Experiment 10: *LR* and Undriven *LRC* Circuits

OBJECTIVES

1. To determine the inductance L and internal resistance R_L of a coil, both with and without an iron core, on the AC/DC Electronics Lab circuit board.

2. To observe electrical oscillations, measure their frequencies, and verify energy relationships in an *LRC* circuit.

INTRODUCTION

(Note: For these lab instructions, and most other uses, the terms "*RLC*" and "*LRC*" are entirely interchangeable. The other four permutations are not as popular.)

Free Oscillations in *LR* and *RLC* circuits

(For purposes of clarity, these experiment instructions will use lower-case letters, $q(t)$ and $i(t)$ to denote time-varying circuit quantities.)
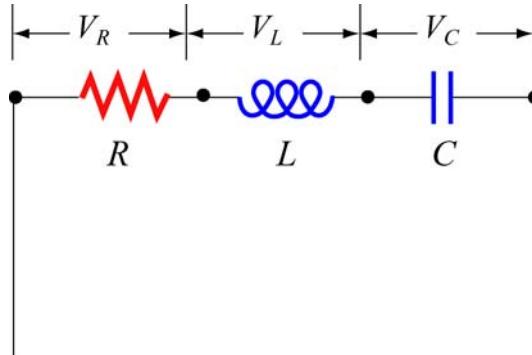
Consider a series *RLC* circuit shown in Figure 15.1.

Figure 15.1 *RLC* Circuit with external voltage removed

Applying the Kirchhoff's voltage rule, the circuit equation for the RLC circuit without any external voltage is

$$0 = L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C}. \tag{10.1}$$

In this experiment, the internal resistance R_L of the inductor is the only resistance. In such cases, when the resistance and inductance are "lumped," a circuit diagram similar to Figure 1 is used for clarity; for our purposes, will use the "total resistance" $R_T = R_L$.

For the situation where the capacitance is not part of the circuit, the current in an *LR* circuit is derived in the 8.02 *Course Notes*, Section 11.4, and given in Equations (11.4.7) and (11.4.13);

$$i(t) = \frac{\mathcal{E}}{R_T}\left(1 - e^{-t/\tau}\right)$$

for an increasing current and

$$i(t) = \frac{\mathcal{E}}{R_T} e^{-t/\tau},$$

for a decreasing current, where the time constant is $\tau = L/R_T$.

For a circuit with capacitance, with appropriately small resistance ("underdamped"), the current can be represented as

$$i(t) = I_0 e^{-\gamma t} \cos \omega t$$

where $\gamma = R_T/2L$, $\omega^2 = \frac{1}{LC} - \frac{R_T^2}{4L^2}$ (see the 8.02 *Course Notes*, **Section 11.6** for a derivation). A plot of $i(t)$ for a typical underdamped circuit is shown in Figure 1.
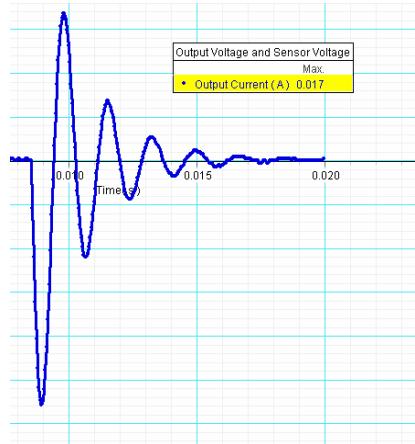

Figure 1 Exponential decay of current oscillations in a typical *RLC* circuit

Since the coefficient of exponential decay, $\gamma = R_T/2L$, is proportional to the resistance, we see that the current will fall off more rapidly as the resistance increases.

