

Topic: Inductors

Related Reading:

Course Notes (Liao et al.): Sections 11.1 – 11.4

Serway & Jewett: Sections 32.1 – 32.4

Giancoli: Sections 30.1 – 30.4

Topic Introduction

Today we continue thinking about Faraday's Law, and move from mutual inductance, in which the changing flux from one circuit induces an EMF in another, to self inductance, in which the changing flux from a circuit induces an EMF in itself.


Self Inductance

Remember that we defined the mutual inductance between two circuits and gave the relation

$\mathcal{E}_2 = -M \frac{dI_1}{dt}$. The self inductance obeys a similar equation: $\mathcal{E} = -L \frac{dI}{dt}$, and the same

concept: when a circuit has a current in it, it creates a magnetic field, and hence a flux, through itself. If that current changes, then the flux will change and hence an EMF will be induced in the circuit. The action of that EMF will be to *oppose the change* in current (if the current is decreasing it will try to make it bigger, if increasing it will try to make it smaller). For this reason, we often refer to the induced EMF as the “back EMF.”

To calculate the self inductance (or inductance, for short) of an object consisting of N turns of wire, imagine that a current I flows through it, and determine how much flux Φ_B that makes through the object itself. The self inductance is defined as $L = N\Phi_B/I$.

An inductor is a circuit element whose main characteristic is its inductance, L . It is drawn as a coil  in circuit diagrams. The strong resemblance to a solenoid is intentional – solenoids make very good inductors both because of their ability to make a strong field inside themselves, and also because the field they produce is fairly well contained, and hence doesn't produce flux (and induce EMFs) in other, nearby circuits.

The role of an inductor is to oppose changing currents. At steady state, in a DC circuit, an inductor is off – it induces no EMF as long as the current through it is constant. As soon as you try to change the current through an inductor though, it will fight back. In this sense an inductor is the opposite of a capacitor. If a capacitor is placed in a steady state current it will eventually fill up and “open” the circuit, whereas an inductor looks like a short in this case. On the other hand, when starting from its uncharged state, a capacitor looks like a short when you first try to move current through it, while an inductor looks like an open circuit, as it prevents the change (from no current to some current).

LR Circuits

We can write down a differential equation for a simple circuit with an inductor, resistor and battery in series using Kirchhoff's loop rule, and using $\mathcal{E} = -L \frac{dI}{dt}$ for the potential drop that

would be measured if you were to walk across the inductor in the direction of current. Just like RC circuits the solutions to this equation are exponential decays down to zero or up to some constant value. Instead of RC, the time constant is now $\tau = L/R$ (a big inductance slows down the circuit as it is more effective at opposing changes, but now a big resistance reduces the size of the current, and hence changes in the current that the inductor will see, and hence decreases the time constant – speeds things up). Just as with RC circuits, you can usually determine what is happening in the circuit just by thinking about what the elements do (e.g. inductors do what they can to keep the current steady – including sourcing current if they see the current decreasing).

Energy in B Fields

Where do inductors get the energy to source current when they need to? In capacitors we found that energy was stored in the electric field between their plates. In inductors, energy is similarly stored, only now its in the magnetic field. Just as with capacitors, where the electric field was created by a charge on the capacitor, we now have a magnetic field created when there is a current through the inductor. Thus, just as with the capacitor, we can discuss

both the energy in the inductor, $U = \frac{1}{2} LI^2$, and the more generic energy density $u_B = \frac{B^2}{2\mu_0}$,

stored in the magnetic field. Again, although we introduce the magnetic field energy density when talking about energy in inductors, it is a generic concept – whenever a magnetic field is created it takes energy to do so, and that energy is stored in the field itself.

Important Equations

Self Inductance, L :
$$L = \frac{N\Phi_B}{I}$$

EMF Induced by Inductor:
$$\mathcal{E} = -L \frac{dI}{dt}$$

Energy stored in Inductor:
$$U = \frac{1}{2} LI^2$$

Energy Density in B Field:
$$u_B = \frac{B^2}{2\mu_0}$$

Time Constant of an LR Circuit:
$$\tau = L/R$$