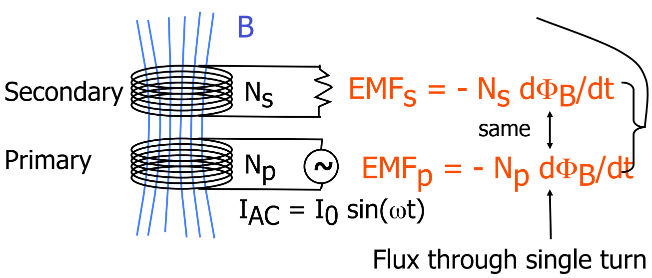
Quiz #4 preparations

- Quiz 4: Wed, 5/4, 10AM,
  - 1 sheet with formulae etc
  - No books, calculators
- Evening review: Tue, 5/3, 7PM
- Tutoring:
  - Angel Solis, Mon + Tue, 5/2, 5-7PM,

Review for Quiz #4

Transformer Action

- Transformer action $EMF_s / EMF_p = N_s / N_p$



Transformer Action

- Transformer action $EMF_s / EMF_p = N_s / N_p$
- Transformers allow change of amplitude for AC voltage
  - ratio of secondary to primary windings
- Constructed such that Φ_B identical for primary and secondary
-

What you need to know

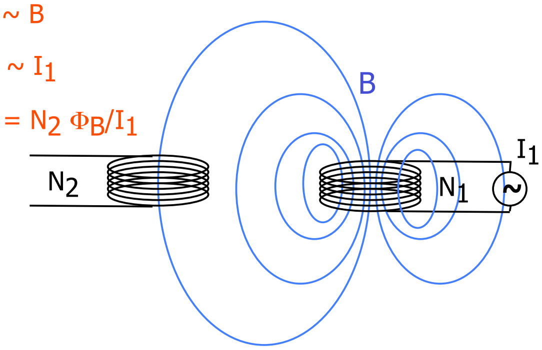
- Transformers
  - Basic principle
  - Transformer in HVPS
  - Relationship between I,V,P on primary/ secondary side
  - Demos
    - **Jacobs Ladder**
    - **Melting nail**

Mutual Inductance

$\Phi_B \sim B$

$B \sim I_1$

Def.: $M_{12} = N_2 \Phi_B / I_1$



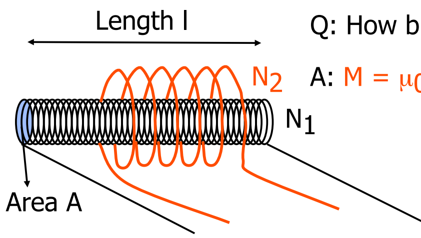
Mutal Inductance

- Coupling is symmetric: $M_{12} = M_{21} = M$
- M depends only on Geometry and Material
- Mutual inductance gives strength of coupling between two coils (conductors):

$$EMF_2 = - N_2\, d\Phi_B/dt = - M\, dI_1/dt$$

- M relates EMF_2 and I_1 (or EMF_1 and I_2)
- Units: $[M] = V/(A/s) = V\, s/A = H$ ('Henry')

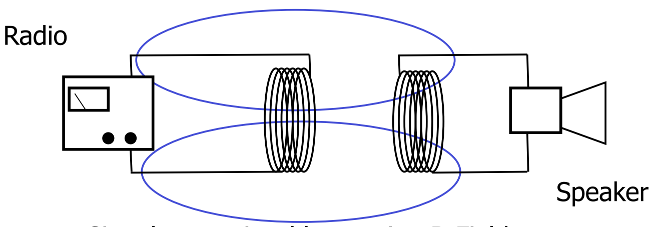
Example: Two Solenoids



Q: How big is $M = N_2\, \Phi_B/I_1$?

