Class 36: Outline

Hour 1:

Concept Review / Overview

PRS Questions – Possible Exam Questions

Hour 2:

Sample Exam

Yell if you have any questions

Before Starting…

All of your grades should now be posted (with possible exception of last problem set). If this is not the case contact me immediately.

Final Exam Topics

Maxwell's Equations:

1. Gauss's Law (and "Magnetic Gauss's Law")
2. Faraday's Law
3. Ampere's Law (with Displacement Current)

   & Biot-Savart & Magnetic moments

Electric and Magnetic Fields:

1. Have associated potentials (you only know E)
2. Exert a force
3. Move as waves (that can interfere & diffract)
4. Contain and transport energy

Circuit Elements: Inductors, Capacitors, Resistors

Test Format

Six Total "Questions"

One with 10 Multiple Choice Questions

Five Analytic Questions

1/3 Questions on New Material

2/3 Questions on Old Material

Maxwell's Equations

Maxwell's Equations

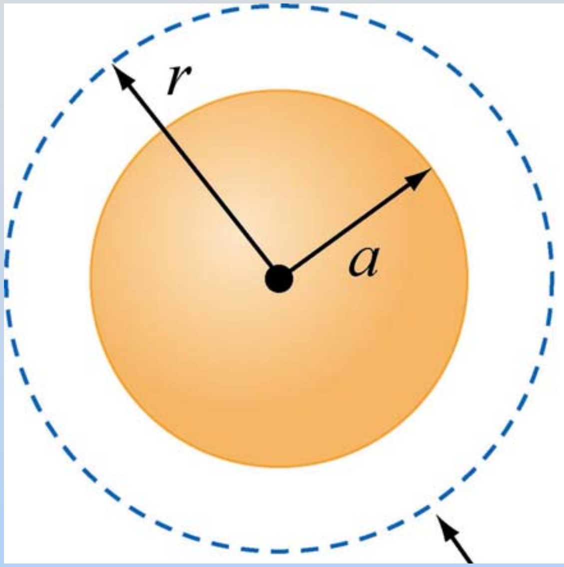
$$\oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{in}}{\varepsilon_0} \qquad \text{(Gauss's Law)}$$

$$\oint_C \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}} = -\frac{d\mathbf{\Phi}_B}{dt} \qquad \text{(Faraday's Law)}$$

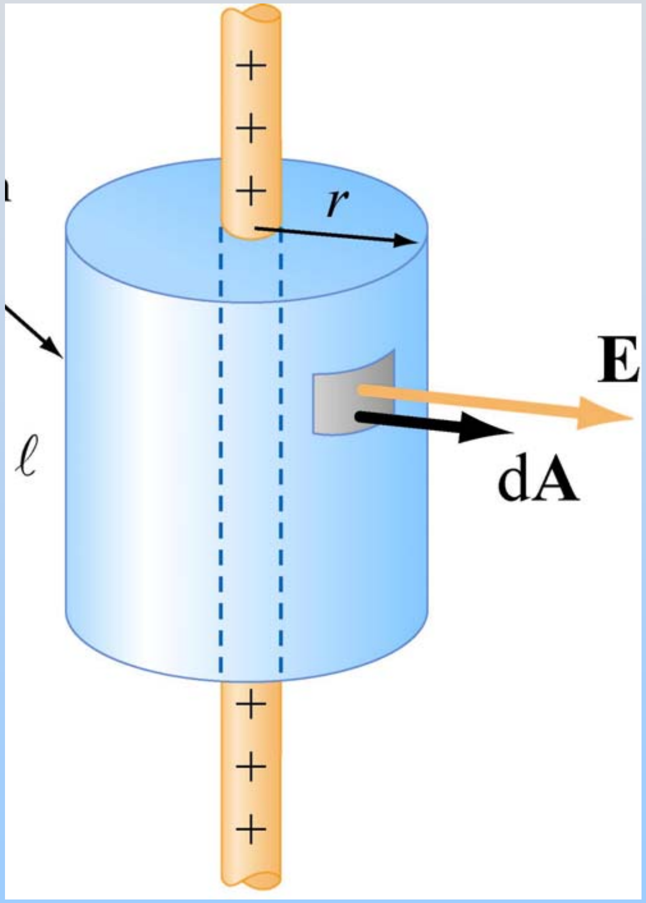
$$\oiint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0 \qquad \text{(Magnetic Gauss's Law)}$$

$$\oint_C \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{s}} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \frac{d\mathbf{\Phi}_E}{dt} \qquad \text{(Ampere-Maxwell Law)}$$

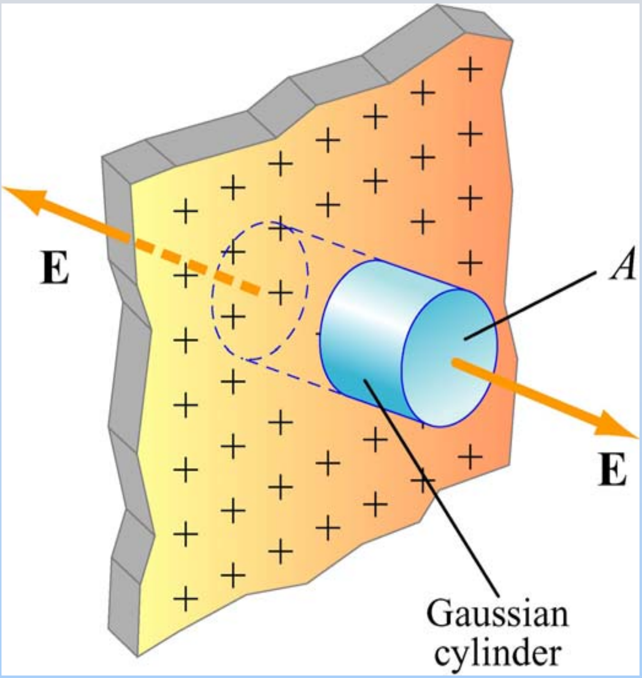
Gauss's Law: $\oint\!\!\oint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{in}}{\varepsilon_0}$



Spherical Symmetry



Cylindrical Symmetry



Planar Symmetry

Maxwell's Equations

$$\oint\!\!\oint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{in}}{\varepsilon_0}$$ (Gauss's Law)

$$\oint_C \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}$$ (Faraday's Law)

$$\oint\!\!\oint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$ (Magnetic Gauss's Law)

$$\oint_C \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{s}} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$ (Ampere-Maxwell Law)

Faraday's Law of Induction

$$\mathcal{E} = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -N\frac{d\Phi_B}{dt}$$

Moving bar, entering field

$$= -N\frac{d}{dt}\left(BA\cos\theta\right)$$

Ramp B

Rotate area in field

Lenz's Law:

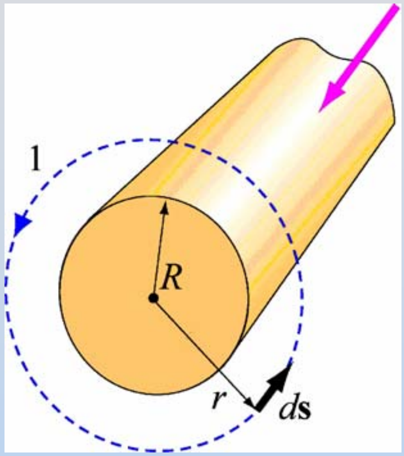
Induced EMF is in direction that ***opposes*** the change in flux that caused it

