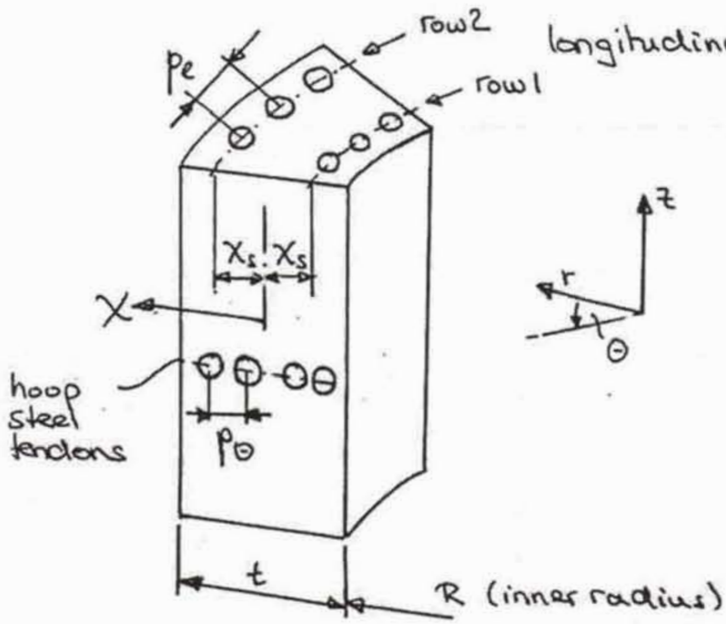
Structural Mechanics in Nuclear Power Technology - Fall 1987

(1.565J, 2.084J, 3.82J, 13.14J, 16.261J, 22.314J)

Calculation - Concept to Problem Set L.54

Containment : reinforced concrete:



subscripts:

s ... steel

c ... concrete

p_ℓ pitch of longitudinal tend

p_θ pitch of hoop tendons

Assumptions:

- $\nu_c = \nu_s = \nu$
- $R \gg t$, handle wall as a plate
- average strains for concrete and steel are the same in longitudinal and azimuthal direction.

$$\bar{\varepsilon}_{\ell c} = \bar{\varepsilon}_{\ell s}$$

$$\bar{\varepsilon}_{\theta c} = \bar{\varepsilon}_{\theta s}$$

with these assumptions we can use the equations from the thin shell theory

$$\frac{dQ_e}{dz} + \frac{N_\theta}{R} - p = 0 \qquad (1)$$

$$\frac{dM_e}{dz} - Q_e = 0 \qquad (2)$$

$$\frac{d^2 w}{dz^2} = \frac{2\varepsilon_{be}}{t} \qquad (3)$$

with Q_e shear force
N_θ normal force
M_e bending moment (around z-axis) } per unit len
w displacement in radial direction
ε_{be} maximum strain due to bending (around z-axi

Assuming a plane state of stress and using a mean value for the stress in radial direction, $\bar{\sigma}_r$, the longitudinal stress in the concrete is given by (approximately)

$$\sigma_{ec} = \frac{E_c}{1-\nu^2}\Big(\underbrace{\bar{\varepsilon}_{ec} - \underbrace{\varepsilon_{be}\left(\tfrac{2x}{t}\right)}_{\text{due to bending}}}_{\varepsilon_{ec}} + \nu\,\bar{\varepsilon}_{\theta c}\Big) + \underbrace{\frac{\nu}{1-\nu}\,\bar{\sigma}_r}_{\text{correction because no plane state of stress}} \qquad (4)$$

(see note on l... page)

The stress in the steel tendons is approximately (for thin tendon

$$\sigma_{es} = E_s\,\varepsilon_{es} \quad , \qquad (5)$$

where again

$$\varepsilon_{es} = \varepsilon_{ec} = \bar{\varepsilon}_{ec} - \varepsilon_{be}\left(\frac{2x}{t}\right) \quad . \qquad (6)$$

