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8.012 Physics I: Classical Mechanics
Fall 2008

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Physics 8.012                                                          Fall 2008

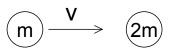> ## Exam 2

**NAME**: _____

Instructions:
1. Do all **FIVE (5)** problems.  You have **90 minutes**.
2. **SHOW ALL WORK**.  Be sure to **circle your final answer**.
3. Read the questions carefully.
4. All work must be done in this booklet in workspace provided.  Extra blank pages are provided at the end if needed.
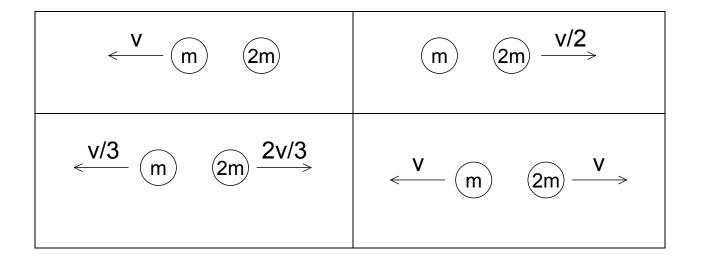5. NO books, notes, calculators or computers are permitted.  **A sheet of useful equations is provided on the last page**.

| Problem | Maximum | Score | Grader |
|:-------:|:-------:|:-----:|:------:|
| 1 | 10 | | |
| 2 | 20 | | |
| 3 | 20 | | |
| 4 | 25 | | |
| 5 | 25 | | |
| **Total** | 100 | | |

Problem 1: Quick Multiple Choice Questions [10 pts]

For each of the following questions circle the correct answer. You do not need to show any work.

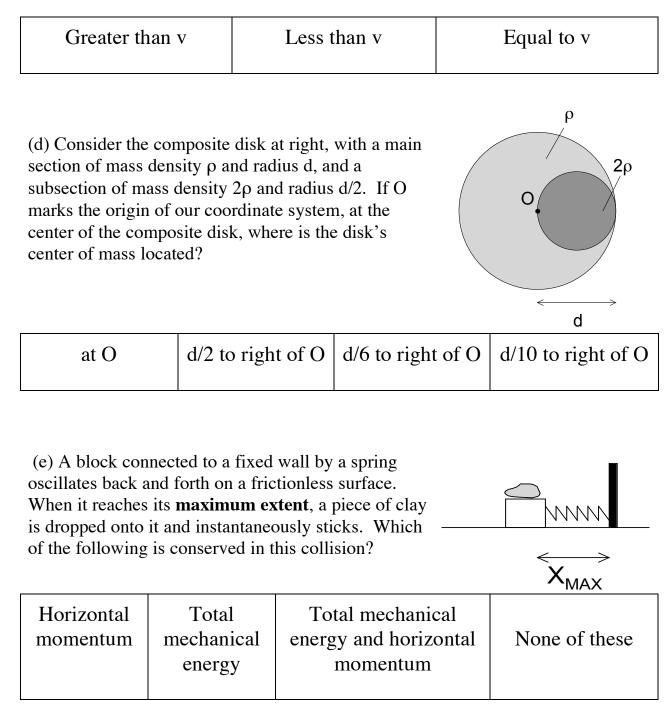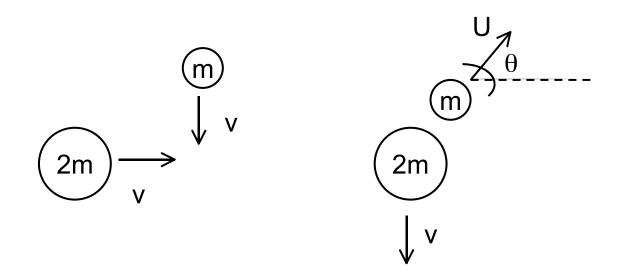(a) Which of the following is the result of an elastic collision between the two isolated objects at right?

$\quad$ (m) $\xrightarrow{\text{V}}$ (2m)

| | |
|---|---|
| $\xleftarrow{\text{V}}$ (m) $\quad$ (2m) | (m) $\quad$ (2m) $\xrightarrow{\text{v/2}}$ |
| $\xleftarrow{\text{v/3}}$ (m) $\quad$ (2m) $\xrightarrow{\text{2v/3}}$ | $\xleftarrow{\text{V}}$ (m) $\quad$ (2m) $\xrightarrow{\text{V}}$ |

(b) Which experiences the **largest** impulse?

| Fully elastic collision of 10 kg ball dropped from 10 m | Fully inelastic collision of 20 kg ball dropped from 10 m | Fully elastic collision of 20 kg ball dropped from 5m | These all have the same impulse |
|---|---|---|---|

(c) A cart traveling at speed v on a frictionless track starts to leak sand. What is the cart's speed at a later time?

| Greater than v | Less than v | Equal to v |
|---|---|---|
| | | |

(d) Consider the composite disk at right, with a main section of mass density ρ and radius d, and a subsection of mass density 2ρ and radius d/2. If O marks the origin of our coordinate system, at the center of the composite disk, where is the disk's center of mass located?



| at O | d/2 to right of O | d/6 to right of O | d/10 to right of O |
|---|---|---|---|
| | | | |

 (e) A block connected to a fixed wall by a spring oscillates back and forth on a frictionless surface. When it reaches its **maximum extent**, a piece of clay is dropped onto it and instantaneously sticks. Which of the following is conserved in this collision?



