22.101 Applied Nuclear Physics
(Fall 2006)

QUIZ No. 1 (closed book) **October 16, 2006**

Problem 1 (20%) In the following sketches, say briefly what you are trying to show.

(a) The wave function of a free particle with energy E (E >0) in a 1D system is written as $\psi = A\sin kx$, $E = \hbar^2 k^2 / 2m$, and A is some constant. Now introduce a potential barrier of height V_o (with $V_o < E$) and range 2L, located at the origin. Sketch the wave function everywhere in the system to show the effects of the potential.

(b) Repeat (a) for a potential well instead of a barrier.

(c) Consider a particle barely bound by the potential well (b). Sketch the wave function everywhere in the system.

Problem 2 (15%)

In solving the 3D wave equation for the bound states for a spherically symmetric well, what new features appear (relative to the 1D problem) in labeling the states and in determining the degeneracy? Sketch what you expect would be the radial wave functions for the ground state and first excited state for a spherical well.

Problem 3 (20%)

The boundary condition $\psi(r) \sim \exp(i\underline{k}\cdot\underline{r}) + f(\theta)e^{ikr}/r$, r >> r_o, is applied to calculate the angular differential scattering cross section. Using the fact that s-wave scattering is spherically symmetric and writing the exterior wave function as $\psi = A\sin(kr+\delta)$, derive the relation between the scattering amplitude $f(\theta)$ and the phase shift δ_o for s-wave.

Problem 4 (45%)

(a) Calculate the phase shift δ_o for s-wave scattering of a particle of mass m and incident kinetic energy E by a potential barrier of height V_o ($V_o > E$) and range r_o.

(b) Simplify your result by taking the low-energy limit, where $kr_o \ll 1$ and $\delta_o \ll 1$. Find the scattering cross section σ. Sketch σ as a function of $k_o r_o$, with $k_o^2 = 2mV_o/\hbar^2$, and show the limiting values of σ for small and large $k_o r_o$.

(c) Consider applying your result to describe neutron-proton scattering, with V_o = 36 MeV and $r_o = 2 \times 10^{-13}$ cm. Give an estimate of the value of σ from (a) and compare it to the experimental value. If there is a discrepancy, what is the implication? Discuss further whether it is appropriate (reasonable) to apply the present model.