

Topics: Mutual Inductance & Transformers; Inductors

Related Reading:

Course Notes (Liao et al.): Chapter 10; Sections 11.1 – 11.4

Topic Introduction

We do a problem solving session on Faraday's Law.

Faraday's Law & Lenz's Law

Recall: Faraday's Law says that a changing magnetic flux generates an EMF $\mathcal{E} = -d\Phi_B/dt$

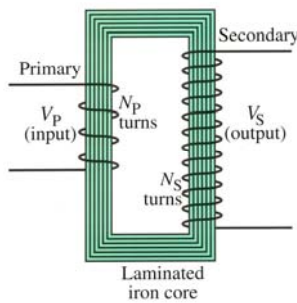
Lenz's Law says that the direction of that EMF is so as to oppose the change in magnetic flux

Mutual Inductance

Since magnetic fields are typically generated by currents, Faraday's law implies that changing currents also generate EMFs. This is the idea of mutual inductance: given any two circuits, a changing current in one will induce an EMF in the other, or, mathematically, $\mathcal{E}_2 = -M dI_1/dt$, where M is the mutual inductance of the two circuits. How does this work?

The current in loop 1 produces a magnetic field (and hence flux) through loop 2. If that current changes in time, the flux through 2 changes in time, creating an EMF in loop 2. The mutual inductance, M , depends on geometry, both on how well the current in the first loop can create a magnetic field and on how much magnetic flux through the second loop that magnetic field will create.

Transformers



A major application of mutual inductance is the transformer, which allows the easy modification of the voltage of AC (alternating current) signals. At left is the schematic of a step up transformer.

An input voltage V_P on the primary coil creates an oscillating magnetic field, which is "steered" through the iron core (recall that ferromagnets like iron act like wires for magnetic fields) and through the secondary coils, which induces an EMF in them. In the ideal case, the amount of flux generated and received is proportional to the number of turns in each coil. Hence the ratio of


the output to input voltage is the same as the ratio of the number of turns in the secondary to the number of turns in the primary. As pictured we have more turns in the secondary, hence this is a "step up transformer," with a larger output voltage than input.

The ease of creating transformers is a strong argument for using AC rather than DC power. Why? Before sending power across transmission lines, voltage is stepped way up (to 240,000 V), leading to smaller currents and losses in the lines. The voltage is then stepped down to 240 V before going into your home.

Self Inductance

Self inductance is very similar to mutual inductance, obeying a similar equation: $\mathcal{E} = -L \frac{dI}{dt}$, and the same concept: when a circuit has a current in it, it creates a magnetic field, and hence a flux, through itself. If that current changes, then the flux will change and hence an EMF will be induced in the circuit. The action of that EMF will be to *oppose the change* in current (if the current is decreasing it will try to make it bigger, if increasing it will try to make it smaller). For this reason, we often refer to the induced EMF as the “back EMF.”

To calculate the self inductance (or inductance, for short) of an object, imagine that a current I flows through it, and determine how much magnetic field and hence flux Φ_B that makes through the object. The self inductance is then $L = \Phi_B / I$.

An inductor is a circuit element whose main characteristic is its inductance, L . It is drawn as a coil  in circuit diagrams. The strong resemblance to a solenoid is intentional – solenoids make very good inductors both because of their ability to make a strong field inside themselves, and also because the field they produce is fairly well contained, and hence doesn't produce much flux (and induce EMFs) in other, nearby circuits.

The role of an inductor is to oppose changing currents. At steady state, in a DC circuit, an inductor is off – it induces no EMF as long as the current through it is constant. As soon as you try to change the current through an inductor though, it will fight back. In this sense an inductor is the opposite of a capacitor. If a capacitor is placed in a steady state current it will eventually fill up and “open” the circuit, whereas an inductor looks like a short in this case. On the other hand, when starting from its uncharged state, a capacitor looks like a short when you first try to move current through it, while an inductor looks like an open circuit, as it prevents the change (from no current to some current).

Important Equations

Faraday's Law:	$\mathcal{E} = -\frac{d\Phi_B}{dt}$
Magnetic Flux:	$\Phi_B = \iint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$
EMF:	$\mathcal{E} = \oint \vec{\mathbf{E}}' \cdot d\vec{\mathbf{s}}$
Mutual Inductance:	$\mathcal{E}_2 = -M \frac{dI_1}{dt}$
Self Inductance, L :	$L = \frac{\Phi_B}{I}$
EMF Induced by Inductor:	$\mathcal{E} = -L \frac{dI}{dt}$