Energy Relationships in *RLC* circuits

As the current oscillates in such circuits, energy may be stored in both the magnetic field of the inductor

$$U_B = \frac{1}{2}Li^2 \tag{10.2}$$

 and in the electric field of the capacitor

$$U_E = \frac{1}{2}CV_C^2 = \frac{1}{2}\frac{q^2}{C}. \tag{10.3}$$

The energy stored in the electric and magnetic fields is simply the sum:

$$U = \frac{1}{2}Li^2 + \frac{1}{2}CV_C^2 \tag{10.4}$$

However, this energy is gradually being lost as heat in the resistor at the rate i^2R_T. Over one period of oscillation T, the dissipated energy is

$$\Delta U = -\left(U(t) - U(t=0)\right) = \int_0^T i^2 R_T \, dt. \tag{10.5}$$

EXPERIMENTAL SETUP

A. Computer

If it is not done already, connect the *Science Workshop* 750 Interface to the computer using the SCSI cable. Connect the power supply to the 750 Interface and turn on the interface power. Always turn on the interface before powering up the computer. Turn on your computer.

B. AC/DC Electronics Lab circuit board

Connect the black banana plug cord from the OUTPUT ground port of the 750 Interface to the banana jack located in the lower right corner on the AC/DC Electronics Lab circuit board. Connect the red banana jack with alligator clip to the positive OUTPUT port of the 750 Interface. In a later part of the experiment, you will also connect the Voltage Sensor to measure the capacitor voltage. For this experiment, in which all of the circuit elements are in series, you will be able to measure and record the Output Current from the 750, so the Current Sensor is not part of this experiment.

MEASUREMENTS

Part 1. Resistance and Inductance of the Coil

Resistance:

Circuit Diagram: Connect the red alligator clip to the right side of your coil. Using a wire, connect the left side of your coil to the banana jack that is connected to the OUTPUT ground port (black) of the 750 Interface (see Figure 2).
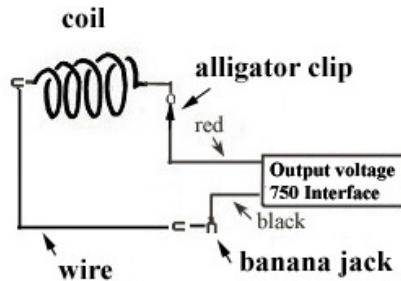

Figure 2 Circuit diagram for measuring the resistance of the coil

DataStudio **File:** Download and open the *Data Studio* file **exp10.ds**. There are more display windows than usual for this experiment; you'll want to minimize or display the windows as you choose. Resize the windows as desired. Not all windows are needed for all parts.
If the following settings have not been made, you will have to change them:

- The Signal Generator should be set to a Positive Square Wave with a frequency of 20 Hz and amplitude of 1 volt. The Sample Rate should be 10000 Hz.

- A graph has been set up to display the Signal Generator Voltage and Output Current.

- Click Start. The Output Current graph displays the familiar behavior of an *LR* circuit (see Figure 3). The maximum voltage and maximum current are recorded internally, and their ratio is given in a Coil Resistance display window.
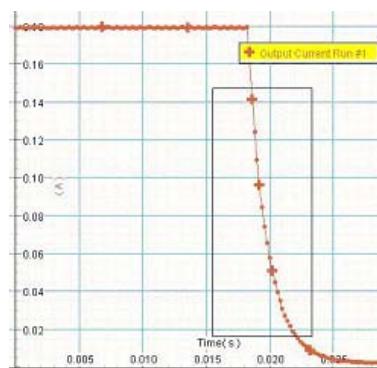

Figure 3 Exponential decay of current in *LR* circuit

Question 1: Record the resistance R_L of your coil on the tearoff sheet.

Inductance (with and without an iron core)

DATA ANALYSIS

When the Positive Square Wave voltage switches to 0 volts, the total resistance is $R_T = R_L$. The current in the circuit decays exponentially and is given by

$$i(t) = i_0 e^{-(L/R_T)t} \,,$$ (10.6)

where $i_0 = \mathcal{E} / R_T$ is the current in the inductor at the time when the voltage drops to zero.

Taking the natural logarithm of both sides of the above equation gives

$$\ln(i) = -(R_T/L)t + \ln(i_0),$$ (10.7)

which means that a graph of $\ln(i)$ $vs. t$ has a slope equal to $-(R_T/L)$.

A display window which shows the logarithm of the current as a function of time is part of the *DataStudio* activity.

