8.012 Physics I: Classical Mechanics
Fall 2008

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.012 **Fall 2005**

MIDTERM EXAM 2
Tuesday, November 8, 2005

Name: ______________________________

MIT ID number: ______________________________

INSTRUCTIONS:
- Do all **FIVE (5)** problems. You have 90 minutes.
- **Show all work**, and circle your answer.
- All work must be done in this booklet.
- No books, notes, or calculators permitted.
- Use only inertial reference frames and real forces.

USEFUL RELATIONS:
- Velocity in polar coordinates: $\dot{\mathbf{r}} = \dot{r}\hat{\mathbf{r}} + r\dot{\theta}\hat{\boldsymbol{\theta}}$
- Acceleration in polar coordinates: $\ddot{\mathbf{r}} = (\ddot{r} - r\dot{\theta}^2)\,\hat{\mathbf{r}} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\hat{\boldsymbol{\theta}}$

Problem	Maximum	Score	Grader
1	15		
2	20		
3	25		
4	20		
5	20		
TOTAL	100		

1. **Problem 1 of 5**
   Two railcars and a cannon. (15 points)

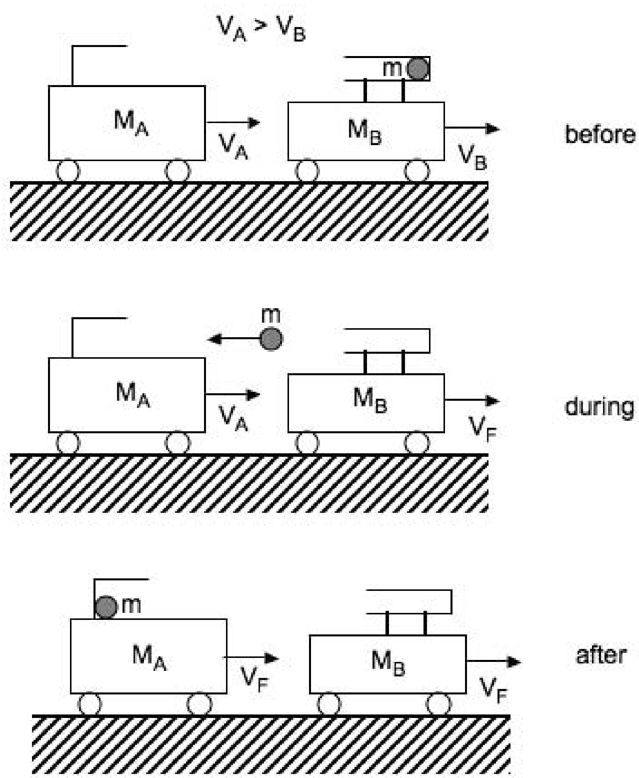
   Two railcars are moving, unpowered, in the same direction along a horizontal track. Car B, which has mass M_B and velocity V_B, carries a projectile of mass m.

   - Initially, car B is being overtaken by car A, which has mass M_A and velocity V_A, with $V_A > V_B$ *(top figure)*.
   - The projectile is then fired from car B to car A with horizontal velocity of magnitude u relative to car B, after which car B moves with velocity V_F *(middle figure)*.
   - Finally, the projectile comes to an abrupt stop after striking car A, after which both cars are travelling at the same velocity V_F *(bottom figure)*.

   You may neglect any vertical motion of the projectile after it is fired.

   (a) (7 points) Calculate car B's final velocity V_F in terms of V_B, u, and the masses.

   (b) (8 points) Calculate the required muzzle velocity u of the projectile such that the final velocity of car A is also V_F. Express your answer in terms of V_A, V_B, and the masses.



2. **Problem 2 of 5**
   Power boat. (20 points)

   A jet boat propels itself by taking up (stationary) water at a constant mass rate $dm/dt = a$ and expelling it with velocity of magnitude u with respect to the boat. The mass M of the boat is constant (you may neglect the fuel used to power the engine).

   (a) (12 points) Neglecting any drag force due to the boat's passage through the water, calculate the velocity of the boat as a function of time, assuming it starts from rest at time $t = 0$. Sketch $v(t)$.

   (b) (8 points) Now, assume the boat starts from rest but experiences a drag force $\mathbf{F}_d = -b\mathbf{v}$ due to its passage through the water, where $\mathbf{v}$ is the boat's velocity vector. The boat will eventually reach a constant terminal velocity v_f. Calculate v_f. [You are not required to calculate $v(t)$ for all t.]