A: $M = \mu_0\, N_1 N_2\, A/l$

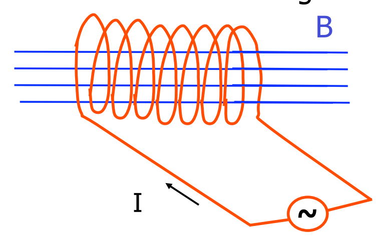
Demo: Two Coils



- Signal transmitted by varying B-Field
- Coupling depends on Geometry (angle, distance)

Self Inductance

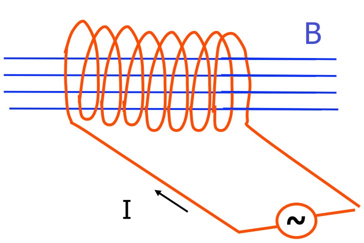
Circuit sees flux generated by it self



Def.: $L = N\, \Phi_B/I$

<u>Self-Inductance</u>

Example: Solenoid



Q: How big is L ?

A: $L = \mu_0\, N^2\, A/L$

Self Inductance

- L is also measured in [H]
- L connects induced EMF and variation in current:

$$EMF = - L\, dI/dt$$

- Remember Lenz' Rule:

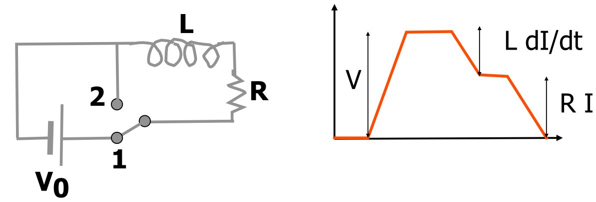
  Induced EMF will 'act against' change in current -> effective 'inertia'

- Delay between current and voltage

What you need to know

- Inductance
  - Mutual Inductance
    - Definition
    - Calculation for simple geometry
  - Self Inductance
    - Definition
    - Calculation for simple geometry
    - Direction of induced EMF (depends only on dI/dt)

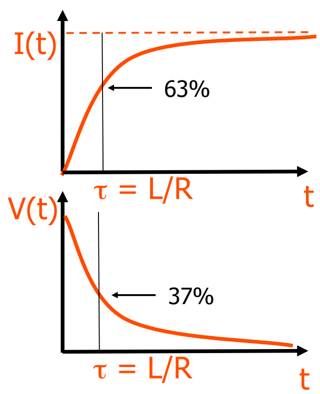
RL Circuits



Kirchoffs Rule: $V_0 + \xi_{ind} = R I \rightarrow V_0 = L\, dI/dt + R I$

Q: What is $I(t)$?

RL Circuits



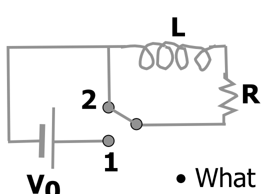
$I(t)=V_0/R\,[1-\exp(-t/\tau)]$

$V(t)=V_0 \exp(-t/\tau)$

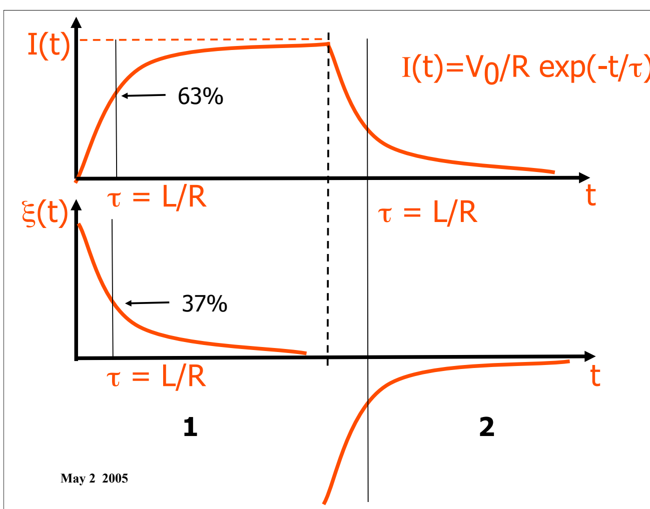
RL Circuits

- Inductance leads to 'delay' in reaction of current to change of voltage V_0
- All practical circuits have some L and R
  - change in I never instantaneous

'Back EMF'



- What happens if we move switch to position 2?