- Bring up this window and identify the part of the plot that is linear. Bring up the Coil Inductance window.
- Use the Zoom Select tool (fourth button from the left) to analyze the linear part of the logarithm plot and then select Linear Fit from the Fit menu option (this may already be done). Move the boundary of the selected region until the best-fit line matches the data as well as possible The slope of this fit line is determined by *DataStudio* and the calculation of the inductance, using the value of R_T calculated by *DataStudio*, is done internally and displayed in the Coil Inductance window. You should see that as you vary the selected data, the displayed calculated value of the inductance changes. When this value changes only slightly as you change the selected data, you will have selected a part of the data for which the linear fit is best. Record this calculated inductance.

- Delete any previous data runs. Insert the iron core into the coil and repeat the above measurements. In order to have the current reach its maximum, you will want to change the frequency from 20 Hz to 15 Hz . You'll know that the current has reached the maximum value if the calculated resistance is the same as that without the coil.

Question 2: Record the value of the inductance with and without the core and record your results on the tearoff sheet.

Part 2: Free Oscillations of the *RLC* Circuit

Circuit Diagram: Now put a $C = 10\,\mu\text{F}$ capacitor in series with the coil without its core, as shown in Figure 4; the circuit resistance will then be the coil resistance found in Part 1. The Voltage Sensor data will be used in Part 3; the circuit itself for Parts 2 and 3 is the same.
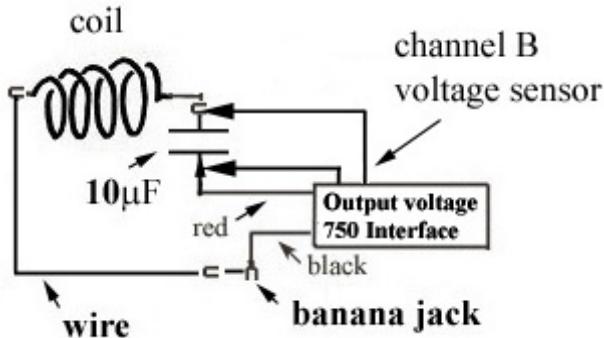

Figure 4 Circuit diagram for the *RLC* circuit

- Delete the previous data runs. Repeat the above procedure to record the current and voltages for this *RLC* circuit. Your current as a function of time graph should resemble Figure 1.
- Use the Zoom Select tool to highlight the part of the data you want to analyze.
- Determine the period *T* of these oscillations by using the SmartTool to measure the time interval between when the current is zero. The period is *twice* the time interval between successive zeroes.
- The SmartTool is not that smart, in that the precision of the time measurement is limited. You may find it more convenient to measure the time for several periods and then divide by the number of periods.

Question 3: Calculate the frequency, $f = 1/T$, of these oscillations and record your results on the tearoff sheet.

Question 4: For "small" values of resistance ($R_T \ll \sqrt{L/C}$), the oscillation frequency is approximately $f \simeq 1/\left(2\pi\sqrt{LC}\right)$. For your circuit parameters, compute the expected value of f and compare it to your measured value. Do you expect your result to be greater, equal, or less than the measured value? Answer on the tearoff sheet.

Part 3: Observe the Energy in the *RLC* Circuit

The *Data Studio* file **exp10.ds** should have a window in which the predicted energies in the capacitor and inductor, and the total electromagnetic energy, are displayed.
[**The calculated energies assume the values** $L = 8.4\,\text{mH}$ **and** $C = 10\,\mu\text{F}$ **; if your circuit parameters are very different you should change these numbers in the calculator window.**]