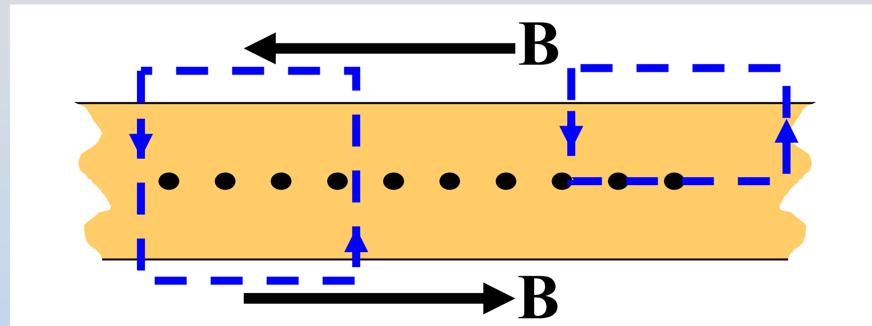
Maxwell's Equations

$$\oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{in}}{\varepsilon_0}$$	(Gauss's Law)
$$\oint_C \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}$$	(Faraday's Law)
$$\oiint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$	(Magnetic Gauss's Law)
$$\oint_C \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{s}} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$	(Ampere-Maxwell Law)

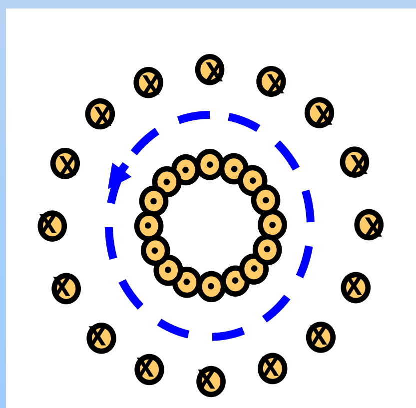
Ampere's Law: $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc}$

Long Circular Symmetry



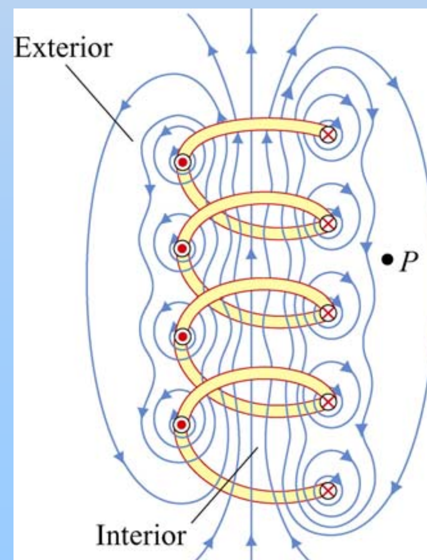


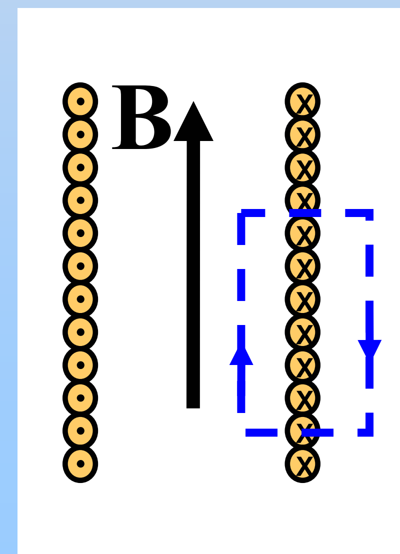
(Infinite) Current Sheet



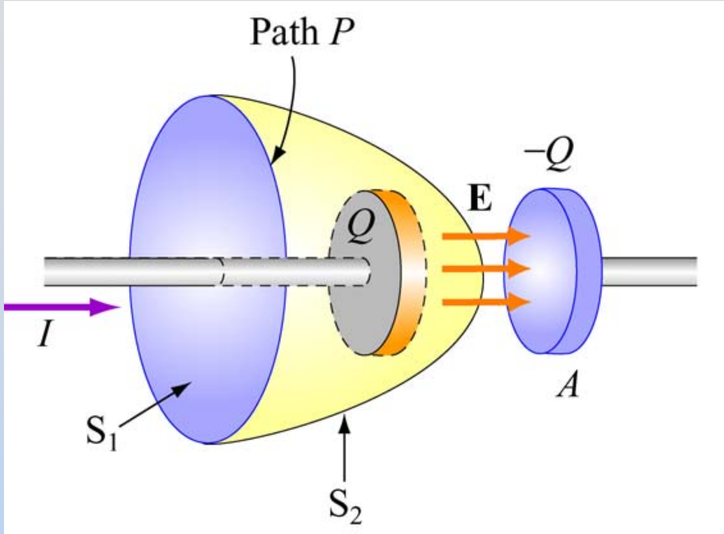
Torus/Coax

Solenoid = 2 Current Sheets





Displacement Current



$$E = \frac{Q}{\varepsilon_0 A} \Rightarrow Q = \varepsilon_0 E A = \varepsilon_0 \Phi_E$$

$$\frac{dQ}{dt} = \varepsilon_0 \frac{d\Phi_E}{dt} \equiv I_d$$

$$\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 (I_{encl} + I_d)$$

Capacitors, EM Waves

$$= \mu_0 I_{encl} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Maxwell's Equations

$$\oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{in}}{\varepsilon_0}$$ (Gauss's Law)

$$\oint_C \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}$$ (Faraday's Law)

$$\oiint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$ (Magnetic Gauss's Law)

$$\oint_C \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{s}} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$ (Ampere-Maxwell Law)

I am nearly certain that you will have one of each

They are very standard – know how to do them all

EM Field Details…

Electric Potential

$$\Delta V = -\int_A^B \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}} = V_B - V_A$$

$$= -Ed$$ (if E constant – e.g. Parallel Plate C)

Common second step to Gauss' Law

$$\vec{\mathbf{E}} = -\nabla V = \text{e.g.} \ -\frac{dV}{dx}\hat{\mathbf{i}}$$

Less Common – Give plot of V, ask for E

Force

Lorentz Force:

$$\vec{\mathbf{F}} = q\left(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}\right)$$

- Single Charge Motion
- Cyclotron Motion
- Cross E & B for no force

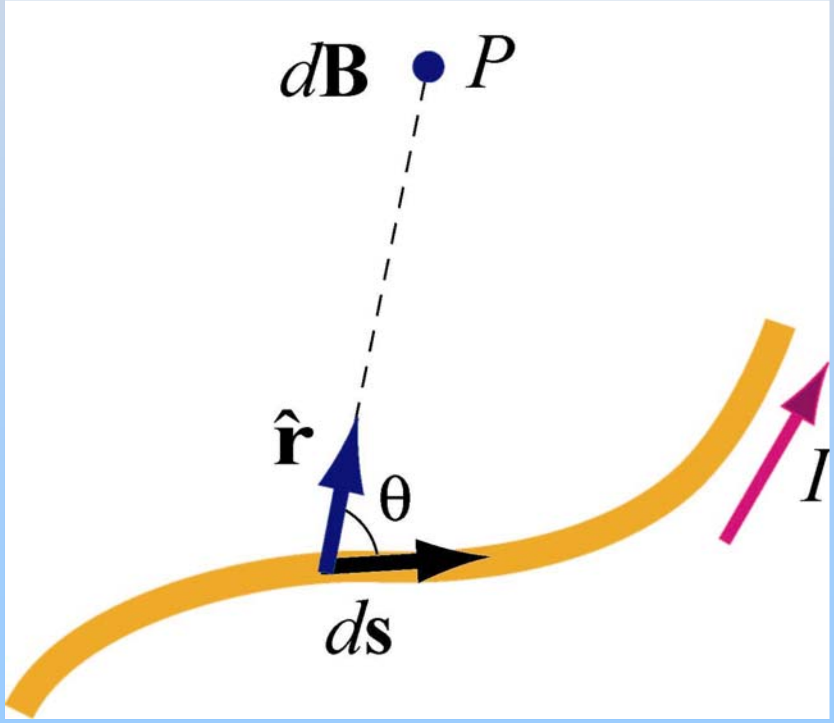
Magnetic Force:

$$d\vec{\mathbf{F}}_B = I d\vec{\mathbf{s}} \times \vec{\mathbf{B}} \Rightarrow \vec{\mathbf{F}}_B = I\left(\vec{\mathbf{L}} \times \vec{\mathbf{B}}\right)$$

