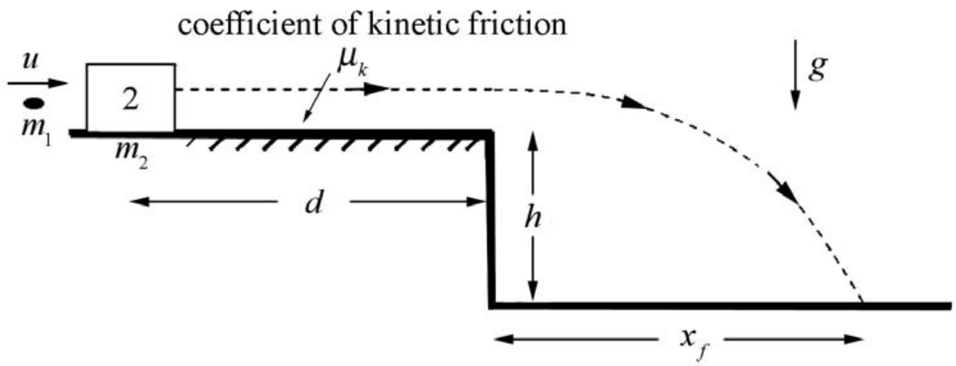
Problem Set 5

1. **Stopping a Bullet**



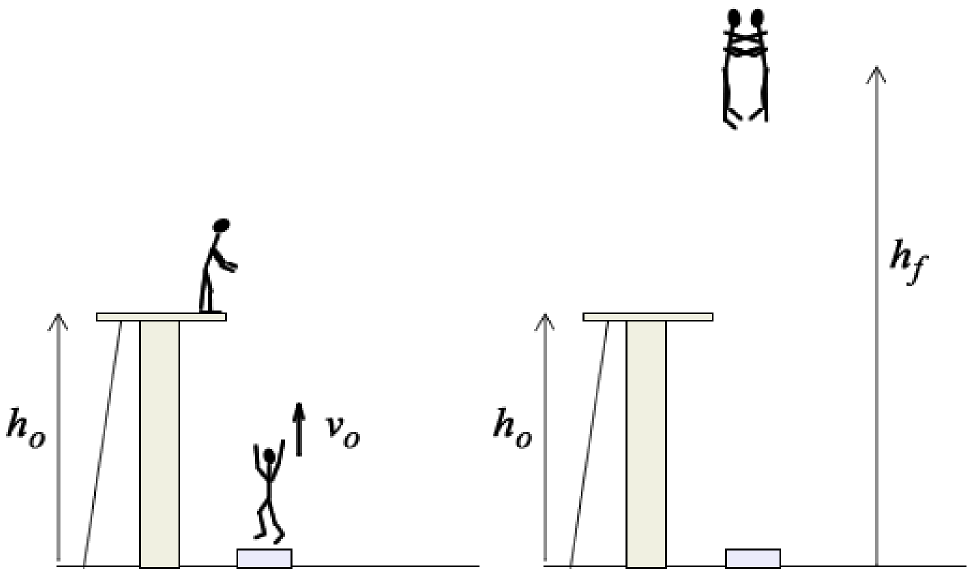
A bullet of mass m_1 traveling horizontally with speed u hits a block of mass m_2 that is originally at rest and becomes embedded in the block. After the collision, the block slides horizontally a distance d on a surface with friction, and then falls off the surface at a height h as shown.

The coefficient of kinetic friction between the block and the surface is μ_k. Assume the collision is nearly instantaneous and all distances are large compared to the size of the block. Neglect air resistance.

(a) What is u_{min}, the minimum speed of the bullet so that the block falls off the surface? Express your answer in terms of some or all of the following: m_1, m_2, μ_k, d, h and g for the gravitational constant.

(b) Assume that the initial speed of the bullet u is large enough for the block to fall off the surface. Calculate x_f, the position where the block hits the ground measured from the bottom edge of the surface. Express your answer in terms of some or all of the following: m_1, m_2, μ_k, u, d, h and g.

2. **Acrobat and Clown** An acrobat of mass m_A jumps upwards off a trampoline with an initial speed v_0. At a height h_0, the acrobat grabs a clown of mass m_B. Assume that the time the acrobat takes to grab the clown is negligibly small.



What is the maximum height h_f reached by the acrobat and clown? Write your answer in terms of some or all of the following: m_A, m_B, g, h_0, and v_0.

3. **Compressive Strength of Bones**

The compressive force per area necessary to break the tibia in the lower leg is about $F/A = 1.6 \times 10^8\ \mathrm{N/m^2}$. The smallest cross sectional area of the tibia, about $3.2\ \mathrm{cm^2}$, is slightly above the ankle. Suppose a person of mass $m = 60$ kg jumps to the ground from a height $h_0 = 2.0$ m and absorbs the shock of hitting the ground by bending the knees. Assume that there is constant deceleration during the collision with the ground, and that the person lowers their center of mass by an amount $d = 1.0$ cm from the time they hit the ground until they stop moving.

(a) What is the collision time Δt_{col}, to 2 significant figures?

(b) Find N_{ave}, the magnitude of the average force exerted by the ground on the person during the collision in Newtons.

(c) What is the ratio of the average force of the ground on the person to the gravitational force on the person? Can we effectively ignore the gravitational force during the collision?

(d) Will the person break his ankle?

4. **Center of Mass of a Rod**

   A thin rod has length L and total mass M lies along the x axis.

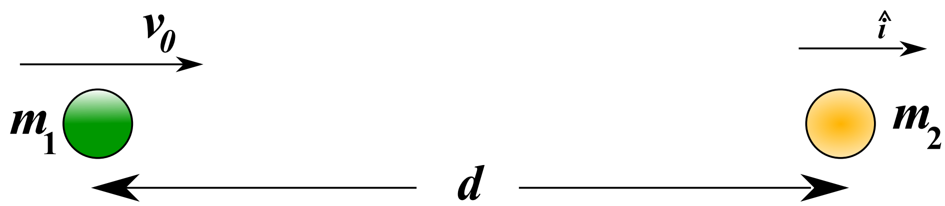
   (a) Suppose the rod is uniform. Find the position of the center of mass with respect to the left end of the rod in terms of L, M, and $\hat{i}$.

   (b) Now suppose the rod is not uniform but has a linear mass density that varies with the distance x from the left end according to

   $$\lambda = \frac{\lambda_0}{L^3} x^3$$

   where λ_0 is a constant and has SI units of kg/m. The total mass of the rod is still M. Find λ_0 and the position of the center of mass with respect to the left end of the rod. Express your answer in terms of some or all of the following: L, M, and $\hat{i}$.

5. **Two Particles Colliding**



Two small particles of mass m_1 and mass m_2 attract each other with a force that varies inversely with the cube of their separation. At time t_0, m_1 has a velocity of magnitude v_0, directed towards m_2, which is at rest a distance d away. At time t_1, the particles collide.

Calculate L, the distance travelled by particle 1 during the time interval $t_1 - t_0$. Express your answer using some or all of the following variables: m_1, m_2, t_0, t_1, v_0, and d.