The average forces in longitudinal direction in the steel tendons are

row 1 : $\bar{F}_{e1} = E_s\,\varepsilon_{es}(-x_s)\,A_s = E_s\left(\bar{\varepsilon}_{ec} + \frac{2x_s}{t}\varepsilon_{be}\right)A_s \qquad (7)$

row 2 : $\bar{F}_{e2} = E_s\,\varepsilon_{es}(x_s)\,A_s = E_s\left(\bar{\varepsilon}_{ec} - \frac{2x_s}{t}\varepsilon_{be}\right)A_s \qquad (8)$

with A_s as the cross-sectional area of the steel tendons

Now, $F_{e1} \neq F_{e2}$ and therefore it exists a bending moment around the z-axis with

$$\text{row1: } M_{e1} = F_{e1} X_s$$
$$\text{row2: } M_{e2} = -F_{e2} X_s \qquad (10)$$

and the total moment per unit length becomes for the steel tendo

$$M_{es} = \frac{M_{e1} + M_{e2}}{p_e}$$

$$\overset{(10),(9)}{\Rightarrow} M_{es} = \left(\frac{2A_s}{t p_e}\right) E_s X_s^2 t \left(\frac{2\varepsilon_{be}}{t}\right)$$

and with

$$X_{es} = \frac{2A_s}{t p_e}, \qquad (11)$$

the fraction of cross-sectional area occupied by longitudinal steel tendons, we get

$$M_{es} = X_{es} E_s X_s^2 \, 2\varepsilon_{be} \qquad (12)$$

The moment of the concrete per unit length can be expressed as

$$M_{ec} = \underbrace{-\int_{-t/2}^{t/2} \sigma_{ec} X dX}_{M_{ec1}} \underbrace{- \Big[- \underbrace{\int \sigma_{ec} X dX}_{\text{row1}} - \underbrace{\int \sigma_{ec} X dX}_{\text{row2}} \Big]}_{M_{ec2}} \qquad (13)$$

We get with eq. (4)

$$M_{ec_1} = -\int_{-\frac{t}{2}}^{\frac{t}{2}} \left(\frac{E_c}{1-\nu^2} \left[(\bar{\varepsilon}_{ec} - \nu \bar{\varepsilon}_{\theta c}) - \frac{2\varepsilon_{be}}{t} X \right] + \frac{\nu}{1-\nu} \bar{\sigma}_r \right) X dX$$

$$M_{ec_1} = 2 \int_0^{t/2} \frac{E_c}{1-\nu^2} \left(\frac{2\varepsilon_{be}}{t}\right) X^2 dX$$

(strains constant in X-direction give no moment)

$$M_{ec_1} = 2 \frac{E_c}{1-\nu^2} \frac{\varepsilon_{be}}{t} \cdot \frac{t^3}{12} \qquad (14)$$

The expression for M_{ce_2} in eq. (13) can be approximated by

$$M_{ec_2} = X_{es} \cdot 2\varepsilon_{be} \cdot X_s^2 \cdot \frac{E_c}{1-\nu^2} \tag{15}$$

Now, from eq. (14) and (15) we get

$$M_{ec} = \frac{2\varepsilon_{be}}{t} \left(\frac{E_c}{1-\nu^2} \right) \left(\frac{t^3}{12} + X_{es} X_s^2 t \right) \tag{16}$$

The total moment of the wall per unit length becomes

$$M_e = M_{es} + M_{ec} = \frac{2\varepsilon_{be}}{t} \left(\frac{E_c}{1-\nu^2} \left(\frac{t^3}{12} + X_{es} X_s^2 t \right) + E_s X_{es} X_s^2 t \right) \tag{17}$$

where we can define the flexural rigidity for the wall to

$$D = \frac{E_c}{1-\nu^2} \left(\frac{t^3}{12} + X_{es} X_s^2 t \right) + E_s X_{es} X_s^2 t \tag{18}$$

and get from eq. (17)

