

Topics: Gauss's Law

Related Reading:

Course Notes (Liao et al.):	Chapter 4
Serway & Jewett:	Chapter 24
Giancoli:	Chapter 22

Topic Introduction

In today's class we will get more practice using Gauss's Law to calculate the electric field from highly symmetric charge distributions. Remember that the idea behind Gauss's law is that, pictorially, electric fields flow out of and into charges. If you surround some region of space with a closed surface (think bag), then observing how much field "flows" into or out of that surface (the flux) tells you how much charge is enclosed by the bag. For example, if you surround a positive charge with a surface then you will see a net flow outwards, whereas if you surround a negative charge with a surface you will see a net flow inwards.

Note: There are only three different symmetries (spherical, cylindrical and planar) and a couple of different types of problems which are typically calculated of each symmetry (solids – like the ball and slab of charge done in class, and nested shells). I strongly encourage you to work through each of these problems and make sure that you understand how to choose your Gaussian surface and how much charge is enclosed.

Electric Flux

For any flat surface of area A , the flux of an electric field \vec{E} through the surface is defined as $\Phi_E = \vec{E} \cdot \vec{A}$, where the direction of \vec{A} is normal to the surface. This captures the idea that the "flow" we are interested in is *through* the surface – if \vec{E} is parallel to the surface then the flux $\Phi_E = 0$.

We can generalize this to non-flat surfaces by breaking up the surface into small patches which are flat and then integrating the flux over these patches. Thus, in general:

$$\Phi_E = \iint_{S_{\square}} \vec{E} \cdot d\vec{A}$$

Gauss's Law

Recall that Gauss's law states that the electric flux through any closed surface is proportional to the total charge enclosed by the surface, or mathematically:

$$\Phi_{E_{\square}} = \oiint_{S_{\square}} \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0}$$

Symmetry and Gaussian Surfaces

Symmetry	System	Gaussian Surface
Cylindrical	Infinite line	Coaxial Cylinder
Planar	Infinite plane	Gaussian "Pillbox"
Spherical	Sphere, Spherical shell	Concentric Sphere

Although Gauss's law is always true, as a tool for calculation of the electric field, it is only useful for highly symmetric systems. The reason that this is true is that in order to solve for the electric field \vec{E} we need to be able to "get it out of the integral." That is, we need to work with systems where the flux integral can be converted into a simple multiplication. This can only be done if the electric field is piecewise constant – that is, at the very least the electric field must be constant across each of the faces composing the Gaussian surface. Furthermore, in order to use this as a tool for calculation, each of these constant values must either be E , the electric field we are trying to solve for, or a constant which is known (such as 0). This is important: in choosing the Gaussian surface you should not place it in such a way that there are two different unknown electric fields leading to the observed flux.

Solving Problems using Gauss's law

- (1) Identify the symmetry associated with the charge distribution, and the associated shape of "Gaussian surfaces" to be used.
- (2) Divide the space into different regions associated with the charge distribution, and determine the exact Gaussian surface to be used for each region. The electric field must be constant and either what we are solving for or known (i.e. zero) across the Gaussian surface.
- (3) For each region, calculate q_{enc} , the charge enclosed by the Gaussian surface.
- (4) For each region, calculate the electric flux $\Phi_{E\Box}$ through the Gaussian surface.
- (5) Equate $\Phi_{E\Box}$ with $q_{\text{enc}} / \epsilon_0$, and solve for the electric field in each region.

Important Equations

Electric flux through a surface S :

$$\Phi_{E\Box} = \iint_{S\Box} \vec{E} \cdot d\vec{A}$$

Gauss's law:

$$\Phi_{E\Box} = \oiint_{S\Box} \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0}$$