RL circuit

- L counteracts change in current both ways
  - Resists increase in I when connecting voltage source
  - Resists decrease in I when disconnecting voltage source
  - 'Back EMF'
- That's what causes spark when switching off e.g. appliance, light

Energy Storage in Inductor

- Energy in Inductor
  - Start with Power $P = V*I = L\, dI/dt\, I = dU/dt$

  -> $dU = L\, dI\, I$

  -> $U = 1/2\, L\, I^2$
- Where is the Energy stored?
  - Example: Solenoid (but true in general)

$U/\text{Volume} = 1/2\, B^2/\mu_0$

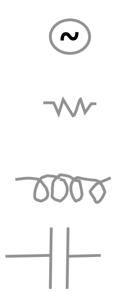
What you need to know

- Inductors
  - I(t) in DC RL circuits
  - Energy storage in inductors
  - Practical use

RLC circuits

- Combine everything we know...
- Resonance Phenomena in RLC circuits
  - Resonance Phenomena known from mechanics (and engineering)
  - Great practical importance

Summary of Circuit Components



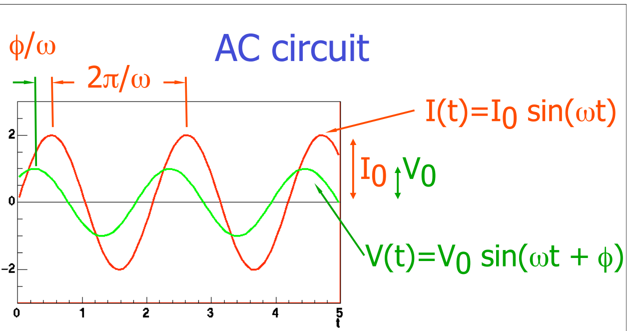
V	**$V(t) = V_0 \cos(\omega t)$**
R	**$V_R = - IR$**
L	**$V_L = - L\, dI/dt$**
C	**$V_C = -Q/C = -1/C \int I dt$**

R,L,C in AC circuit

- AC circuit
  - $I(t) = I_0 \sin(\omega t)$
  - $V(t) = V_0 \sin(\omega t + \phi)$

  } same ω!
- Relationship between V and I can be characterized by two quantities
  - Impedance $Z = V_0/I_0$
  - Phase-shift ϕ

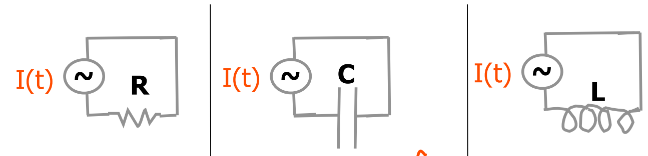
AC circuit



Impedance $Z = V_0/I_0$

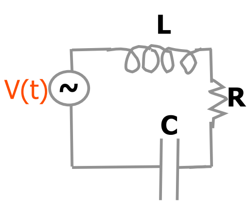
Phase-shift ϕ

First: Look at the components

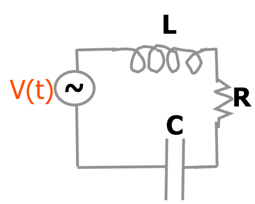


R	C	L
$V = I R$	$V = Q/C = 1/C \int I dt$	$V = L\, dI/dt$
$Z = R$, $\phi = 0$	$Z = 1/(\omega C)$, $\phi = -\pi/2$	$Z = \omega L$, $\phi = \pi/2$
V and I in phase	V lags I by 90^o	I lags V by 90^o

RLC circuit



RLC circuit

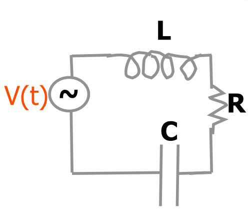


$V - L\, dI/dt - IR - Q/C = 0$

$L\, d^2Q/dt^2 = -1/C\, Q - R\, dQ/dt + V$

2^{nd} order differential equation

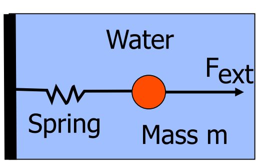
RLC circuit



$V - L\, dI/dt - IR - Q/C = 0$

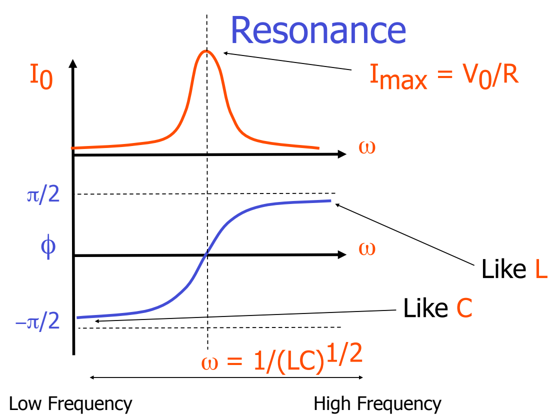
$L\, d^2Q/dt^2 = -1/C\, Q - R\, dQ/dt + V$

