22.101 Applied Nuclear Physics
(Fall 2006)

Solutions to Quiz 3 Dec. 15, 2006

Prob 1

(a) $P(\Omega_c) = C \quad 0 \le \theta_c \le \pi/2$ $\qquad \int d\Omega_c P(\Omega_c) = 1 \quad 2\pi C \int_0^1 d\mu = 1 \Rightarrow C = 1/2\pi$

$\qquad\quad 0 \quad$ otherwise

(b) $G(\theta_c)\,d\theta_c = \int_{\varphi=0}^{2\pi} d\varphi \sin\theta_c \, d\theta_c \, P(\Omega_c) = \sin\theta_c \, d\theta_c \qquad G(\theta_c) = \sin\theta_c$

$F(E\to E')\,dE' = G(\theta_c)\,d\theta_c \qquad F(E\to E') = G(\theta_c) \left| \frac{d\theta_c}{dE'} \right|$

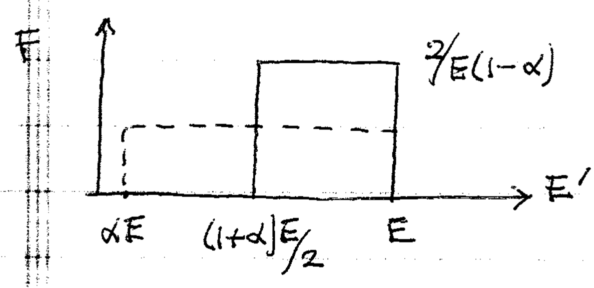
with $E' = \frac{E}{2}\left[(1+\alpha) + (1-\alpha)\cos\theta_c\right]$ $\qquad \theta_c = \pi/2, \ E' = \frac{E(1+\alpha)}{2}$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad = 0, \ E' = E$

$F(E\to E') = \frac{2}{E(1-\alpha)} \qquad \frac{E}{2} \le E' \le E$

$\qquad\qquad\quad 0 \qquad\qquad$ otherwise

(c)



present : ———

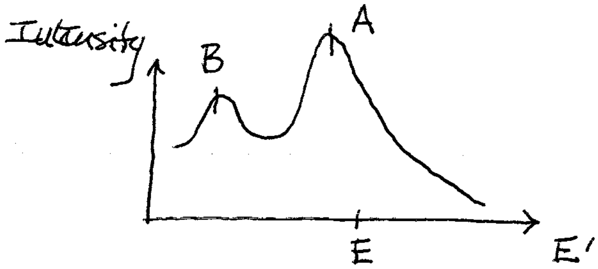
sph symmetry : - - - -

(d) Range : present range $= E - \frac{E}{2}(1+\alpha) = \frac{E(1-\alpha)}{2}$

sph symmetry range $= E - \alpha E \quad = E(1-\alpha)$

When angular range is restricted, expect the energy range to be also restricted. Reduction is a factor of 2 in the present case.

Prob 2



(a) Peak A : elastic scattering of thermal neutron $E' \sim E$
dominant process is Bragg diffraction in a crystal

Peak B : (lattice) inelastic scattering, $E' < E$ (downscattering by exciting lattice vibration – phonon emission)

Intensity variation with T and θ —
peak A will vary with θ (Bragg condition, $\lambda = 2d \sin\theta$) but not with T
peak B will not vary much with T or θ, although the phonon absorption (upscattering by de-exciting lattice vibration) will be sensitive to T (intensity will increase with increasing T)

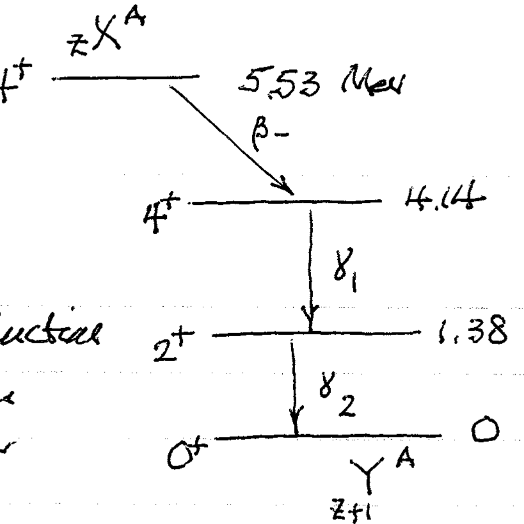
(b) peak A : elastic photon scattering
B : inelastic photon scattering
} both are Compton scattering, while the elastic component → Thomson scattering

$\alpha \equiv E/m_e c^2 \to 0$ Compton → Thomson (Thomson dominates)
$\alpha \gg 1$ Compton dominates

position of peak B will vary with θ according to $\omega' = \dfrac{\omega}{1 + \alpha(1-\cos\theta)}$

$E = \hbar\omega$
$E' = \hbar\omega'$

Prob 3



(a) $_ZX^A$ does not undergo β_+ decay (given), so β_+ must come from pair production by the two γ's indicated in the diagram (provided $E > 1.02$ MeV $(2m_ec^2)$)

[Note, another process "internal pair conversion" also can occur – one person in the class mentioned this]

(b) End point energies

$E(\gamma_1) = 4.14 - 1.38 = 2.76$ $\quad T_{max}(\beta_+) = 2.76 - 1.02 = 1.74$ MeV

$E(\gamma_2) = 1.38$ $\quad = 1.38 - 1.02 = 0.36$

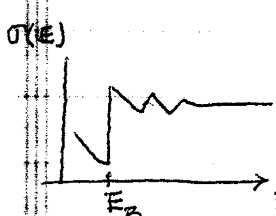
(c) Decay modes : β_- $4^+ \to 4^+$ allowed, F and G-T

γ_1 $4^+ \to 2^+$ E2

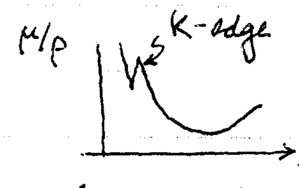
γ_2 $2^+ \to 0^+$ E2 unique

Prob 4

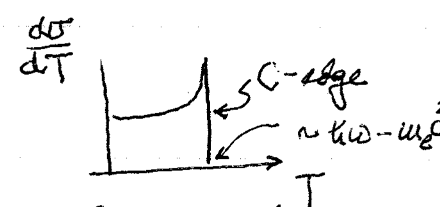
(a)



thermal neutron c.x. for scattering by crystal 'Bragg cutoff'

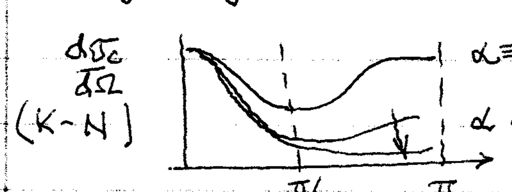


γ attenuation coeff high Z (Pb), K-shell photoelectric effect



Compton electron distribution

(b)

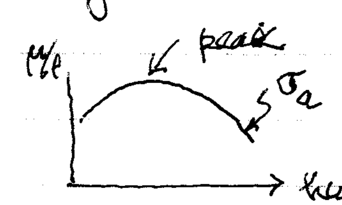


$\alpha \equiv \hbar\omega/m_ec^2 \to 0$ (Thomson)

α increasing (forward scattering bias)

(c) $\sigma_a \equiv \sigma_c - \sigma_{sc}$, $\sigma_c = \int d\Omega \frac{d\sigma_c}{d\Omega}$, $\sigma_{sc} = \int d\Omega \frac{\omega'}{\omega}\frac{d\sigma_c}{d\Omega}$

$\sigma_c \sim 1 - 2\alpha$, $\sigma_{sc} \sim 1 - 3\alpha$, $\therefore \sigma_a \sim \alpha$ ← this means there will be a peak in σ_a



(d) Both selection rules are governed by conservation of angular momentum (orbital & spin) and parity

β-decay $\quad \underline{I}_p = \underline{I}_D + \underline{L}_\beta + \underline{S}_\beta \qquad L_\beta = 0, 1, \ldots$

$\qquad \pi_p = (-1)^{L_\beta}\, \pi_D \qquad S_\beta = 0 \text{ or } 1$

γ-decay $\quad \underline{I}_i = \underline{I}_f + \underline{L}_\gamma \qquad L_\gamma = 1, 2, \ldots$

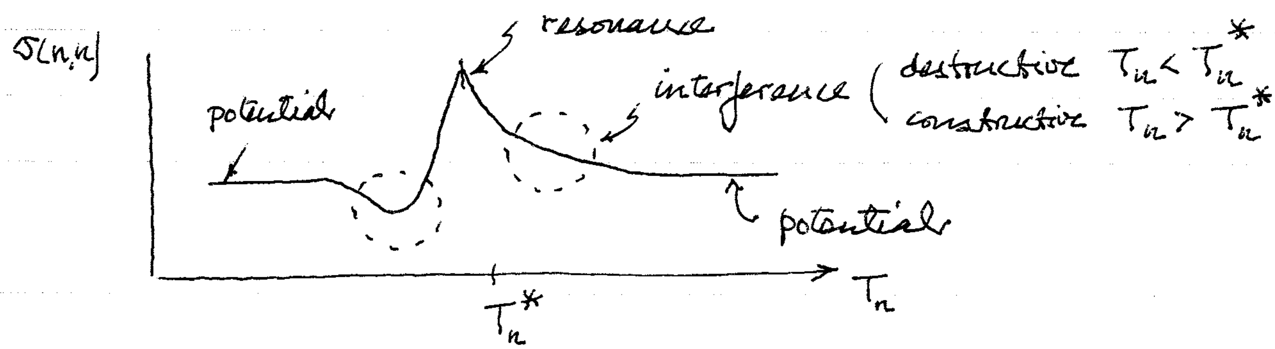
$\qquad \pi_i = \pi_\gamma\, \pi_f$

$\pi_\gamma = (-1)^{L_\gamma}$ E transition

$ = -(-1)^{L_\gamma}$ M

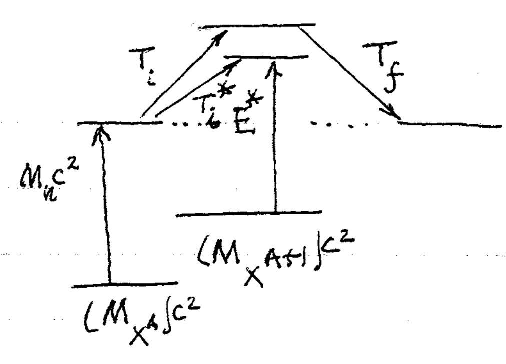
angular momentum and parity aspects are expressed differently

(e)



$$n + {}_Z X^A \rightarrow ({}_Z X^{A+1})^* \longrightarrow n + {}_Z X^A$$

CN



at resonance $T_i = T_i^*$ ($T_n = T_n^*$)
CN is at energy E^* (one of its resonance levels)

$Q = 0$ (elastic scattering)