22.02 Intro to Applied Nuclear Physics

Mid-Term Exam

Thursday March 18, 2010 Name: ....

Problem 1: Short Questions *40 points*

1. What is an observable in quantum mechanics and by which mathematical object is represented?
2. How are the possible values of measurement outcomes of an observable determined?
3. What is the probability of finding a particle described by the wavefunction $\psi(x, y, z)$ in a small volume $dV = dx\,dy\,dz$ around the position $\vec{r} = \{\tilde{x}, \tilde{y}, 0\}$?
4. Is the evolution of a quantum mechanical system a stochastic (random) process?
5. For a system described by the wavefunction $\psi(x, t = t_1) = a_1 w_1(x) + a_{91} w_{91}(x)$ (where w_n are energy eigenfunctions with eigenvalues E_n) what is the probability of measuring an energy $E = E_{91}$ at time $t = t_1$? What about at time $t = 2t_1$?
6. Consider a particle described by the wavefunction $\varphi(x)$ and an Hamiltonian with eigenvalues E_n and corresponding eigenfunctions $w_n(x)$. Is the probability of obtaining the energy E_n given by $p(E_n) = \int_{-\infty}^{\infty} dx\, w_n^*(x)\varphi(x)$?
7. A particle is in the quantum state $\psi = B\cos(\sqrt{2}x)$.
   a) What are the possible results of a momentum measurement?
   b) What are the probabilities of each possible momentum measurement?
8. A particle is in the quantum state, $\psi = Ae^{-i79kx}$.
   a) What are the possible results of a momentum measurement?
   b) What are the probabilities of each possible momentum measurement?
   c) What physical situation is represented by this quantum state?
9. A particle is in the angular momentum eigenstate, $\psi = |l, m_z\rangle = |5, -4\rangle$.
   a) What would a measurement of the total angular momentum, L^2, yield?
   b) What would a measurement of the z-component of angular momentum, L_z, yield?
   c) What would a measurement of the x-component of angular momentum, L_x, yield?
10. A nucleus consists of two spin 1/2 nucleons, $s_1 = \frac{1}{2}$, and, $s_2 = \frac{1}{2}$. Both nucleons are in the orbital angular momentum $l = 0$.
    a) How many spin states are there for each nucleon?
    b) How many spin states does the system have (based on the uncoupled representation)?
    c) Which quantum numbers would you use to label the coupled representation states?

Problem 2: Eigenvalue problem *15 points*

The quantum mechanical observable *Grades on the 22.02 Mid-Term* has the eigenvalue problem,

$$\hat{G}\varphi_k = g_k\varphi_k, \qquad k = 1, 2, 3, 4$$

with g_1 =A, g_2 =B, g_3 =C, g_4 =D and where φ_k are normalized eigenfunctions. If the normalized state of the system is

$$\psi_{\text{class}} = c_1\varphi_1 + \frac{1}{\sqrt{5}}\varphi_2 + \frac{2}{5}\varphi_3 + \frac{1}{5}\varphi_4$$

and assuming that the usual rules of Quantum Mechanics apply and that there are only 4 possible grades on the exam,

a) What is the probability of an A grade outcome?

b) What is the average grade of the exam? (you can take A=5, B=4, C=3, D=2).

c) Consider now the operator $\widehat{PF}$ (Pass/Fail), that obeys the following rules:

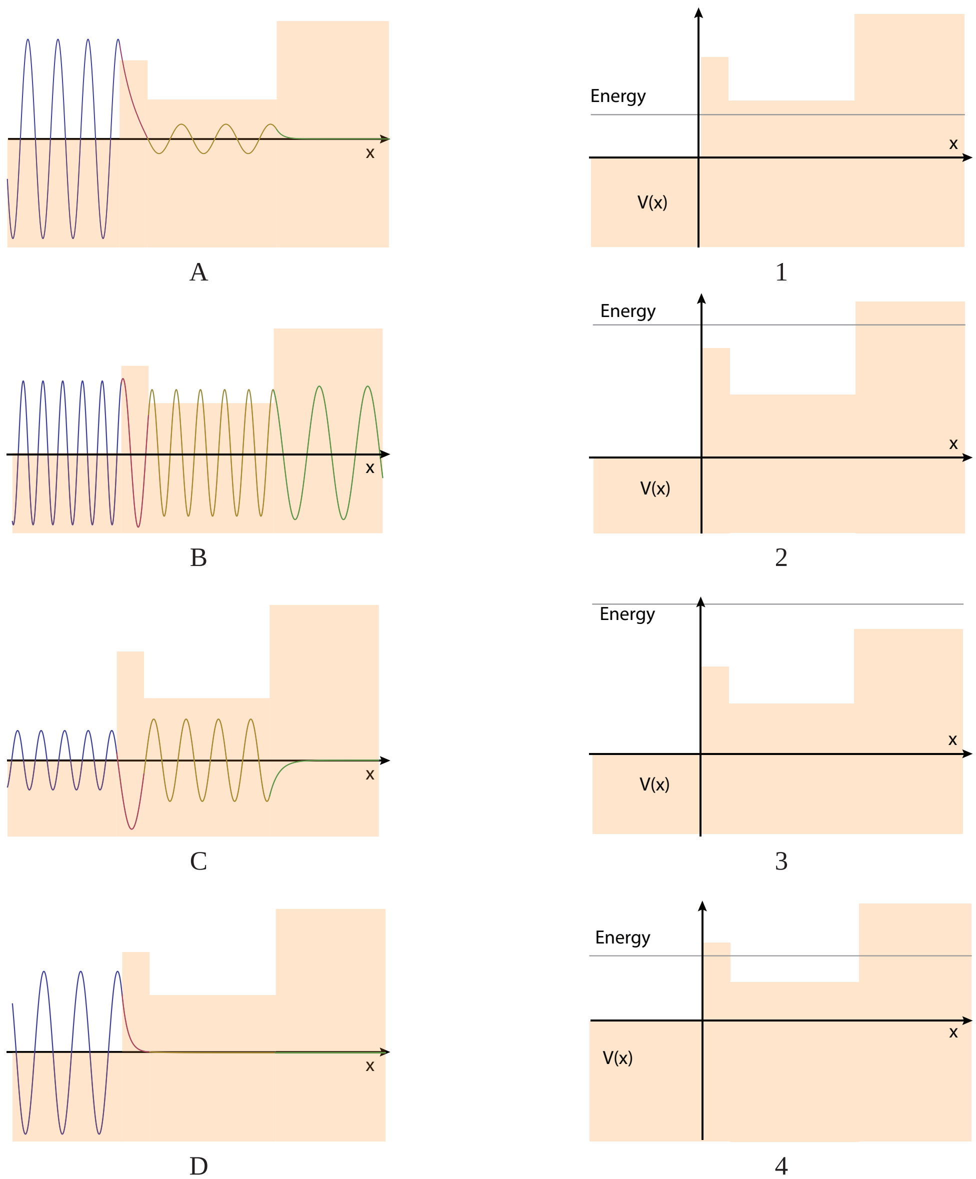
$$\widehat{PF}\varphi_k = P\varphi_k, \qquad \text{for } k = 1, 2, 3$$

$$\widehat{PF}\varphi_4 = F\varphi_4$$

where P and F are real number and φ_k are the same functions as above. Do the operators $\hat{G}$ and $\widehat{PF}$ commute? Give a mathematical reason for your answer.

Problem 3: Scattering *25 points*

a) (15 points) In the following figures you can see for a given potential profile a set of eigenfunctions (left column) and a set of possible total energy values (right column) describing different scenarios of scattering for a particle incoming from the left side (here I plot the real part of the eigenfunction). You should match each figure in the left column with the corresponding one in the right column (that is, indicate what energy of the particle gives rise to which scattering behavior). To get full credit you need to add a **SHORT** description of each situation such as ”wavelength is shorter in region I compared to region II. . . amplitude is . . .. in various regions ..., the wavefunctions is a traveling wave/decays exponentially/is reflected..., ” .



b) (10 points) Match the 1D scattering energy eigenfunctions on the left with the correct potential profile (if any). Provide an explanation for your answer. (Notice: here I plot the real part of the eigenfunction).