- Parallel Currents Attract
- Force on Moving Bar (w/ Faraday)

The Biot-Savart Law

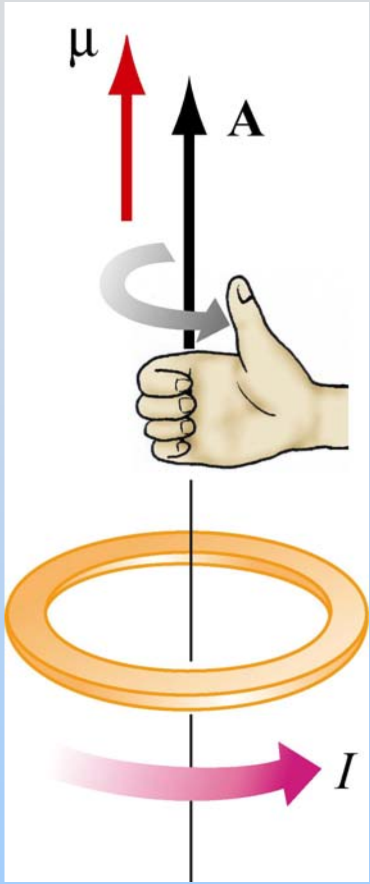
Current element of length ds carrying current I (or equivalently charge q with velocity v) produces a magnetic field:



$$\vec{\mathbf{B}} = \frac{\mu_o}{4\pi} \frac{q\,\vec{\mathbf{v}} \times \hat{\mathbf{r}}}{r^2}$$

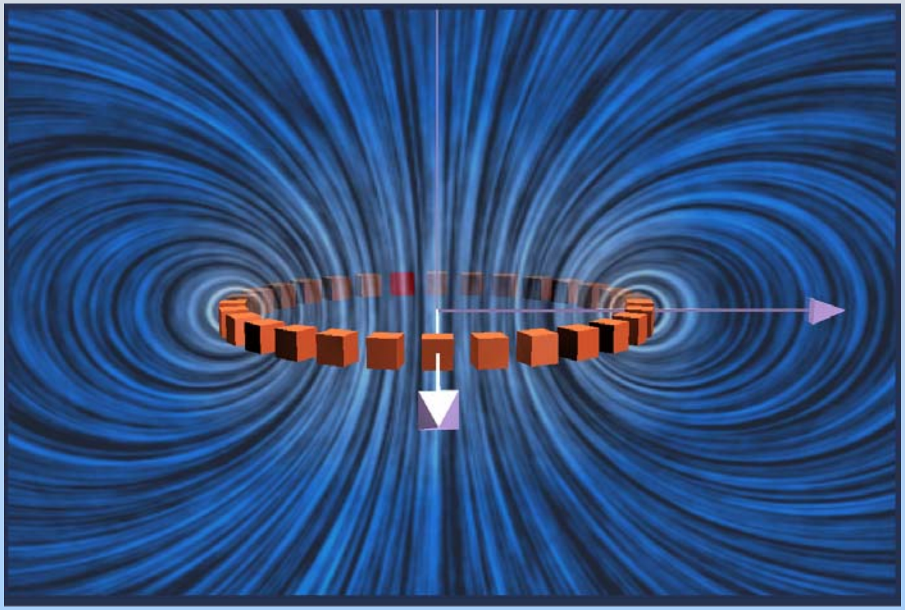
$$\mathbf{d}\vec{\mathbf{B}} = \frac{\mu_0}{4\pi} \frac{I\,d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$$

Magnetic Dipole Moments



$$\vec{\mu} \equiv IA\hat{\mathbf{n}} \equiv I\vec{\mathbf{A}}$$

Generate:



Feel:

1) Torque aligns with external field $\vec{\tau} = \vec{\mu} \times \vec{\mathbf{B}}$
2) Forces as for bar magnets

Traveling Sine Wave

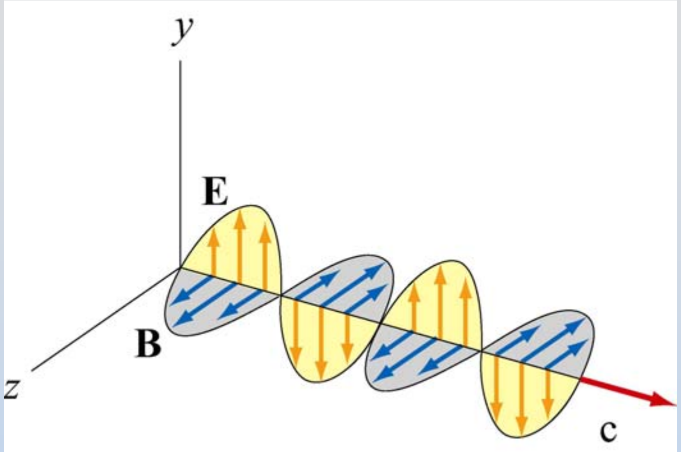
$$\vec{\mathbf{E}} = \hat{\mathbf{E}} E_0 \sin(kx - \omega t)$$

- Wavelength: λ
- Frequency : f
- Wave Number: $k = \frac{2\pi}{\lambda}$
- Angular Frequency: $\omega = 2\pi f$
- Period: $T = \frac{1}{f} = \frac{2\pi}{\omega}$
- Speed of Propagation: $v = \frac{\omega}{k} = \lambda f$
- Direction of Propagation: $+x$

Good chance this will be one question!

EM Waves

Travel (through vacuum) with speed of light



$$v = c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3\times 10^8 \frac{m}{s}$$

At every point in the wave and any instant of time, E and B are in phase with one another, with

$$\frac{E}{B} = \frac{E_0}{B_0} = c$$

E and B fields perpendicular to one another, and to the direction of propagation (they are **transverse**):

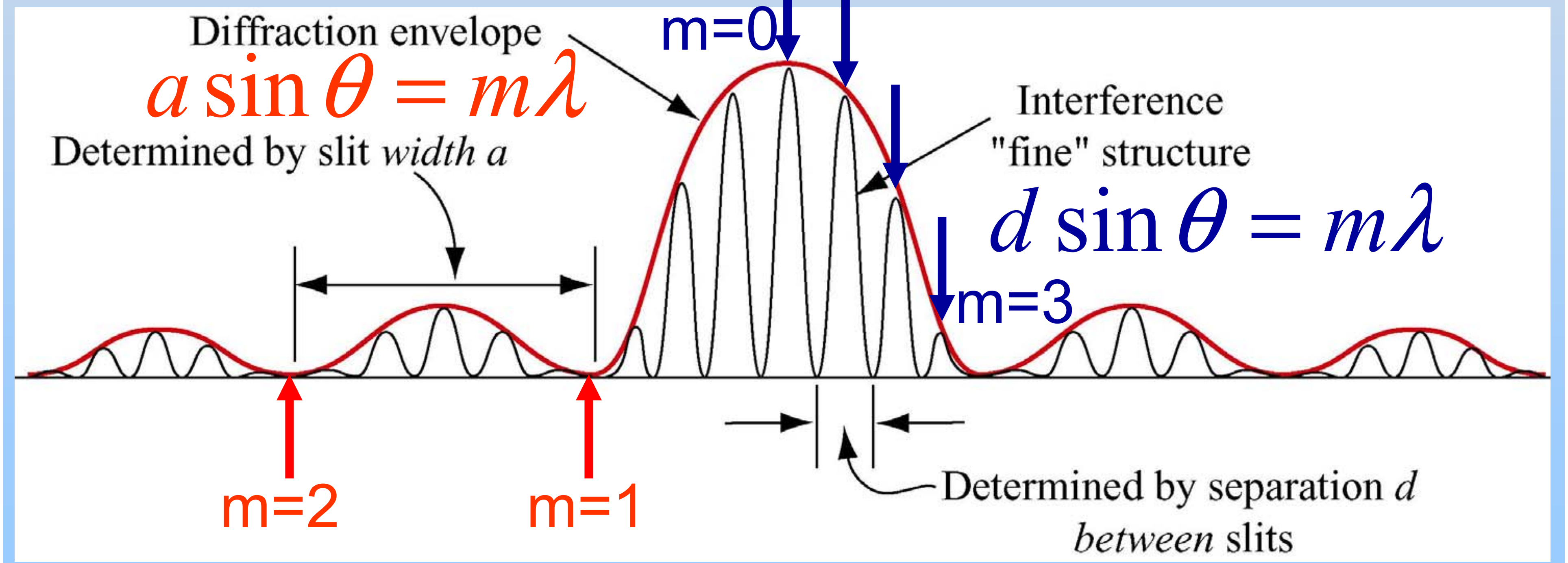
$$\text{Direction of propagation} = \text{Direction of } \vec{\mathbf{E}} \times \vec{\mathbf{B}}$$

