MIT Course 8.033, Fall 2006, Formula Sheet

(Dated: August 24, 2006)

- 4-vectors:

$$\mathbf{X} \equiv \begin{pmatrix} x \\ y \\ z \\ ct \end{pmatrix}, \quad \mathbf{U} \equiv \frac{d\mathbf{X}}{d\tau} = \gamma_u \begin{pmatrix} u_x \\ u_y \\ u_z \\ c \end{pmatrix},$$

$$\mathbf{K} \equiv \gamma_u \begin{pmatrix} k_x \\ k_y \\ k_z \\ w/c \end{pmatrix}, \quad \mathbf{P} \equiv m_0 \mathbf{U} = \begin{pmatrix} p_x \\ p_y \\ p_z \\ E/c \end{pmatrix}$$

$$\mathcal{F} \equiv \frac{d}{d\tau}\mathbf{P} = \gamma_u \begin{pmatrix} \mathbf{F} \\ P/c \end{pmatrix}, \quad \mathbb{J} \equiv \rho_0 \mathbf{U} = \begin{pmatrix} J_x \\ J_y \\ J_z \\ \rho c \end{pmatrix}$$

- Lorentz transformation:

$$\mathbf{\Lambda}(\hat{\mathbf{x}}v) = \begin{pmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{pmatrix}, \quad \begin{pmatrix} x' \\ y' \\ z' \\ ct' \end{pmatrix} = \begin{pmatrix} \gamma(x - \beta ct) \\ y \\ z \\ \gamma(ct - \beta x) \end{pmatrix}$$

- Parallel velocity addition:

$$v'' = \frac{v + v'}{1 + \frac{vv'}{c^2}}$$

- Aberration & Doppler effect:

$$\cos\theta' = \frac{\cos\theta - \beta}{1 - \beta\cos\theta}, \quad \omega' = \omega\gamma(1 - \beta\cos\theta)$$

- Energy:

$$E = m_0\gamma c^2 = \sqrt{(m_0 c^2)^2 + (cp)^2}$$

- Electromagnetism:

$$\begin{aligned} \mathbf{F} &= q(\mathbf{E} + \frac{1}{c}\mathbf{u} \times \mathbf{B}) \\ E'_x &= E_x \\ E'_y &= \gamma(E_y - \beta B_z) \\ E'_z &= \gamma(E_z + \beta B_y) \\ B'_x &= B_x \\ B'_y &= \gamma(B_y + \beta E_z) \\ B'_z &= \gamma(B_z - \beta E_y). \end{aligned}$$

- Euler-Lagrange equation:

$$\frac{\partial f}{\partial x} - \frac{d}{dt}\frac{\partial f}{\partial \dot{x}} = 0$$

- Metrics (Minkowski, Newtonian, FRW, Schwartzschild), $c = G = 1$, $d\Omega^2 \equiv d\theta^2 + \sin^2\theta d\varphi^2$:

$$\begin{aligned} d\tau^2 &= dt^2 - dx^2 - dy^2 - dz^2 \\ &= dt^2 - dr^2 - r^2 d\Omega^2, \\ d\tau^2 &= (1 + 2\phi)dt^2 - r^2 d\Omega^2, \\ d\tau^2 &= dt^2 - a(t)^2\left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2\right), \\ d\tau^2 &= \left(1 - \frac{2M}{r}\right)dt^2 - \left(1 - \frac{2M}{r}\right)^{-1} dr^2 - r^2 d\Omega^2 \end{aligned}$$

- Friedmann equation:

$$H^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}, \quad H \equiv \frac{\dot{a}}{a}$$

- Shell coordinates:

$$\begin{aligned} dt_{\text{shell}} &= \gamma_r^{-1} dt, \quad dr_{\text{shell}} = \gamma_r dr \\ \gamma_r &\equiv \frac{1}{\sqrt{1 - \beta_r^2}}, \quad \beta_r \equiv \left(\frac{2M}{r}\right)^{1/2} \end{aligned}$$

- Schwarzschild orbits:

$$\begin{aligned} \left(\frac{dr}{d\tau}\right)^2 &= \tilde{E}^2 - \tilde{V}(\tilde{L}, r)^2, \\ \frac{d\varphi}{d\tau} &= \frac{\tilde{L}}{r^2}, \\ \frac{dt}{d\tau} &= \gamma_r^2 \tilde{E}, \\ \tilde{V}(\tilde{L}, r)^2 &\equiv \left(1 - \frac{2M}{r}\right)\left(1 + \frac{\tilde{L}^2}{r^2}\right) \end{aligned}$$

- Just kidding:

$$\begin{aligned} R_{\mu\nu} &- \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}, \\ R &\equiv g^{\mu\nu} R_{\mu\nu}, \\ R_{\mu\nu} &\equiv R^{\alpha}_{\mu\alpha\nu}, \\ R^{\alpha}_{\mu\nu\beta} &\equiv \Gamma^{\alpha}_{\nu\beta,\mu} - \Gamma^{\alpha}_{\mu\beta,\nu} + \Gamma^{\gamma}_{\mu\beta}\Gamma^{\alpha}_{\nu\gamma} - \Gamma^{\gamma}_{\nu\beta}\Gamma^{\alpha}_{\mu\gamma}, \\ \Gamma^{\alpha}_{\mu\nu} &\equiv \frac{1}{2} g^{\alpha\sigma}\left(g_{\sigma\mu,\nu} + g_{\sigma\nu,\mu} - g_{\mu\nu,\sigma}\right) \end{aligned}$$ ☺