$$M_e = \frac{2\varepsilon_{be}}{t} \cdot D \tag{19}$$

Using eq. (19) for eq. (3) yields

$$\frac{d^2 w}{dz^2} = \frac{M_e}{D} , \tag{20}$$

and plugging this in eq. (2) gives

$$D \frac{d^3 w}{dz^3} = Q_e \tag{21}$$

Using eq. (21) for eq. (1) gives

$$D \frac{d^4 w}{dz^4} + \frac{N_\theta}{R} - p = 0 \tag{22}$$

Now, the normal force N_θ is composed of the normal forces in the steel tendons and in the concrete, which are

$$N_{\theta s} = X_{\theta s} \bar{\sigma}_{\theta s} \cdot t \tag{23}$$

$$N_{\theta c} = (1 - X_{\theta s}) \bar{\sigma}_{\theta c} \cdot t \tag{24}$$

$$N_\theta = N_{\theta s} + N_{\theta c} = [X_{\theta s} \bar{\sigma}_{\theta s} + (1 - X_{\theta s}) \bar{\sigma}_{\theta c}]\, t \tag{25}$$

with $\chi_{\Theta s} = \frac{4A_s}{t p_t}$ the fraction of the cross sectional area occupied by the hoop steel tendons.

Now, analogous to eq. (4) and (5) we get

$$\bar{\sigma}_{\Theta c} = \frac{E_c}{1-\nu^2} \left(\bar{\varepsilon}_{\Theta c} + \nu \bar{\varepsilon}_{\varrho c} \right) + \frac{\nu}{1-\nu} \bar{\sigma}_r \tag{26}$$

$$\bar{\sigma}_{\Theta s} = E_s \bar{\varepsilon}_{\Theta s} \tag{27}$$

Plugging eq. (26) and (27) in eq. (25) yields

$$N_\Theta = \left[\chi_{\Theta s} E_s \bar{\varepsilon}_{\Theta s} + (1-\chi_{\Theta s}) \frac{E_c}{1-\nu^2} (\bar{\varepsilon}_{\Theta c} + \nu \bar{\varepsilon}_{\varrho c}) \right] t + (1-\chi_{\Theta s}) \cdot \frac{\nu t}{1-\nu} \bar{\sigma}_r \tag{28}$$

We can obtain the strains in Θ-direction by

$$\bar{\varepsilon}_{\Theta s} \approx \varepsilon_\Theta = \frac{w}{R} \tag{29}$$

with the displacement w in radial direction.

But unknown in eq. (28) remains $\bar{\varepsilon}_{\varrho c}$ (we want to calculate N_Θ).

The necessary second equation can now be obtained by using the same considerations for the normal force N_ϱ as before for N_Θ.

$$N_\varrho = N_{\varrho s} + N_{\varrho c} = \left[\chi_{\varrho s} \bar{\sigma}_{\varrho s} + (1-\chi_{\varrho s}) \bar{\sigma}_{\varrho c} \right] t \tag{30}$$

Similarly to N_Θ, with eq. (4) and (5) we get

$$N_\varrho = \left[\chi_{\varrho s} E_s \bar{\varepsilon}_{\varrho s} + (1-\chi_{\varrho s}) \left[\frac{E_c}{1-\nu^2} (\bar{\varepsilon}_{\varrho c} + \nu \bar{\varepsilon}_{\Theta c}) + \frac{\nu}{1-\nu} \bar{\sigma}_r \right] \right] t \tag{31}$$

Using the thin wall approximation yields

$$\sigma_\varrho = \frac{pR}{2t} = \frac{N_\varrho}{t}$$

and thereby

$$N_\varrho = \frac{pR}{2} \tag{32}$$

Now, eq. (31) can be used to calculate $\bar{\varepsilon}_{\varrho c}$, using $\bar{\varepsilon}_{\varrho c} \approx \bar{\varepsilon}_{\varrho s}$

$$\bar{\varepsilon}_{\ell c} = \frac{\frac{N_\ell}{t} - (1-X_{\ell s})\left[\frac{E_c}{1-\nu^2}\nu\bar{\varepsilon}_{\theta c} + \frac{\nu}{1-\nu}\bar{\sigma}_r\right]}{X_{\ell s}E_s + (1-X_{\ell s})\frac{E_c}{1-\nu^2}} \quad (33)$$