'Inertia' 'Spring' 'Friction'



$m\, d^2x/dt^2 = -k\, x - f\, dx/dt + F_{ext}$

Resonance



RLC circuit

$V_0 \sin(\omega t) = I_0\{[\omega L - 1/(\omega C)] \cos(\omega t - \phi) + R \sin(\omega t - \phi)\}$

Solution (requires two tricks):

$I_0 = V_0/([\omega L - 1/(\omega C)]^2 + R^2)^{1/2} = V_0/Z$

$\tan(\phi) = [\omega L - 1/(\omega C)]/R$

-> For $\omega L = 1/(\omega C)$, Z is minimal and $\phi = 0$

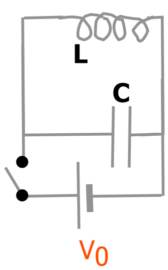
i.e. $\omega 0 = 1/(LC)^{1/2}$ Resonance Frequency

Resonance

- Practical importance
  - 'Tuning' a radio or TV means adjusting the resonance frequency of a circuit to match the frequency of the carrier signal

LC-Circuit

- What happens if we open switch?



$-L\, dI/dt - Q/C = 0$

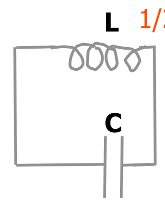
$L\, d^2Q/dt^2 + Q/C = 0$

↕

$d^2x/dt^2 + \omega_0^2 x = 0$

Harmonic Oscillator!

LC-Circuit



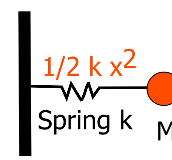
$1/2\ L I^2$

$1/2\ Q^2/C$

Energy in E-Field

↕ Oscillation

Energy in B-Field



$1/2\ k x^2$

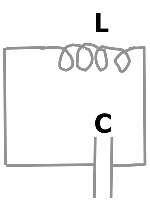
$1/2\ m v^2$

Potential Energy

↕ Oscillation

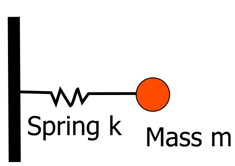
Kinetic Energy

LC-Circuit



$d^2Q/dt^2 + 1/(LC)\, Q = 0$

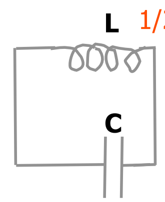
$\omega_0^2 = 1/(LC)$



$d^2x/dt^2 + k/m\, x = 0$

$\omega_0^2 = k/m$

LC-Circuit



$1/2\ L I^2$

$1/2\ Q^2/C$

Energy in E-Field

↕ Oscillation

Energy in B-Field

- Total energy U(t) is conserved:

$Q(t) \sim \cos(\omega t)$

$dQ/dt \sim \sin(\omega t)$

$U_L \sim (dQ/dt)^2 \sim \sin^2$

$U_C \sim Q(t)^2 \sim \cos^2$

$\cos^2(\omega t) + \sin^2(\omega t) = 1$

Electromagnetic Oscillations

- In an LC circuit, we see oscillations:

Energy in E-Field
↕
Energy in B-Field

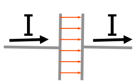
- Q: Can we get oscillations without circuit?
- A: Yes!
  - **Electromagnetic Waves**

What you need to know

- RLC Circuits
  - How to obtain diff. equ (but not solve it)
  - Definition of impedance, phase shift
  - Phaseshift for C,R,L AC circuits
  - Impedance, phase shift at resonance
  - Limiting behavior of RLC circuit with frequency
  - LC, RLC analogy with mechanical systems
  - LC oscillations: Frequency, role of E,B energy

Displacement Current

- Ampere's Law broken – How can we fix it?