Interference (& Diffraction)

$$\Delta L = m\lambda \quad \Rightarrow \quad \text{Constructive Interference}$$

$$\Delta L = \left(m + \tfrac{1}{2}\right)\lambda \Rightarrow \quad \text{Destructive Interference}$$

Likely multiple choice problem?



Energy Storage

Energy is stored in E & B Fields

$$u_E = \frac{\varepsilon_o E^2}{2}$$

: Electric Energy Density

In capacitor: $U_C = \frac{1}{2} C V^2$

In EM Wave

$$u_B = \frac{B^2}{2\mu_o}$$

: Magnetic Energy Density

In inductor: $U_L = \frac{1}{2} L I^2$

In EM Wave

Energy Flow

Poynting vector: $\vec{\mathbf{S}} = \dfrac{\vec{\mathbf{E}} \times \vec{\mathbf{B}}}{\mu_0}$

- (Dis)charging C, L
- Resistor (always in)
- EM Radiation

For EM Radiation

Intensity: $I \equiv \langle S \rangle = \dfrac{E_0 B_0}{2\mu_0} = \dfrac{E_0^2}{2\mu_0 c} = \dfrac{c B_0^2}{2\mu_0}$

Circuits

There will be no quantitative circuit questions on the final and no questions regarding driven RLC Circuits

Only in the multiple choice will there be circuit type questions

BUT….

Circuit Elements

NAME	Value	V / ε	Power / Energy
Resistor	$R = \dfrac{\rho \ell}{A}$	IR	$I^2 R$
Capacitor	$C = \dfrac{Q}{\lvert \Delta V \rvert}$	$\dfrac{Q}{C}$	$\frac{1}{2} C V^2$
Inductor	$L = \dfrac{N \Phi}{I}$	$-L \dfrac{dI}{dt}$	$\frac{1}{2} L I^2$

Circuits

For "what happens just after switch is thrown":

Capacitor: Uncharged is short, charged is open

Inductor: Current doesn't change instantly!

Initially looks like open, steady state is short

RC & RL Circuits have "charging" and "discharging" curves that go exponentially with a time constant:

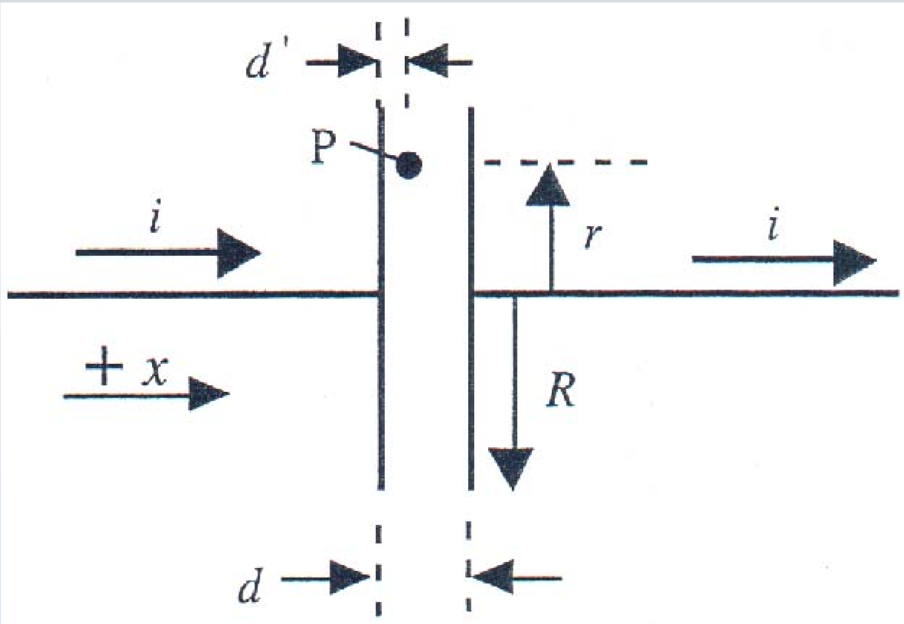
LC & RLC Circuits oscillate:

$$V, Q, I \propto \cos(\omega t) \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

SAMPLE EXAM

F2002 #5, S2003 #3, SFB#1, SFC#1, SFD#1

Problem 1: Gauss's Law

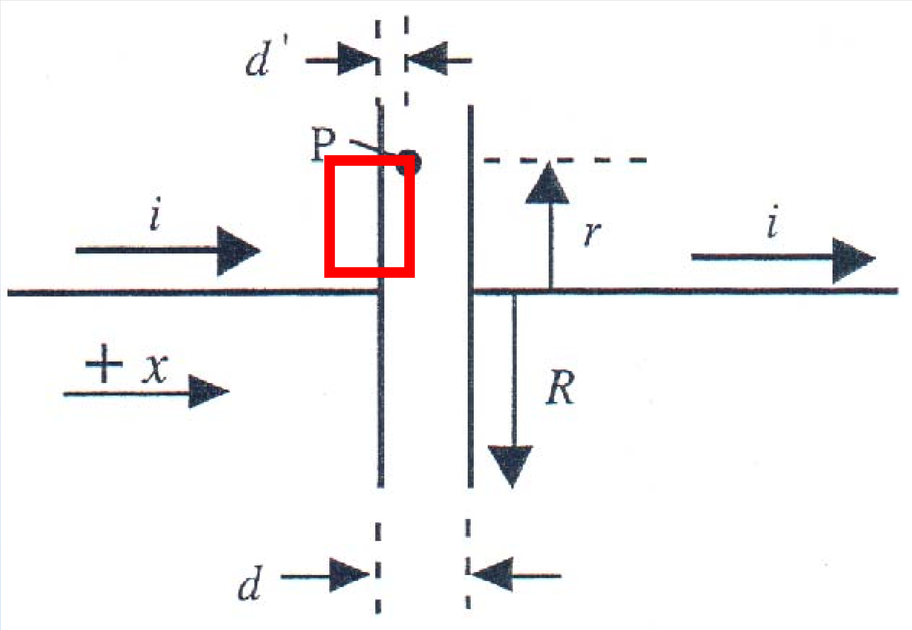


A circular capacitor of spacing *d* and radius *R* is in a circuit carrying the steady current *i* shown.

