experiments turn out to show that it is and they have.

The oddity here for a biographer of Einstein opposed to a physicist is given what you know about Einstein between 1879 when he's born and say, 1925 or so when he completes the last of his really great physics. How could-- I mean, I actually keep-- I've been working on Einstein off and on for years and years. I keep finding out new ways in which he's just inconceivably bright and on target and with a nose for the right problem and insightful.

And yet by the 1935, 10 years later, he's still a relatively young man. He's in his '50s. Which being in my '50s I think is an extremely young man. Just 10 years after doing work that's right on the edge of modern quantum mechanics that is essential to its foundation. That's really extraordinary.

10 years after that, he's saying, no reasonable definition of reality should be permitted to behave this way. Where does that come from? Well the first thing I want to tell you-- again, this is all going to be a really cartoon version. Because there's not much time, I understand. Is that Einstein-- I mean, how much are you aware of Einstein's role in the creation of the quantum theory? A lot. I mean, a lot? None? OK.

I mean, I'm going to make a claim that except for Heisenberg, Schrodinger, maybe Bohr. Maybe Born. Maybe a couple of others. There's no one more important to the quantum theory than Einstein. And you could maybe even argue that from a sort of foundational point of view that without Einstein, rigorous thinking about quantum mechanics would have taken much, much longer. I mean, he's really central to it.

Planck in 1900 publishes as an ad hoc solution to the black body problem the first quantum theory. In 1905, Einstein says it's not an ad hoc thing. If you look at the photoelectric effect is the particular problem he's dealing with in explanation. But that's really-- the behavior of the photoelectric effect is really presented as the confirmation of this idea that light exists as quanta with particular kinds of behavior.

And from 1905 on, he spends probably more time on quantum problems than he did on any other physics problems. Certainly more than on relativity. Though he spent enormous energy on special and general relativity. One of most amazing things about Einstein, in fact, is that despite the fact that he's seen and appears by 1935 to be this hidebound old guy who can't accommodate himself to the new world is he had an extraordinary capacity to do what the Red Queen did in *Alice in Wonderland.* and believe two impossible things before breakfast.

Just think in 1905. April, he publishes The Quantum Theory Of Light. June he publishes Special Relativity, which treats light as a wave. And makes no mention of his revolutionary-- I mean, he called it revolutionary in a private letter in 1905. So he knew what he had in the quantum theory of light. But in special relativity, go read the special relativity paper. It's actually lovely reading. And you'll see he doesn't even nod in that direction. He doesn't say, you know this is a hero-- he says nothing.

So he's capable of doing excellent-- and there's a reason that year is called the annus mirabilis, the year of miracles. And in part it's because Einstein is able to actually really focus on these things. And I realize class is almost over. So there's several more steps in Einstein's quantum journey.

But you know, what you should take away is that Einstein's ability to deal with the problems of quantum pictures extends to the point-- he's the first person to suggest there might be a problem with causality in quantum mechanics. He does this in 1917, eight years before quantum mechanics is invented. When he starts looking at what the quote "classical" quantum theory tells you about the emission of radiation from an excited atom. He realized you can't predict it precisely. Radioactive decay has the same problem. He says, well-- he writes in a letter. I don't want to give up causality, but we may have to.

So he's aware of these things. So I see class is over now at 12:30. OK, sorry. So juts to finish off, the question here is why does Einstein give up on this. And the answer, I think, is because in addition to his-- as he started at the beginning of his career, he says with the quantum theory of light and with special relativity, ignore your physical pictures. Try and look at the phenomena and explain those.

And by 1935, that becomes very difficult for him. Because the phenomenology becomes too strange. One of the things that quantum mechanics does is it takes away the immediate ability to visualize physical systems. There. English is my first language sometimes.

And that's an aesthetic failure on Einstein's part. He had the intellectual capacity and explicitly said, quantum mechanics is a logically consistent theory that incredibly powerfully describes lots of problems. He said that in print. He nominated Schrodinger and Heisenberg for Nobel Prizes twice. I mean, he wasn't stupid. He was Albert Einstein. But he was aesthetically incapable of pursuing this new physics in ways that were possible under the research possibilities of the time. And that is what I would leave you with. Physics is an aesthetic as well as an intellectual pursuit. So thank you all.

[APPLAUSE]