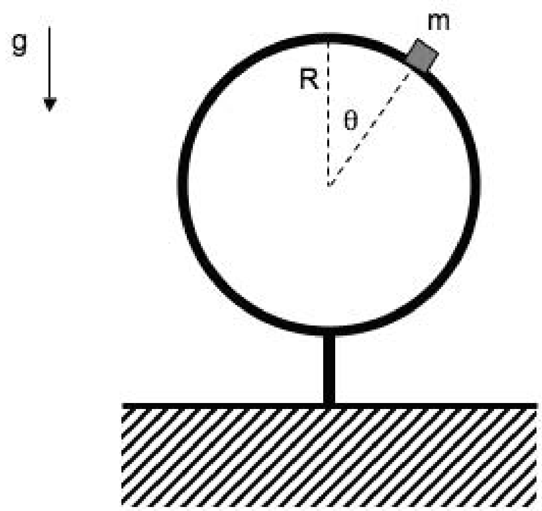
3. **Problem 3 of 5**
Mass on a sphere. (25 points)

A mass m rests on top of a fixed sphere of radius R. The mass is released with negligible speed to the right, and it slowly starts to slide down the sphere (with increasing angle θ). A frictional force $F_0 \sin\theta$ opposes the motion as long as the mass is in contact with the sphere. Gravity acts downward.

(a) (9 points) Find the total mechanical energy E as a function of θ, taking $E = 0$ at the top of the sphere (where $\theta = 0$). (Hint: Your answer should *not* have any explicit dependence on velocity.)

(b) (8 points) Find the kinetic energy as a function of θ.

(c) (8 points) Calculate the angle at which the mass loses contact with the sphere.



4. **Problem 4 of 5**
   Particle in a 1-D potential. (20 points)

   A particle of mass M moves in one dimension along the x-axis. Its potential energy is given by $U(x) = ax^3 - bx$, where a and b are positive constants.

   (a) (4 points) Calculate the force on the particle, $F(x)$. Find the position of all equilibrium points, and identify them as stable or unstable.

   (b) (4 points) Draw an energy diagram showing the potential energy U, the kinetic energy K, and the total mechanical energy E for bound motion. Show the location of the stable and unstable equilibrium points on your diagram.

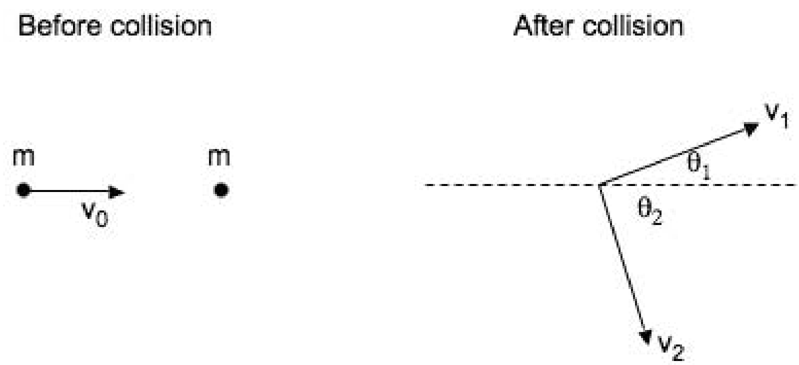
   (c) (4 points) What is the minimum kinetic energy required for the particle to be unbound?

   (d) (4 points) Find the *period* of small oscillations around the stable equilibrium.

   (e) (4 points) Estimate the maximum displacement from equilibrium for which the small oscillation approximation is valid to 1 percent accuracy.

5. **Problem 5 of 5**
Particle proofs. (20 points)

(a) **Two-body collision in two dimensions.** (10 points) A moving particle collides elastically with an equally massive particle at rest. After the collision, neither of the two particles is moving parallel to the initial velocity. Assume that there are no external forces acting on the particles. Show that the two particles must move at right angles to one another after the collision (i.e., show that in the figure below, $\theta_1 + \theta_2 = 90°$, or equivalently $\mathbf{v}_1 \cdot \mathbf{v}_2 = 0$).



(b) **Angular momentum of a system of particles.** (10 points) Show that if the total linear momentum of a system of particles is zero, then the total angular momentum of the system is the same with respect to all origins.

Useful vector relation: $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c} = (\mathbf{a} \times \mathbf{c}) + (\mathbf{b} \times \mathbf{c})$