Problem 4: Alpha Particle Tunneling *20 points*

A beam of alpha particles is directed at a potential barrier as indicated in the figure below. The beam has a high flux of, $\Gamma = 6 \times 10^{22}$ particles/sec. The energy of the alpha particles is $E = 5$ MeV , and the potential barrier height is, $V_B = 85$ MeV and its width is $L = 10$ fm. You may assume the alpha rest mass to be $m_\alpha c^2 = 4000$ MeV (and remember that $\hbar c \approx 200$ MeV fm, with $c \approx 3 \times 10^8$ m s^{-1} the speed of light).

a) Make a drawing of the eigenfunction for this problem, assuming the alpha particles are shot in from the left.

b) Estimate the probability of tunneling, P_{tun}, through the barrier. Write the generic approximate formula for tunneling and then estimate the numbers quantitatively (the plot below can help you with the numerical estimate).

c) If the incoming beam is described by the wavefunction, $\psi(x) = Ae^{ikx}$ calculate A that gives the flux $\Gamma = p(x)v$ (where $p(x)dx$ is the probability of finding the particle at $\{x, x + dx\}$ and v the particle velocity).

d) Assuming the flux of alphas emerging from the barrier is $\Gamma_{tun} = P_{tun}\Gamma$, how long should you wait to see an alpha particle exit from the barrier? (you need to calculate the time τ such that $\Gamma_{tun}\tau = 1$)

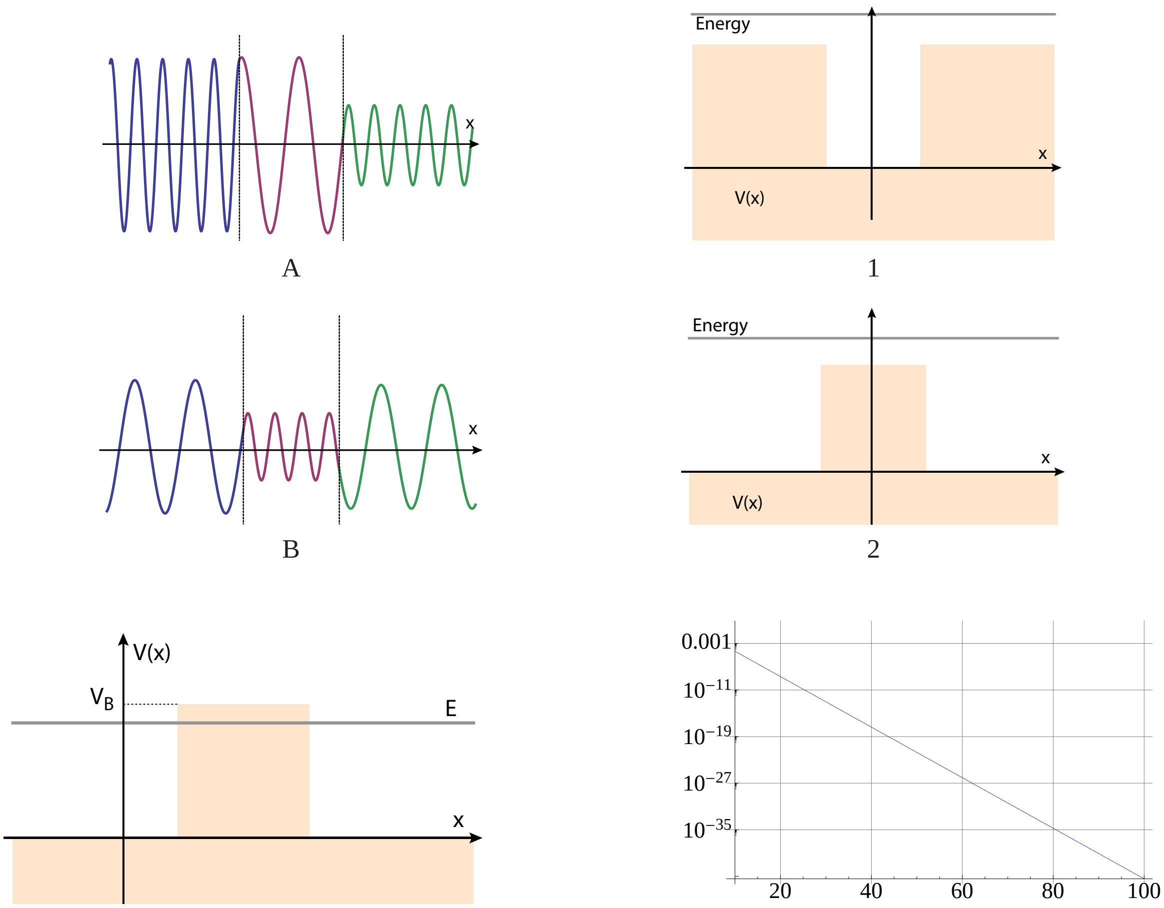


Figure 1: Left: Scattering potential. Right: e^{-x}.