MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Problem Solving 3: Gauss's Law

REFERENCE: Section 4.2, 8.02 Course Notes.

Introduction

When approaching Gauss's Law problems, we described a problem solving strategy summarized below (see also, Section 4.7, 8.02 Course Notes):

Summary: Methodology for Applying Gauss's Law

Step 1: Identify the 'symmetry' properties of the charge distribution.

Step 2: Determine the direction of the electric field

Step 3: Decide how many different regions of space the charge distribution determines

For each region of space…

Step 4: Choose a Gaussian surface through each part of which the electric flux is either constant or zero

Step 5: Calculate the flux through the Gaussian surface (in terms of the unknown $\boldsymbol{E}$)

Step 6: Calculate the charge enclosed in the choice of the Gaussian surface

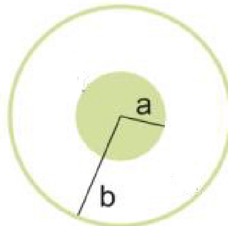
Step 7: Equate the two sides of Gauss's Law in order to find an expression for the magnitude of the electric field

Then…

Step 8: Graph the magnitude of the electric field as a function of the parameter specifying the Gaussian surface for all regions of space.

You should now apply this strategy to the following problem.

Question: ***Concentric Cylinders***



A long very thin non-conducting cylindrical shell of radius b and length L surrounds a long solid non-conducting cylinder of radius a and length L with $b > a$. The inner cylinder has a uniform charge $+Q$ distributed throughout its volume. On the outer cylinder we place an equal and opposite to charge, $-Q$. The region $a < r < b$ is empty.

Step 1 Question: (*Answer on the tear-sheet at the end!*) What is the 'symmetry' property of the charge distribution here?

Step 2 Question: (*Answer on the tear-sheet at the end!*) What is the direction of the electric field?

Step 3 Question: (*Put your answer on the tear-sheet at the end!*) How many different regions of space does the charge distribution determine (in other words, how many different formulae for **E** are you going to have to calculate?)

Step 4 Question: (*Put your answer on the tear-sheet at the end!*) For each region of space, describe your choice of a Gaussian surface. What variable did you choose to parameterize your Gaussian surface? What is the range of that variable?

Step 5 Question: (*Put your answer on the tear-sheet at the end!*) For the region for $r < a$, calculate the flux through your choice of the Gaussian surface. Your expression should include the unknown electric field for that region.

Step 6 Question: (*Put your answer on the tear-sheet at the end!*) For the region for $r < a$, write the charge enclosed in your choice of Gaussian surface (this should be in terms of Q, r & a, NOT **E**).

Step 7 Question 1: (*Put your answer on the tear-sheet at the end!*) For the region for $r < a$, equate the two sides of Gauss's Law that you calculated in steps 5 and 6, in order to find an expression for the magnitude of the electric field.

Step 7 Question 2: (*Put your answer on the tear-sheet at the end!*) Repeat the same procedure in order to calculate the electric field as a function of r for the regions $a < r < b$.

Step 8 Question: (*Put your answer on the tear-sheet at the end!*) Make a graph in the space below of the magnitude of the electric field as a function of the parameter specifying the Gaussian surface for all regions of space.

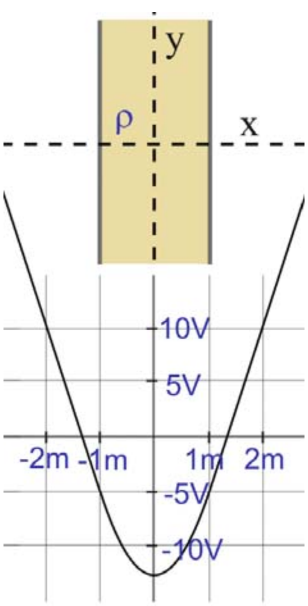
Penultimate Question (*Put your answer on the tear-sheet at the end!*) What is the potential difference between $r = a$ and $r = 0$? That is, what is $\Delta V = V(a) - V(0)$?

Final Question: (*Put your answer on the tear-sheet at the end!*) What is the potential difference between $r = b$ and $r = a$? That is, what is $\Delta V = V(b) - V(a)$?

Sample Exam Question (If time, try to do this by yourself, closed notes)

An semi-infinite (infinite in y- and z-, bounded in x) slab of charge carries a charge per unit volume ρ. The lower plot shows the electric potential $V(x)$ due to this slab as a function of horizontal distance x from the center of the slab. It is linear for $x < -1$ m and $x > 1$ m, and given by $V(x) = \frac{15}{2}x^2 - \frac{25}{2}$ for -1 m $< x < 1$ m.

(a) What is the x-component of the electric field in the region $x < -1$ m?
(b) What is the x-component of the electric field in the region $x > 1$ m?
(c) What is the x-component of the E field in the region -1 m $< x < 1$ m?
(d) Use Gauss's Law and your answers above to find the charge density ρ of the slab.



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Tear off this page and turn it in at the end of class !!!!

Note:
Writing in the name of a student who is not present is a COD offense.

Group ____________________________________ **(e.g. L02 10A Please Fill Out)**

Names ____________________________________

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Problem Solving 3: Using Gauss's Law

Step 1 Question: What is the 'symmetry' property of the charge distribution here?

Step 2 Question: What is the direction of the electric field?

Step 3 Question: How many different regions of space does the charge distribution determine?

Step 4 Question: For each region of space, describe your choice of the Gaussian surface. What variable did you choose to parameterize your Gaussian surface? What is the range of that variable?

Step 5 Question: For the region for $r < a$, calculate the flux through your choice of the Gaussian surface. Your expression should include the unknown electric field for that region.

Step 6 Question: For the region for $r < a$, calculate the charge enclosed in your choice of the Gaussian.

Step 7 Question 1: For the region for $r < a$, equate the two sides of Gauss's Law that you calculated in steps 5 and 6, in order to find an expression for the magnitude of the electric field.

Step 7 Question 2: Repeat the same procedure in order to calculate the electric field as a function of r for the regions $a < r < b$.

Step 8 Question: Make a graph in the space below of the magnitude of the electric field as a function of the parameter specifying the Gaussian surface for all regions of space.

Penultimate Question: What is the potential difference between $r = a$ and $r = 0$? That is, what is $\Delta V = V(a) - V(0)$?

Final Question: What is the potential difference between $r = b$ and $r = a$? That is, what is $\Delta V = V(b) - V(a)$?