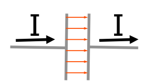


Q = C V

Displacement Current $I_D = \varepsilon_0 \, d\Phi_E/dt$

Displacement Current

- Extension of Ampere's Law:



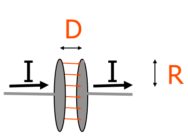
Q = C V

Displacement Current $I_D = \varepsilon_0 \, d\Phi_E/dt$

Changing field inside C also produces B-Field!

Displacement Current

- Example calculation: B(r) for r > R



Q = C V

-> $B(r) = R^2/(2rc^2) \, dV/dt$

Maxwell's Equations

$$\oint_{A_{closed}} \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$\oint_{L_{closed}} \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\oint_{A_{closed}} \vec{B} \cdot d\vec{A} = 0$$

$$\oint_{L_{closed}} \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} + \mu_0 \, \epsilon_0 \frac{d\Phi_E}{dt}$$

- Symmetry between E and B
  - although there are no magnetic monopoles
- Basis for radio, TV, electric motors, generators, electric power transmission, electric circuits etc

Maxwell's Equations

$$\oint_{A_{closed}} \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$\oint_{L_{closed}} \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\oint_{A_{closed}} \vec{B}\cdot d\vec{A} = 0$$

$$\oint_{L_{closed}} \vec{B}\cdot d\vec{l} = \mu_0 I_{encl} + \mu_0\,\epsilon_0 \frac{d\Phi_E}{dt}$$

$1/c^2$

- M.E.'s **predict** electromagnetic waves, moving with speed of light
- Major triumph of science

What you need to know

- Displacement current
  - Definition
  - Calculation for simple geometry
  - It's not a current
- Maxwells equations
  - Meaning in words

Reminder on waves

- Types of waves
  - Transverse
  - Longitudinal
    - compression/decompression

Reminder on waves

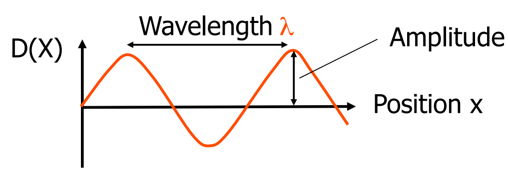
- For a travelling wave (sound, water)
  Q: What is actually moving?
- **-> Energy!**
- Speed of propagation: $v = \lambda f$
- Wave equation:

$$\frac{\partial^2 D(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D(x,t)}{\partial t^2}$$

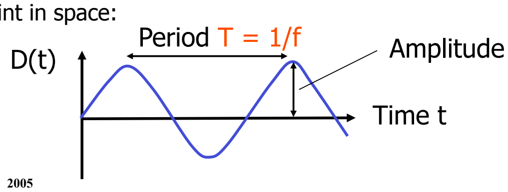
Couples variation in time and space

Reminder on waves

At a moment in time:



At a point in space:



Wave Equation

- Wave equation:

$$\frac{\partial^2 D(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D(x,t)}{\partial t^2}$$

Couples variation in time and space

- Speed of propagation: $v = \lambda f$
- We can derive a wave equation from Maxwells equations **(NOT IN QUIZ)**

Plane waves

- Example solution: Plane waves

$$\begin{aligned} E_y &= E_0 \cos(kz - \omega t) \\ B_x &= B_0 \cos(kz - \omega t) \\ \text{with } k &= \frac{2\pi}{\lambda}, \omega = 2\pi f \text{ and } f\lambda = c. \end{aligned}$$

E.M. Wave Summary

- $\vec{E} \perp \vec{B}$ and perpendicular to direction of propagation
- Transverse waves
- Speed of propagation $v = c = \lambda f$
- $|\vec{E}|/|\vec{B}| = c$
- E.M. waves travel without medium

What you need to know

- Waves
  - What is a wave?
  - Types of waves
  - Relationships between wavelength, frequency wave speed
- E.M. waves
  - Properties
  - Connection to demos (speed, polarisation)
  - Relative direction of E,B,v

AMP Experiment

- Understand general idea/purpose
- Understand voltage dividers
- Undertand need for negative feedback loop