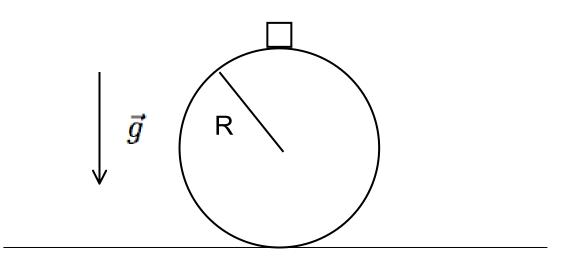| Horizontal momentum | Total mechanical energy | Total mechanical energy and horizontal momentum | None of these |
|---|---|---|---|
| | | | |

Problem 2: Collision [20 pts]



Two balls, one of mass m and one of mass 2m, approach from orthogonal directions with identical speeds v and collide. After the collision, the more massive ball moves with the same speed v but downward (orthogonal to its original direction) and the less massive ball moves with speed U at an angle θ with respect to horizontal. Assume that no external forces act during the collision.

(a) [10 pts] Calculate the final speed U of the less massive ball and the angle θ.

(b) [10 pts] Determine how much kinetic energy is lost or gained by the two balls during the collision. Is this collision elastic, inelastic or superelastic?
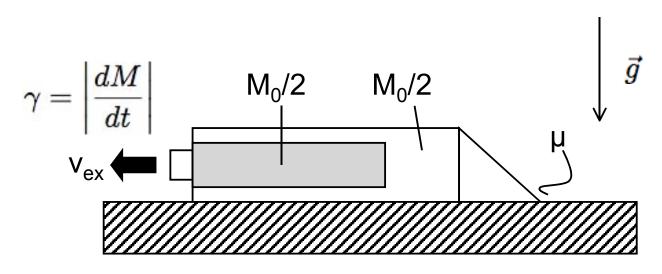
Problem 3: Cube on a Sphere [20 pts]



A small block starts from rest and slides down from the top of a fixed sphere of radius R, where R >> size of the block. The surface of the sphere is frictionless and constant gravitational acceleration g acts downward.

(a) [10 pts] Determine the speed of the block as a function of angle from the top while it remains in contact with the sphere.

(b) [10 pts] At what angle does the block lose contact with the sphere?

Problem 4: Tabletop Rocket [25 pts]

$$\gamma = \left| \frac{dM}{dt} \right|$$

$M_0/2$     $M_0/2$

$\vec{g}$

$v_{ex}$

$\mu$

A rocket of total mass $M_0$, half of which is fuel, starts at rest on a long horizontal table. The coefficient of friction between the rocket and table surfaces is $\mu$. At time $t = 0$, the rocket is ignited, ejecting fuel out at a constant rate $\gamma = |dM/dt|$ with velocity $v_{ex}$ relative to the rocket. Constant gravitational acceleration g acts downward.

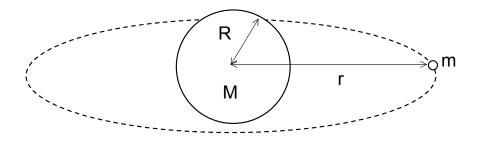(a) [10 pts] What condition must be met for the rocket to start moving at $t = 0$?

(b) [10 pts] Assuming that the rocket satisfies this requirement, what is the maximum speed $V_{MAX}$ achieved by the rocket?

(c) [5 pts] How far does the rocket go after it runs out of fuel? You can express your answer in terms of $V_{MAX}$.

(d) [BONUS 5 pts] How far does the rocket travel in total? For this you will need to make use of the following integral:

$$\int \ln u \, du = u \ln u - u$$

Problem 5: Planet Orbit [25 pts]



A small planet of mass m is in a circular orbit of radius r around a star of mass M and radius R in otherwise empty space (assume M >> m so the star is stationary).

(a) [10 pts] Determine the potential energy U(r), the kinetic energy K(r) and the total mechanical energy E(r) of the planet in terms of G, M and r assuming U→0 as r→∞.

(b) [5 pts] Determine the minimum amount of mechanical energy that must be added to the planet to cause it to escape from the star (i.e., r→∞).  By what <u>factor</u> must the speed of the planet be increased to cause it to escape?

(c) [5 pts] Now assume that the planet in subject to a viscous force of the form

$$\vec{F} = -Amv^2\hat{v}$$

where A is a constant and $\hat{v}$ is the direction of motion.  Compute the loss of mechanical energy in one orbital period in terms of G, M, r and A. Assume that this loss is small enough that neither the orbital radius nor speed of the planet changes appreciably in one orbit.

(d) [5 pts] Building from (c), compute the change in radius of the planet in one orbital period due to the viscous force and the corresponding radial velocity based on the assumptions above, in terms of G, M, r and A.  Does the planet fall into the star or away from it?