We now would like to generalize our concept of work to the motion of an object that goes in more than one dimension.

And so, if we have our object here.

And we're applying a force.

And the object is moving.

And let's say the path is a little bit more complicated.

So our object, a little bit later, has moved.

And we're going to call that displacement.

We now want to generalize our concept to work to handle this type of motion in more than one dimension.

In order to do that, we'll need a new mathematical operation, which we're going to call the scalar product.

So what I'd like to do first, is define this in terms of vectors and then we'll apply it to the concept of work.

So suppose I have two vectors, a, and another vector, b.

And they're separated by an angle, theta.

And in this case, our theta, we're going to just take it between 0 and pi.

Now, I'd like to talk about how much one vector projects in the direction of the other.

So if I call this the parallel component of b.

Then I want to define a quantity which we're going to call the scalar product of a dot b.

Which is the magnitude of a times the amount of b that is in the direction parallel to a.

From our geometric diagram, you can see that b parallel is equal to the magnitude of b times the cosine of theta.

Remember, theta's going from 0 to pi.

So this quantity, b parallel, can both be positive or negative.

If theta is between 0 and pi/2, then in that range, the cosine goes from 1 to 0.

And this quantity, b, will be greater or equal to 0.

b parallel.

If theta is going from pi/2, theta to pi, then we have that b parallel is less than or equal to 0.

And, in particular, at the value pi/2, b parallel is 0 because when theta is pi/2, the b vector is completely perpendicular to a.

And this is what we're going to define to be the scalar product.

But geometrically, we can also look at-- let's draw our pictures again-- a and b.

Let's consider how much of the a is parallel to b.

So that's a parallel.

And if this is the angle theta, we see that a parallel-- and I'll write it as a parallel-- is the magnitude of a times cosine of theta.

And so, I can also define a dot b as equal to how much of how much of a is parallel to b times the magnitude of b.

In both of these instances, because b parallel is the magnitude of b cosine theta, and a parallel is a magnitude of a cosine theta, what we're writing is that a dot b is the magnitude of a cosine theta magnitude of b.

And now you can see that this quantity is a parallel.

And this quantity is b parallel.

And this is our geometric definition of a dot product.

One thing that we want to consider are two important rules for dot products.

And they're the following.

That if we take a vector and multiply a by a quantity-- c, here, is a scalar quantity-- then this is equal to c times a dot b.

Geometrically, this is very easy to see.

Because if we draw a and we draw b.

And if I multiply a by a scalar, and let's make, in this particular picture, our scalar bigger than 1, then the new vector a is multiplied by c and it's c times a.

The scalar product is just ca dot b, is the magnitude of ca times how much of b is parallel.

And when you multiply this together, you get c times a times to b parallel.

c of a dot b is equal to c times the magnitude of a times b parallel.

And you can see that these two expressions are equal.

If you multiply a dot product by a scalar, it satisfies this rule.

And there's one other crucial rule that we'll need when we look at vectors.

That if you take a vector a and you add to it, a vector b, and you dotted in a vector c, then this is vector addition and multiplication.

This vector addition distributes over vector multiplication by a dot c plus b dot c.

So these two facts are crucial to our development of vectors.

One thing that we can say is-- we'll leave this as a little exercise for you to prove this result.

It's a pretty straightforward vector construction.

And to just give you a little bit of a hint.

If we have a vector a and another vector b and we have a similar vector c, then if I draw the vector a dot b plus b and I want to take a dot c.

So I have how much of a is parallel to c.

And if I look at how much of b is parallel to c, then as part of the exercise, make sure that you can see that how much of a plus b parallel to c agrees to this distributive rule.

So these are our definition of scalar product.

And the two key facts that we'll need when we apply it in our example of work.