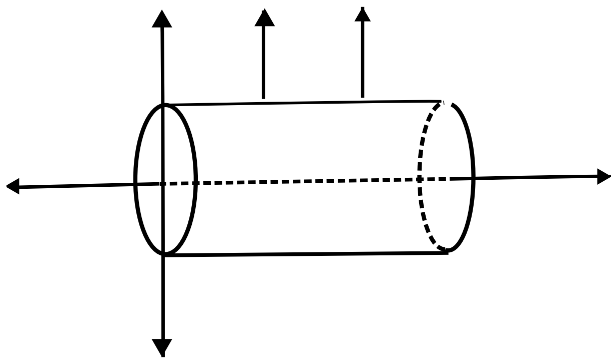
8.022 Lecture Notes Class 9 - 09/20/2006

Given line of charge with density x

Line Charge

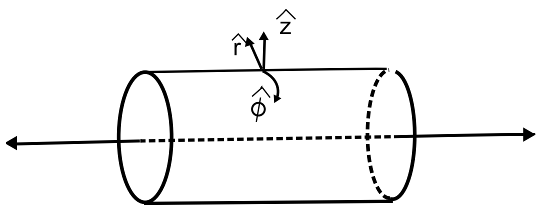
Find $\vec{E}(\vec{r})$ everywhere



$$\int_S \vec{E} \cdot d\vec{a} = \frac{1}{\epsilon_0} Q_{enclosed} \qquad Q_{enclosed} = \lambda L$$

$$L 2\pi r \cdot \vec{E}(\vec{r}) = \frac{\lambda L}{\epsilon_0}$$

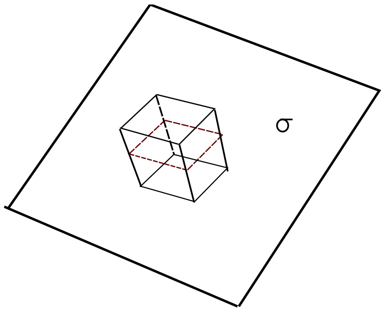
$$\vec{E}(\vec{r}) = \frac{\lambda}{2\pi\epsilon_0 r}\hat{r}$$



$E(r)\alpha\frac{1}{r}$

No $\hat{\phi}$ because $\vec{\nabla} \times \vec{E} = 0$.
No $\hat{z}$ because vectors cancel out
$E(r) \propto \frac{1}{r}$
Like a plane : Cross sections reveal that field lines spread only perpendicular to line.

N-dimensional $\rightarrow \frac{1}{r^{N-1}}$(Surface area of N-D object is (N-1)D)



Plane Charge

$$Q_{enclosed} = \sigma \cdot A^2$$

$$\int \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\epsilon_0}$$

$$\oint\!\!\oint\!\!\oint_S \vec{E} \cdot dA = \frac{\sigma \cdot A^2}{\epsilon_0}$$

$$\int_0^A \int_0^A \hat{z}\vec{E}(\vec{r})dxdy + \int_0^A \int_0^A -\vec{E}(\vec{r})(-\hat{z})dxdy = \frac{\sigma \cdot A^2}{\epsilon_0}$$

$$2A^2 E_z(\vec{r}) = \frac{\sigma A^2}{\epsilon_0}$$

$$E_z(\vec{r}) = \frac{\sigma}{2\epsilon_0}\vec{E} = \hat{n} \cdot \frac{\sigma}{2\epsilon_0}$$

Common Electric Fields

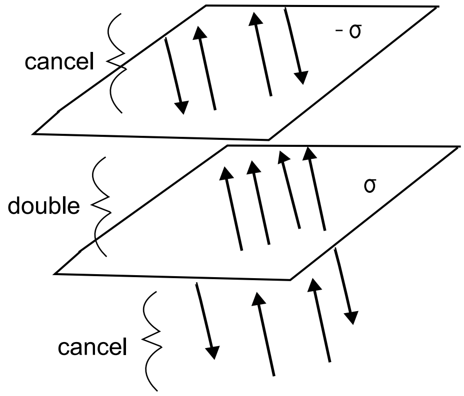
Line:

$$\vec{E}(\vec{r}) = \frac{\lambda}{2\pi\epsilon_0 r}\hat{r}$$

Plane:

$$\vec{E}(\vec{r}) = \hat{n} \cdot \frac{\sigma}{2\epsilon_0}$$

Double Plane:



$$\vec{E}(\vec{r}) = \hat{n} \cdot \frac{\sigma}{\epsilon} \qquad \text{(inside)}$$
$$\vec{E}(\vec{r}) = \vec{0} \qquad \text{(outside)}$$

$$\int_{\vec{a}}^{\vec{b}} \vec{E} \cdot d\vec{l} \qquad \vec{E} = \tfrac{1}{4\pi\epsilon_0}\tfrac{q}{r^2}\hat{r}$$
$$= \quad \tfrac{1}{4\pi\epsilon_0}\int_{\vec{a}}^{\vec{b}} \tfrac{q}{r^2} dr$$
$$= \ -\tfrac{1}{4\pi\epsilon_0}\left(\tfrac{q}{r(\vec{b})} - \tfrac{q}{r(\vec{a})}\right)$$

$$\oint \vec{E} \cdot d\vec{l} = 0 \qquad \text{because } r(\vec{b}) = r(\vec{a})$$

$$\Rightarrow \vec{\nabla} \times \vec{E} = \vec{0}$$