In this "Energies" window, the energies are plotted. Because the energies are very small, they have been multiplied by a million, that is, the units are micro-joules. You will want to expand the scale, using the Zoom Select tool. Figure 5 shows an example of what you should expect to find.
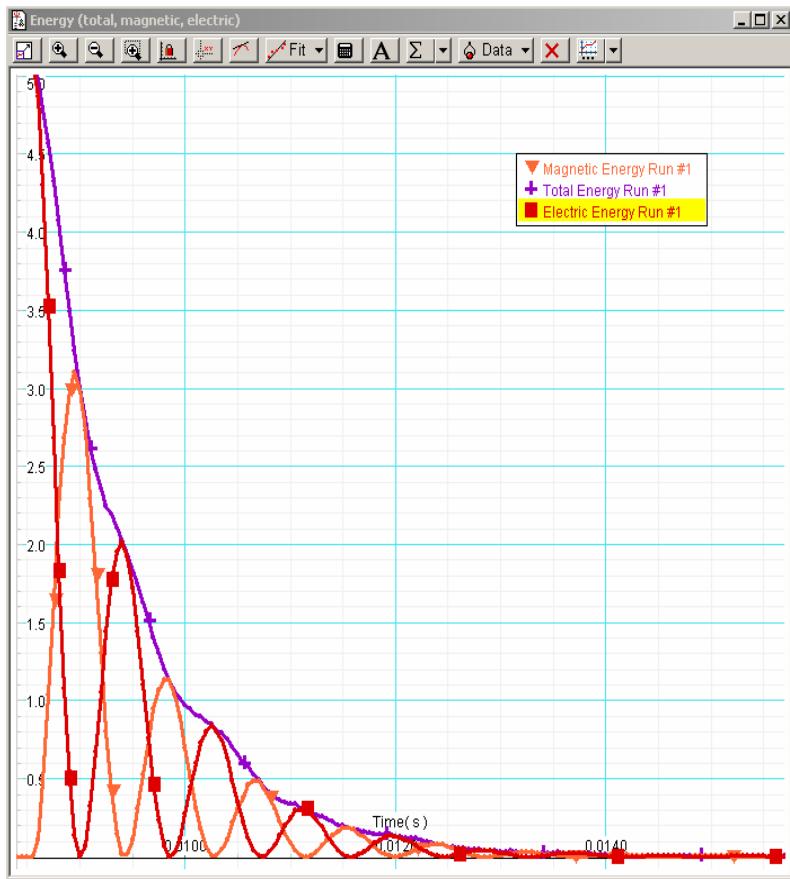

Figure 5 Energy in an *RLC* circuit

Question 5 (*answer on the tearoff sheet*): The circuit is losing energy most rapidly at times when the graph of total energy is steepest; these times occur at about the same times that the magnetic energy reaches a local maximum. Briefly explain why.

OPTIONAL:

Without deleting the previous data run, insert the iron core and take another data run. Note the similarities and differences in the current graphs.
The energy graph will be messy. Delete the previous runs. In the Calculator window, set the value of "L" to that of the coil with the iron core inserted. Make sure you click "Accept" for the new value of the inductance. Take another data run and look at the energy graph. Note the similarities and differences in the graphs with and without the core.

Tear off this page and turn it in at the end of class.
Note: **Writing in the name of a student who is not present is a Committee on Discipline offense.**

Experiment Summary 10: Undriven *RLC* Circuit

Group and Section _____ **(e.g. 10A, L02: Please Fill Out)**

Names _____

_____

_____

MEASUREMENTS

Part 1. Resistance and Inductance of the Coil

Question 1: Measure and record the resistance R_L of your coil.

Coil Resistance: $R_L = $ ____ Ω

Question 2: Record the inductance of your coil, with and without the iron core:

Coil inductance without core: $L = $ ____ mH

Coil inductance with core: $L = $ ____ mH

Part 2: Free Oscillations of the *RLC* circuit

Question 3: Calculate the frequency, $f = 1/T$, of these oscillations and record your results here.

Answer: $f =$ ____ Hz

Question 4: For your circuit parameters, compute the expected value of $f_{predicted}$ and compare it to your measured value. Do you expect your result to be greater, equal, or less than the measured value?

Answer: $f_{predicted} =$ ____ Hz $f_{predicted} - f =$ ____ Hz

Part 3: Observe the Energy in the *RLC* Circuit

Question 5: The circuit is losing energy most rapidly at times when the graph of total energy is steepest; these times occur at about the same times that the magnetic energy reaches a local maximum. Briefly explain why.

Answer: