22.101 Applied Nuclear Physics (Fall 2006)

Solutions to Quiz 2 Nov. 15, 2006

Prob 1

(a) $H = \frac{p^2}{2\mu} + V(r) + V_{ss}(r)\, \underline{S}_n \cdot \underline{S}_p$

(b) $\underline{S}_t \equiv \underline{S}_n + \underline{S}_p \qquad S_t^2 = S_n^2 + S_p^2 + 2\, \underline{S}_n \cdot \underline{S}_p$

(c) $|S_t, m_{S_t}, S_n, S_p\rangle \equiv |\rangle$

(d) $S_t^2 |\rangle = \hbar^2 S_t(S_t+1) |\rangle, \quad |S_n - S_p| \le S_t \le S_n + S_p$

$(S_t)_z |\rangle = \hbar^2 m_{S_t} |\rangle, \quad -S_t \le m_{S_t} \le S_t$

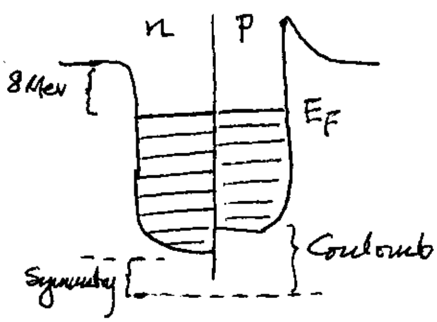
$S_n = 1/2, \; S_p = 1/2$

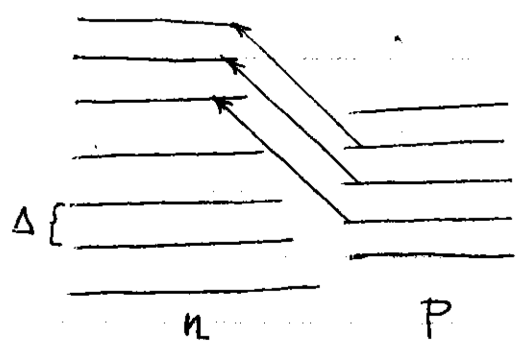
$0 \le S_t \le 1 \quad \therefore \; S_t = 0, 1$

$m_{S_t} = -1, 0, 1$

Prob 2

(a)





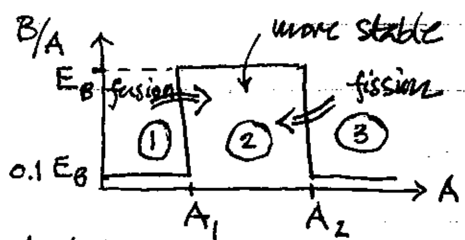
(b) transform ν (= 3 in sketch) protons into neutrons

$\left(\frac{A}{2}, \frac{A}{2}\right) \to (N, Z) \qquad N = \frac{A}{2} + \nu, \quad Z = \frac{A}{2} - \nu \qquad \Big\}\; \nu = \frac{1}{2}(N - Z)$

energy req'd $= \nu * \nu\Delta = \frac{1}{4}(N-Z)^2 \Delta, \quad \Delta \sim E_F/A$

$\therefore$ asymmetry energy $= a_a (N-Z)^2 / A$

Prob 3



(a) Nuclei in ② more stable than ① & ③

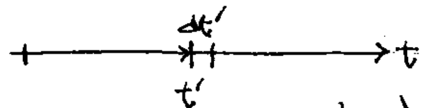
(b) fission is favorable in ③, products in ②

(c) fusion is favorable in ①, products in ②

Prob 4

prob of no decay to t', then decay in dt' interval

$$= e^{-\lambda t'} \lambda \, dt'$$



$\lambda = \lambda_\beta + \lambda_{EC}$

(a) prob of decay bet. t_1 & t_2

$$= \lambda \int_{t_1}^{t_2} e^{-\lambda t'} dt' = e^{-\lambda t_1} - e^{-\lambda t_2}$$

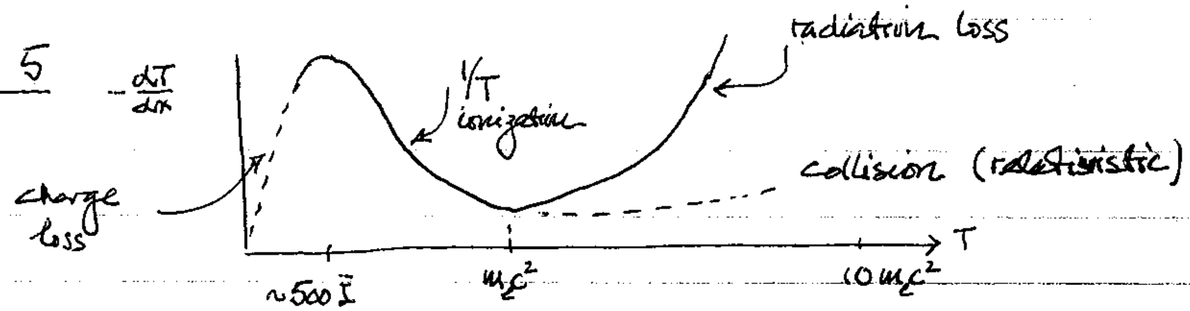
(b) fraction in (a) decaying by β^+ is $\dfrac{\lambda_\beta}{\lambda_\beta + \lambda_{EC}}$

(c) fraction decaying by β^+ during (t_1, t_2) $= \dfrac{\lambda_\beta}{\lambda}\left[e^{-\lambda t_1} - e^{-\lambda t_2} \right]$

(d) result is reasonable, rewrite (a) as

$$\underbrace{e^{-\lambda t_1}}_{\text{no decay to } t_1} \underbrace{\left[1 - e^{-\lambda (t_2 - t_1)} \right]}_{\text{decay during } (t_1, t_2)}$$

Prob 5



Prob 6

(a) $\sqrt{E_3}$ has to be real and positive

(b) $s^2 + t \geq 0$

there will be a threshold value for E_1 which is larger than $|Q|$