22.251 Systems Analysis of the Nuclear Fuel Cycle

Fall 2009

HOMEWORK SET #7

Basic facts about xenon

Xe135 is one of the fission products with a high "cumulative yield". Although its direct yield is only about 0.003 (about 0.003 of all fissions give rise to a xenon fission product), the yield of a Xe135 precursor I135 is about 0.061 for fissions in U235, so the total cumulative yield is 0.064. Xenon has a very large thermal capture cross section; its 2200 m/s cross section is about 2.66 million barns, and because of its very non-1/v behavior, the maxwellian average cross section at 68°F is larger – 2.72 million barns. Because of this, xenon typically holds down more than 2% reactivity in LWRs.

Furthermore, xenon has a half life of 9.21 hours and the iodine precursor has a half life of 6.71 hours. Because the xenon is created by fission but removed by decay and by neutron capture, we expect its concentration to reach an equilibrium after 6 to 10 half lives. However, since most of the creation by fission comes via the iodine, interesting time-dependent transients occur when the flux level (i.e., the power level) is changed. In particular, a power decrease leads to an immediate rise in xenon concentration because the iodine producing most of it is still present and decaying to xenon, but the flux responsible for part of its removal has decreased.

The objective of this homework problem is to study these transient and equilibrium effects and to do this as a further example of the matrix exponential method that is employed in the ORIGEN code.

Equations describing time behavior of xenon

$$\frac{dI}{dt} = \gamma_I \Sigma_f \varphi - \lambda_I I$$
$$\frac{dX}{dt} = \gamma_X \Sigma_f \varphi + \lambda_I I - \lambda_X X - \sigma_X \varphi X \tag{1}$$

Here I and X are iodine and xenon concentrations, γ is fission yield, and the other variables have their usual significance.

Solution of transient xenon equations by matrix exponential

To solve the equations by the matrix exponential method, we first note that they are a set of ordinary differential equations that, for a constant flux, have constant

coefficients. (Obviously we could also solve them by a number of other methods, but let's use the matrix exponential here.)

The first step is to write the equations as a matrix-vector set, using the time dependent iodine and xenon concentrations as the unknown vector:

$$\frac{d}{dt}\begin{bmatrix} I(t) \\ X(t) \end{bmatrix} = -\begin{bmatrix} \lambda_I & 0 \\ -\lambda_I & \lambda_X + \sigma_X \phi \end{bmatrix}\begin{bmatrix} I(t) \\ X(t) \end{bmatrix} + \begin{bmatrix} \gamma_I \Sigma_f \phi \\ \gamma_X \Sigma_f \phi \end{bmatrix} \qquad (2)$$

We could have made ϕ time dependent here, but we choose to treat it as a constant anticipating that we will step ourselves ahead in our solution and so can change ϕ at each time step if necessary; we also want to retain the character of a differential equation with constant coefficients.

Letting Y denote the iodine-xenon vector, A the coefficient matrix, and S the rightmost term that provides the iodine and xenon, se rewrite this in simplified form as:

$$\frac{dY}{dt} = -AY + S \qquad (3)$$

we can now bomb blithely ahead and solve this as a scalar differential equation:

$$Y(t) = \{Y(0) - \frac{S}{A}\}e^{-At} + \frac{S}{A} \qquad (4)$$

However for this to make sense where A is a matrix and S is a vector, we have to rewrite it slightly as:

$$Y(t) = e^{-At}\{Y(0) - A^{-1}S\} + A^{-1}S \qquad (5)$$

where A⁻¹ represents the inverse of the A matrix. Note that e^{-At} is also a matrix.

Input data

	$\boldsymbol{\lambda}$	$\boldsymbol{\sigma_a}$	$\boldsymbol{\gamma}$
Iodine	2.87E-5 sec⁻¹	0	0.061
Xenon	2.09E-5 sec⁻¹	1.2E6 b	0.003
Σ_f for this problem	0.109		
ϕ for this problem	3E13		

The cross sections here represent typical values at 800 deg C in a PWR lattice.

1. By setting the time derivatives of eq (1) to zero, show that the equilibrium concentrations of iodine and xenon are:

$$I^E = \gamma_I \Sigma_f \varphi$$

$$X^E = \frac{(\gamma_I + \gamma_X)\Sigma_f \varphi}{\lambda_X + \sigma_X \varphi}$$

(6)

Evaluate these numerically.

2. Evaluate the individual elements of the A matrix numerically. Then evaluate the individual elements of the matrix "At" for t = 1 hour. Multiplication of a matrix by a scalar multiplies each element of the matrix by that scalar. Watch your units here!

3. Evaluate the matrix e⁻ᴬᵗ for t = 1 hour by expanding in the usual Taylor series. Include enough terms to get the individual elements correct to three places.

4. Calculate and plot the xenon number density during a startup transient using eq (5) iteratively. That is, use eq (5) with your numerical At matrix values for t=1 hour and at each step replace Y(0) with the Y(t) you got from the previous step. The initial Y(0) is zero because this is a startup. Use enough steps to get to equilibrium. How many hours did this take? Do the equilibrium concentrations of iodine and xenon match the values you got in part 1?

5. Starting from the equilibrium values, calculate and plot a shutdown transient until a few hours past the peak. In this case the S vector is zero because the flux is zero. As before, use a 1 hour time step. If the equilibrium xenon is worth 2.5% reactivity, what is the peak xenon worth? Assume that xenon worth is proportional to xenon concentration, a good approximation.

6. **Extra credit!!!** Verify that eq (5) is correct at equilibrium by finding the symbolic inverse of the A matrix (that is, in terms of the λ's and σₓφ), and then showing that Y(∞) from eq (5) gives the same symbolic result as Equation (6).

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