At time t=0 it is uncharged

1. Find the electric field **E**(t) at P vs. time t (mag. & dir.)
2. Find the potential at P, V(t), given that the potential at the right hand plate is fixed at 0
3. Find the magnetic field **B**(t) at P
4. Find the total field energy between the plates U(t)

Solution 1: Gauss's Law



1.Find the electric field **E**(t):

Assume a charge *q* on the left plate (-*q* on the right)

Gauss's Law:

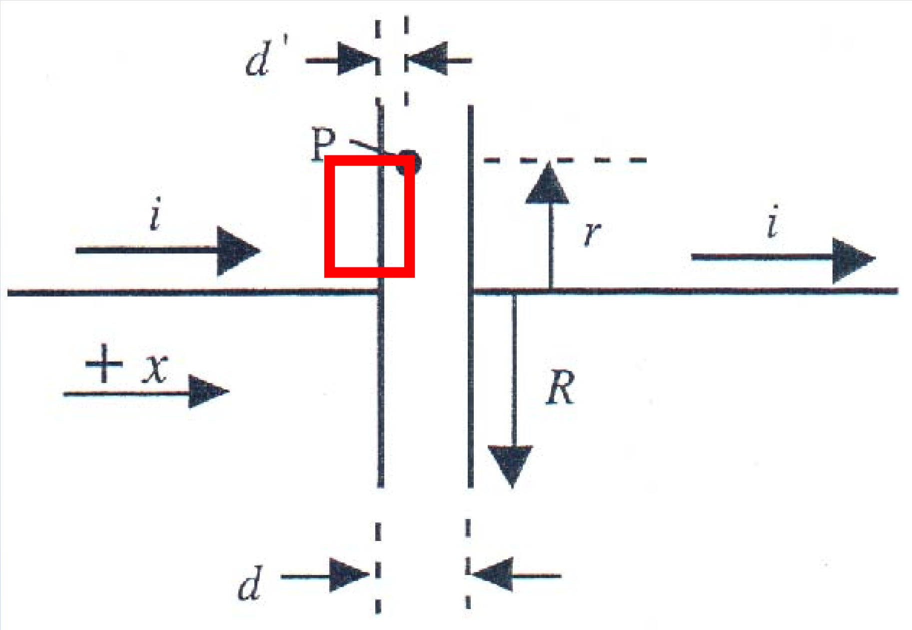
$$\oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = EA = \frac{Q_{in}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$$

$$E = \frac{\sigma}{\varepsilon_0} = \frac{q}{\pi R^2 \varepsilon_0}$$

Since $q(t=0) = 0$, $q = it$

$$\boxed{\vec{\mathbf{E}}(t) = \frac{it}{\pi R^2 \varepsilon_0} \text{ to the right}}$$

Solution 1.2: Gauss's Law



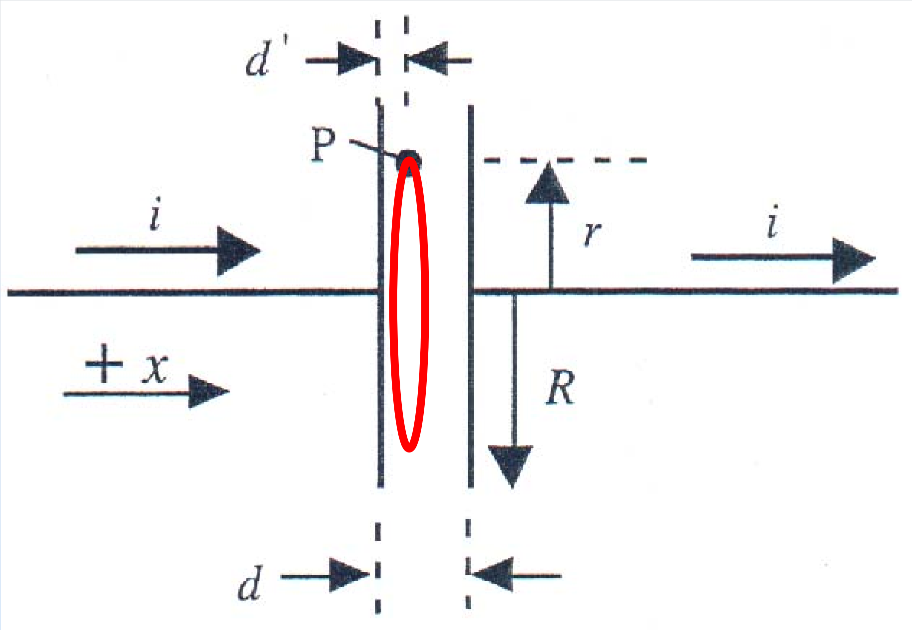
2.Find the potential V(t):

Since the E field is uniform,
V = E * distance

$$V(t) = \left|\vec{\mathbf{E}}(t)\right|\left(d - d'\right) = \frac{it}{\pi R^2 \varepsilon_0}\left(d - d'\right)$$

Check: This should be positive since its between a positive plate (left) and zero potential (right)

Solution 1.3: Gauss's Law



3.Find **B**(t):

Ampere's Law:

$$\oint_C \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{s}} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

$$\Phi_E = EA = \left(\frac{it}{\pi R^2 \varepsilon_0}\right)\pi r^2$$

$$2\pi r B = 0 + \mu_0 \varepsilon_0 \frac{r^2 i}{R^2 \varepsilon_0}$$

$$\frac{d\Phi_E}{dt} = \frac{r^2 i}{R^2 \varepsilon_0}$$

$$\vec{\mathbf{B}}(t) = \frac{\mu_0 i r}{2\pi R^2} \text{ out of the page}$$

Solution 1.4: Gauss's Law

4. Find **Total Field Energy** between the plates

E Field Energy Density: $u_E = \frac{\varepsilon_o E^2}{2} = \frac{\varepsilon_o}{2}\left(\frac{it}{\pi R^2 \varepsilon_0}\right)^2$

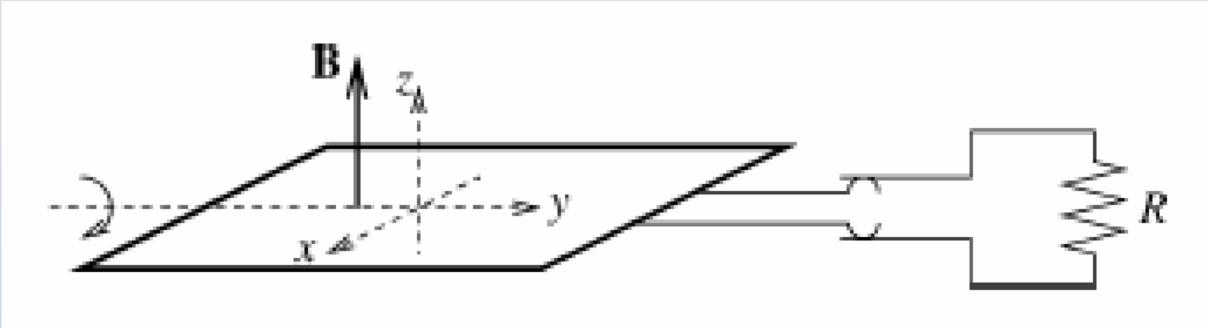
B Field Energy Density: $u_B = \frac{B^2}{2\mu_o} = \frac{1}{2\mu_o}\left(\frac{\mu_0 ir}{2\pi R^2}\right)^2$

Total Energy $U = \iiint (u_E + u_B)\, dV$ (Integrate over cylinder)

$$= \frac{\varepsilon_o}{2}\left(\frac{it}{\pi R^2 \varepsilon_0}\right)^2 \bullet \pi R^2 d + \frac{1}{2\mu_o}\left(\frac{\mu_0 i}{2\pi R^2}\right)^2 \int r^2 \bullet d \cdot 2\pi r \; dr$$

$$= \frac{(it)^2}{2}\frac{d}{\varepsilon_0 \pi R^2} + \frac{1}{2}\frac{\mu_o d}{8\pi} i^2 \qquad \left(= \frac{q^2}{2C} + \frac{1}{2} L i^2 \right)$$

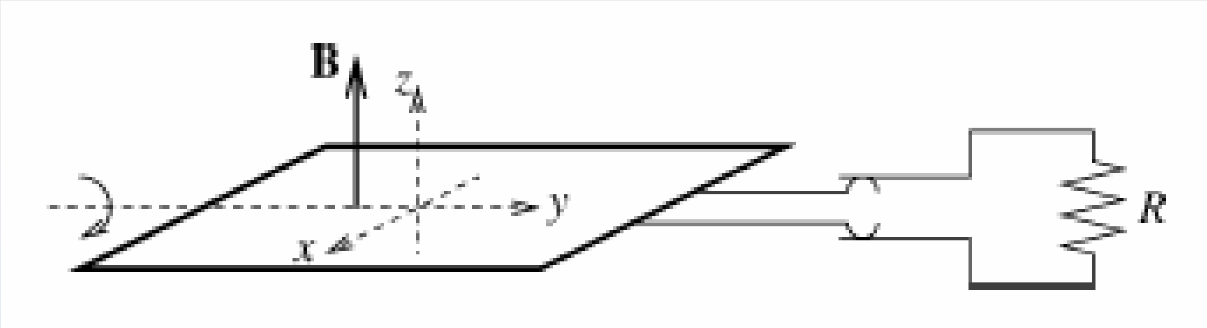
Problem 2: Faraday's Law



A simple electric generator rotates with frequency *f* about the y-axis in a uniform *B* field. The rotor consists of n windings of area S. It powers a lightbulb of resistance R (all other wires have no resistance).

1. What is the maximum value I_{max} of the induced current? What is the orientation of the coil when this current is achieved?
2. What power must be supplied to maintain the rotation (ignoring friction)?

Solution 2: Faraday's Law



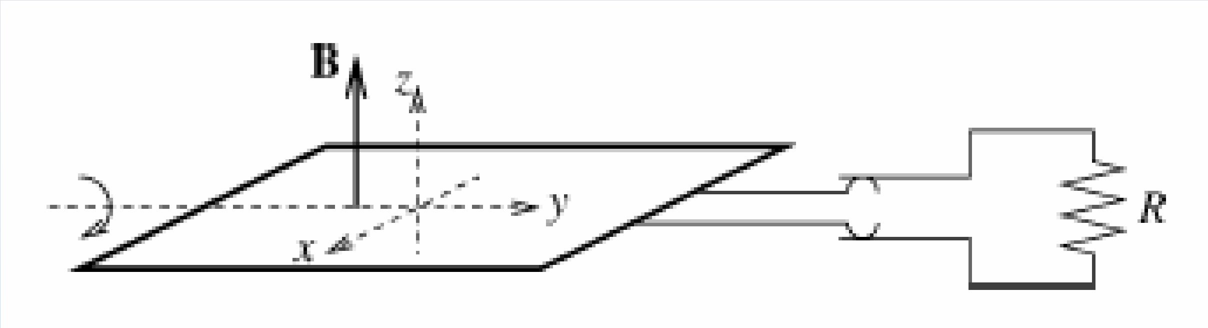
Faraday's Law: $\oint_C \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}$

$$I = \frac{\varepsilon}{R} = -\frac{1}{R}\frac{d\Phi_B}{dt} = -\frac{1}{R}\frac{d}{dt}\left(nBS\cos\left(\omega t\right)\right) = \frac{nBS}{R}\omega\sin\left(\omega t\right)$$

$$\boxed{I_{\max} = \frac{nBS}{R}\omega = \frac{nBS}{R}2\pi f}$$

Max when flux is changing the most – at 90º to current picture

Solution 2.2: Faraday's Law



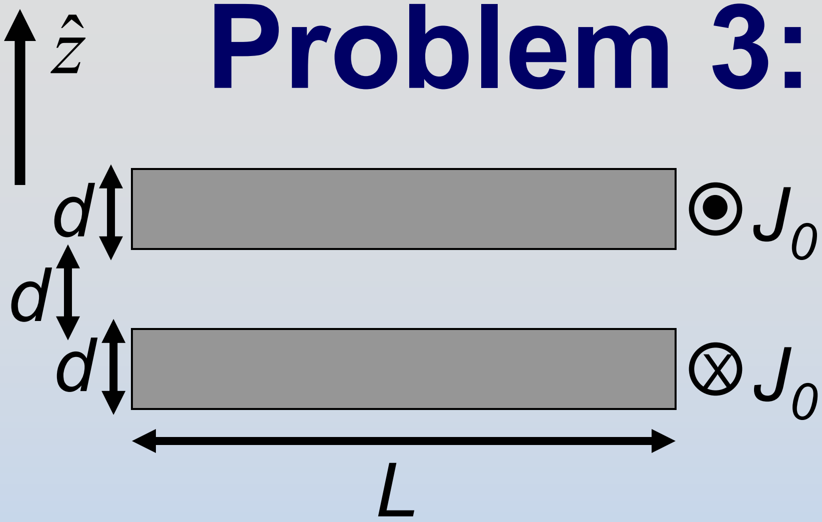
2. Power delivered?

Power delivered must equal power dissipated!

$$P = I^2 R = \left(\frac{nBS}{R} 2\pi f \sin(\omega t) \right)^2 R = R \left(\frac{nBS}{R} 2\pi f \right)^2 \sin^2(\omega t)$$

$$\langle P \rangle = \frac{R}{2} \left(\frac{nBS}{R} 2\pi f \right)^2$$

Problem 3: Ampere's Law



Consider the two long current sheets at left, each carrying a current density J_0 (out the top, in the bottom)

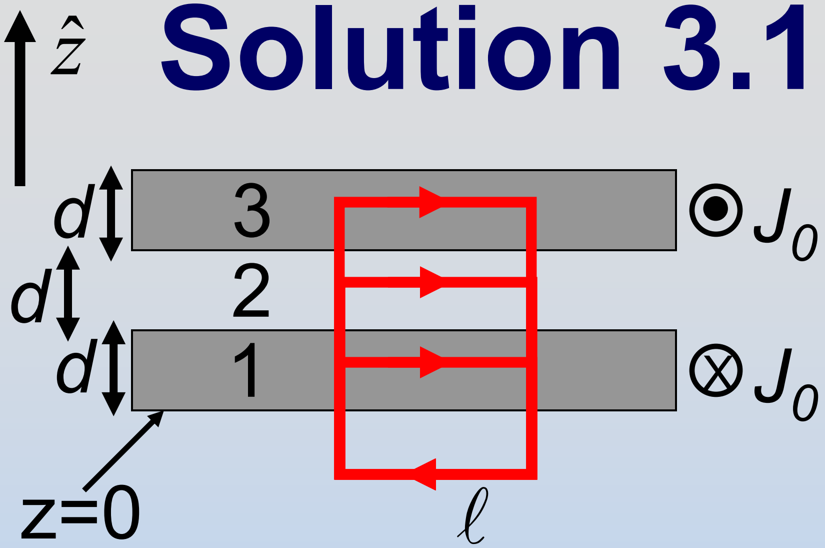
a) Use Ampere's law to find the magnetic field for all z. Make sure that you show your choice of Amperian loop for each region.

At t=0 the current starts decreasing: $J(t)=J_0 - at$

b) Calculate the electric field (magnitude and direction) at the bottom of the top sheet.

c) Calculate the Poynting vector at the same location

Solution 3.1: Ampere's Law



By symmetry, above the top and below the bottom the B field must be 0.

Elsewhere B is to right

Region 1:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc} \Rightarrow B\ell = \mu_0 J_0 z \ell \Rightarrow \boxed{B = \mu_0 J_0 z}$$

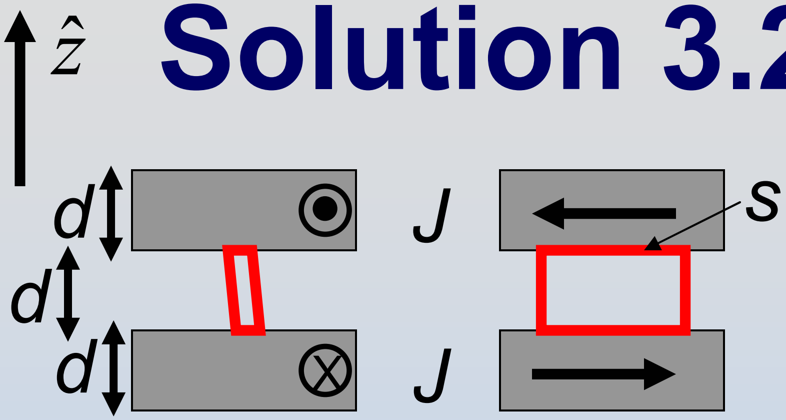
Region 2:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc} \Rightarrow B\ell = \mu_0 J_0 d \ell \Rightarrow \boxed{B = \mu_0 J_0 d}$$

Region 3:

$$B\ell = \mu_0 \left(J_0 d - J_0 \left(z - 2d\right)\right)\ell \Rightarrow \boxed{B = \mu_0 J_0 \left(3d - z\right)}$$

Solution 3.2: Ampere's Law



Why is there an electric field?

Changing magnetic field → Faraday's Law!

$$\oint_C \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}$$

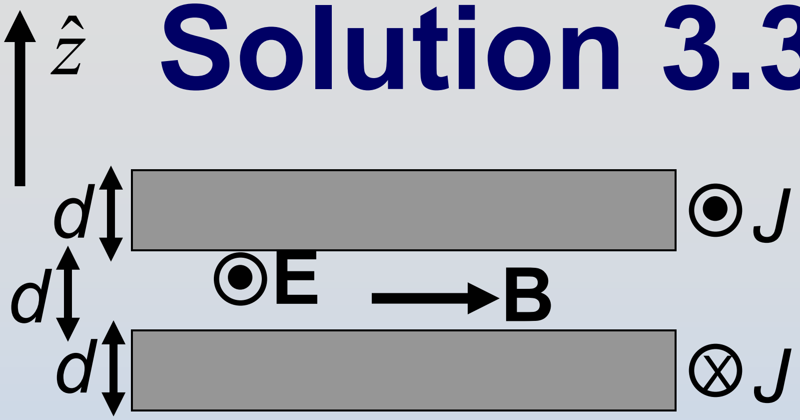
Use rectangle of sides *d, s* to find **E** at bottom of top plate

J is decreasing → B to right is decreasing → induced field wants to make B to right → E out of page

$$2sE = \frac{d}{dt}\left(Bsd\right) = sd\frac{d}{dt}\left(\mu_0 dJ\right) = sd^2\mu_0\frac{dJ}{dt}$$

$$\Rightarrow \vec{\mathbf{E}} = \tfrac{1}{2}d^2\mu_0 a \text{ out of page}$$

Solution 3.3: Ampere's Law



Recall

$$\vec{\mathbf{E}} = \tfrac{1}{4} d^2 \mu_0 a \text{ out of page}$$

$$\vec{\mathbf{B}} = \mu_0 J d \text{ to the right}$$

Calculate the Poynting vector (at bottom of top plate):

$$\vec{\mathbf{S}} = \frac{1}{\mu_0} \vec{\mathbf{E}} \times \vec{\mathbf{B}} = \frac{1}{\mu_0} \left(\tfrac{1}{4} d^2 \mu_0 a \right) \left(\mu_0 J d \right) \hat{\mathbf{z}}$$

That is, energy is leaving the system (discharging)

If this were a solenoid I would have you integrate over the outer edge and show that this = d/dt(1/2 LI^2)

Problem 4: EM Wave

The magnetic field of a plane EM wave is:

$$\vec{\mathbf{B}} = 10^{-9}\cos\left(\left(\pi\,\mathrm{m}^{-1}\right)y + \left(3\pi\times10^{8}\,\mathrm{s}^{-1}\right)t\right)\hat{\mathbf{i}}\ \text{Tesla}$$

(a) In what direction does the wave travel?
(b) What is the wavelength, frequency & speed of the wave?
(c) Write the complete vector expression for **E**
(d) What is the time-average energy flux carried in the wave? What is the direction of energy flow? (μ_o = 4 π x l0^{-7} in SI units; retain fractions and the factor π in your answer.)

Solution 4.1: EM Wave

$$\vec{\mathbf{B}} = 10^{-9}\cos\left(\left(\pi\,\mathrm{m}^{-1}\right)y + \left(3\pi\times10^{8}\,\mathrm{s}^{-1}\right)t\right)\hat{\mathbf{i}}\ \text{Tesla}$$

(a) Travels in the $-\hat{\mathbf{j}}$ direction (-y)

(b) $$k = \pi\,\mathrm{m}^{-1} \Rightarrow \lambda = \frac{2\pi}{k} = 2\,\mathrm{m}$$

$$\omega = 3\pi\times10^{8}\,\mathrm{s}^{-1} \Rightarrow f = \frac{\omega}{2\pi} = \frac{3}{2}\times10^{8}\,\mathrm{s}^{-1}$$

$$v = \frac{\omega}{k} = \lambda f = 3\times10^{8}\,\frac{\mathrm{m}}{\mathrm{s}}$$

(c)

$$\vec{\mathbf{E}} = -3\times10^{-1}\cos\left(\left(\pi\,\mathrm{m}^{-1}\right)y + \left(3\pi\times10^{8}\,\mathrm{s}^{-1}\right)t\right)\hat{\mathbf{k}}\ \text{V/m}$$

Solution 4.2: EM Wave

$$\vec{\mathbf{B}} = 10^{-9}\cos\left(\left(\pi\,\mathrm{m}^{-1}\right)y + \left(3\pi\times10^{8}\,\mathrm{s}^{-1}\right)t\right)\hat{\mathbf{i}}\ \text{Tesla}$$

(d) $$\left\langle \vec{\mathbf{S}} \right\rangle = \left\langle \frac{1}{\mu_0}\vec{\mathbf{E}}\times\vec{\mathbf{B}} \right\rangle$$

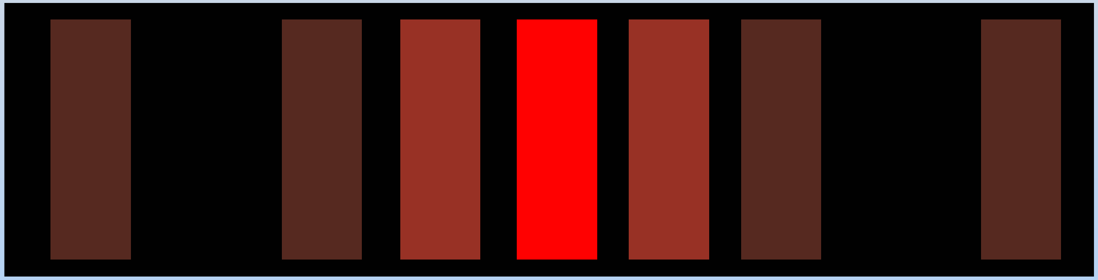
S points along direction of travel: - $\hat{\mathbf{j}}$

$$= \frac{1}{2}\frac{1}{\mu_0}E_0 B_0$$

$$= \frac{1}{2}\left(\frac{1}{4\pi\times10^{-7}}\right)\left(3\times10^{-1}\right)\left(10^{-9}\right)\frac{\mathrm{W}}{\mathrm{m}^2\mathrm{s}}$$

Problem 5: Interference

In an experiment you shine red laser light (λ=600 nm) at a slide and see the following pattern on a screen placed 1 m away:

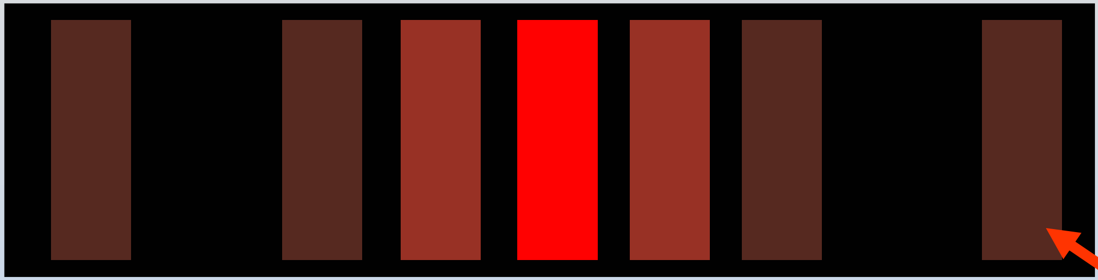


You measure the distance between successive fringes to be 20 mm

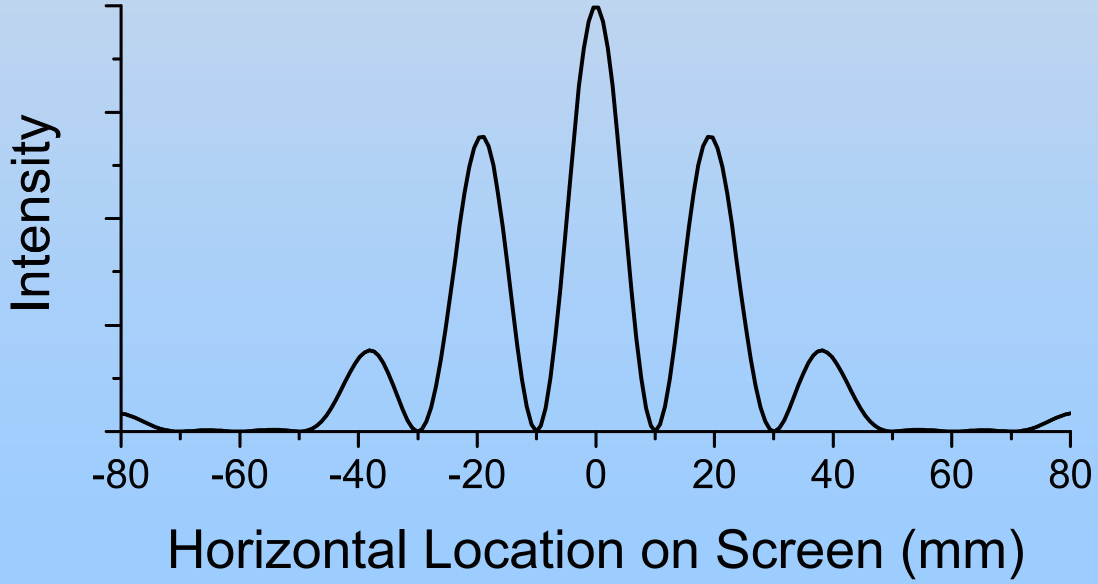
a) Are you looking at a single slit or at two slits?

b) What are the relevant lengths (width, separation if 2 slits)? What is the orientation of the slits?

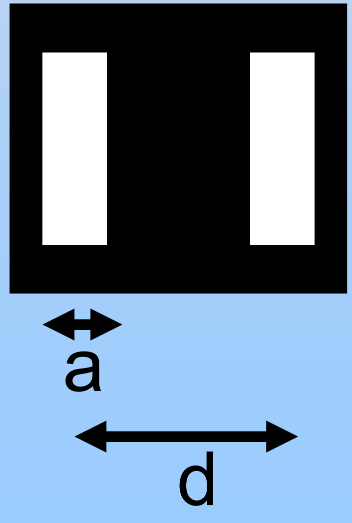
Solution 5.1: Interference



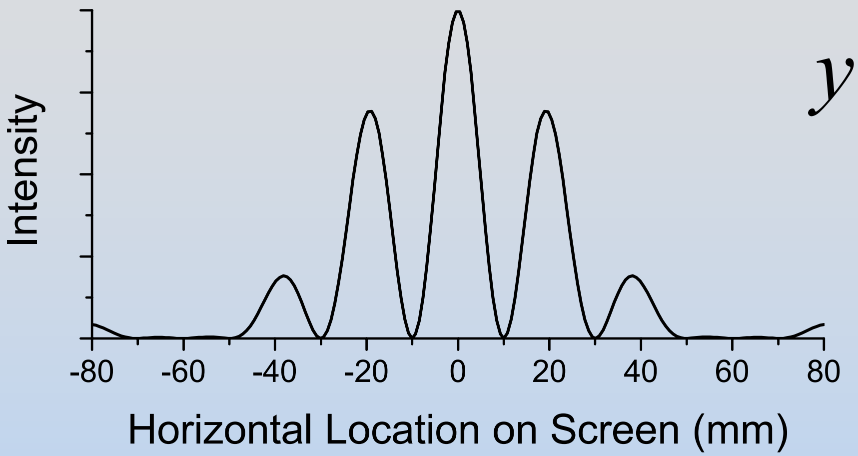
First translate the picture to a plot:

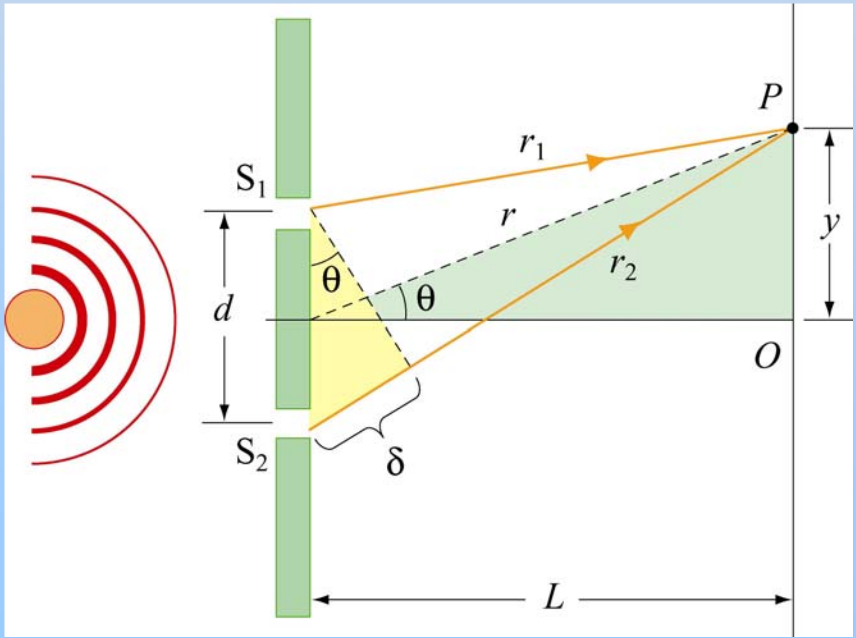


(a) Must be two slits



Solution 5.2: Interference





$$y = L\tan\theta \approx L\sin\theta = L\frac{m\lambda}{d}$$

$$d = L\frac{m\lambda}{y} = (1\text{m})\frac{(1)(600\text{nm})}{(20\text{mm})}$$

$$= (1\text{m})\frac{\left(6\times10^{-7}\right)}{\left(2\times10^{-2}\right)} = \boxed{3\times10^{-5}\text{m}}$$

<u>At 60 mm…</u>

$$a\sin\theta = (1)\lambda$$

$$d\sin\theta = (3)\lambda$$

$$\Rightarrow \frac{a}{d} = \frac{1}{3}$$

$$\boxed{a = 10^{-5}\text{m}}$$

Why is the sky blue?

400 nm Wavelength 700 nm

Small particles preferentially scatter small wavelengths

You also might have seen a red moon last fall – during the lunar eclipse.

When totally eclipsed by the Earth the only light illuminating the moon is diffracted by Earth's atmosphere