Define: $E^* = X_{\ell s}E_s + (1-X_{\ell s})\frac{E_c}{1-\nu^2} = X_{\ell s}E_s + E_\ell$

$$E_\ell = (1-X_{\ell s})\frac{E_c}{1-\nu^2}$$

$$E_\theta = (1-X_{\theta s})\frac{E_c}{1-\nu^2}$$

Using this expressions, eq. (33) yields

$$\bar{\varepsilon}_{\ell c} = \frac{\frac{N_\ell}{t} - \left(\nu E_\ell \bar{\varepsilon}_{\theta c} + (1-X_{\ell s})\frac{\nu}{1-\nu}\bar{\sigma}_r\right)}{E^*} \quad (34)$$

Plugging eq. (34) in eq. (28) gives:

$$\frac{1}{t}N_\theta = \left[X_{\theta s}E_s + E_\theta\left(1 - \frac{\nu^2 E_\ell}{E^*}\right)\right]\bar{\varepsilon}_{\theta c} + \nu\frac{E_\theta}{E^*}\frac{N_\ell}{t}$$

$$+ \frac{\nu}{1-\nu}\bar{\sigma}_r\left(1 - X_{\theta s} - (1-X_{\ell s})\frac{\nu E_\theta}{E^*}\right) \quad (35)$$

Using eq (29) we can write

$$N_\theta = \alpha w + \gamma \quad (36)$$

with

$$\alpha = \frac{t}{R}\left[X_{\theta s}E_s + E_\theta\left(1 - \frac{\nu^2 E_\ell}{E^*}\right)\right]$$

$$\gamma = \frac{\nu E_\theta}{E^*}N_\ell + \frac{\nu t}{1-\nu}\bar{\sigma}_r\left(1 - X_{\theta s} - (1-X_{\ell s})\frac{\nu E_\theta}{E^*}\right) \quad (37)$$

Now we can plug in eq. (36) in eq. (22):

$$D\frac{d^4 w}{dz^4} + \frac{\alpha}{R}w = p - \frac{\gamma}{R} \quad (38)$$

This equation can be rewritten by defining

$$\beta^4 = \frac{\alpha}{4DR} \quad (39)$$

$$\frac{d^4 w}{dz^4} + 4\beta^4 w = \frac{1}{D}\left(p - \frac{\gamma}{R}\right) \quad (40)$$

The solution of this differential equation is

$$w = \underset{\text{homogenous}}{w_H} + \underset{\text{particular}}{w_p} = e^{-\beta z}(c_1 \cos\beta z + c_2 \sin\beta z) + \frac{1}{4\beta^4 D}\left(p - \frac{k}{R}\right) \quad (41)$$

The boundary conditions are the "built-in" conditions at $z=0$, where the containment joins the base mat.

$$w(0) = 0$$

$$\frac{dw}{dz}(0) = 0 \ .$$

This gives $c_1 = c_2 = w_p = \frac{1}{4\beta^4 D}\left(p - \frac{k}{R}\right) = \frac{R}{\alpha}\left(p - \frac{k}{R}\right) \quad (42)$

and eq. (41) becomes

$$w(z) = w_p\left(1 - e^{-\beta z}(\cos\beta z + \sin\beta z)\right) \quad (43)$$

The maximum displacement occurs for $z \to \infty$ and is

$$w_{max} = w_p \ .$$

Calculating conservative, we can use

$$\bar{\varepsilon}_{\Theta c} = \frac{w_{max}}{R} = \frac{w_p}{R} \quad (44)$$

for our further calculations.

The maximum stress in longitudinal direction occurs at $X = -\frac{t}{2}$ for $\bar{\varepsilon}_{\Theta c_{max}}$ and $\bar{\varepsilon}_{\ell c_{max}}$ (max. for $z \to \infty$, too)

$$\sigma_{\ell c_{max}} = \frac{E_c}{1-\nu^2}\left(\bar{\varepsilon}_{\ell c_{max}} + \nu \bar{\varepsilon}_{\Theta c_{max}} + \frac{2t}{2t}\varepsilon_{b\ell}\right) + \frac{\nu}{1-\nu}\bar{\sigma}_r \quad (45)$$

Now, the maximum longitudinal stress $\sigma_{\ell c_{max}}$ shall be offset by the tendon prestress to get zero net concrete stress upon pressurization.

This yields to:

$$\sigma_{\ell c_{max}}(1 - X_{es}) = X_{es}\,\sigma_{es\,prestress}$$

and thereby

$$\sigma_{es\,prestress} = \frac{1 - X_{es}}{X_{es}}\,\sigma_{\ell c_{max}}$$

Similarly, the maximum stress in hoop direction is given by eq. (26), using $\bar{\varepsilon}_{\theta c\,max}$ and $\bar{\varepsilon}_{\ell c\,max}$:

$$\sigma_{\theta c\,max} = \frac{E_c}{1-\nu^2}\left(\bar{\varepsilon}_{\theta c_{max}} + \nu\,\bar{\varepsilon}_{\ell c_{max}}\right) + \frac{\nu}{1-\nu}\,\bar{\sigma}_r$$

Now we get the condition

$$\sigma_{\theta c\,max}\,(1-X_{\theta s}) = X_{\theta s}\,\sigma_{\theta s\,prestress}$$

and thereby

$$\sigma_{\theta s\,prestress} = \frac{(1-X_{\theta s})}{X_{\theta s}}\,\sigma_{\theta c\,max}\,.$$

To obtain the maximum tensile stresses in the rebars upon pressurization, we only have to add the elastic stresses to the prestresses :

$$\sigma_{\ell s} = \sigma_{\ell s\,prestress} + E_s\,(\bar{\varepsilon}_{\ell s} + \varepsilon_{be})$$

$$\sigma_{\theta s} = \sigma_{\theta s\,prestress} + E_s\,\bar{\varepsilon}_{\theta s}$$

Derivation of eq. (4), Problem Set L.54

Assume basically plane state of stress.
Then we can write

$$\sigma_{\ell\,plane} = \frac{E}{1-\nu^2}\left(\varepsilon_\ell + \nu\,\varepsilon_\Theta\right) \qquad (1)$$

In this equation we have neglected the influence of σ_r and ε_r, but we have considered ε_ℓ and ε_Θ.
Now, for the real 3-dimensional state of stress we get

$$\tilde{\sigma}_\ell = \frac{E}{(1+\nu)\;(2\nu-1)}\left[(\nu-1)\underset{0}{\underset{\shortparallel}{\varepsilon_\ell}} - \nu\left(\underset{0}{\underset{\shortparallel}{\varepsilon_\Theta}} + \varepsilon_r\right)\right]$$

$\Rightarrow$ because already considered in (1)

$$\Rightarrow \quad \tilde{\sigma}_\ell = \frac{E}{(1+\nu)(2\nu-1)}\left(-\nu\,\varepsilon_r\right) \qquad (2)$$

Similarly, we can reduce the equation for σ_r for three dimensional state of stress to

$$\sigma_r = \frac{E}{(1+\nu)(2\nu-1)}\,(\nu-1)\,\varepsilon_r \quad , \qquad (3)$$

taking again into account $\varepsilon_\ell = 0$, $\varepsilon_\Theta = 0$.
Now, eq. (2) and (3) can be combined to

$$\tilde{\sigma}_\ell = \frac{\nu}{1-\nu}\,\sigma_r \quad , \qquad (4)$$

which is approximately the stress in longitudinal direction due to σ_r.
Combining eq. (4) and (1) gives the total stress in longitudinal direction

$$\sigma_\ell = \frac{E}{1-\nu^2}\left(\varepsilon_\ell + \nu\,\varepsilon_\Theta\right) + \frac{\nu}{1-\nu}\,\sigma_r \quad .$$

Analogous to this, we get the total stress in Θ-direction

$$\sigma_\Theta = \frac{E}{1-\nu^2}\left(\varepsilon_\Theta + \nu\,\varepsilon_\ell\right) + \frac{\nu}{1-\nu}\,\sigma_r \quad .$$