22.02 Intro to Applied Nuclear Physics

Mid-Term Exam

Wednesday, March 21 2012   Name: . . . .

Problem 1:   Short Questions   *24 points*

*These short questions require only **short answers** (but even for yes/no questions give a brief explanation)*

a) In the SEMF, the volume term gives the most important contribution to the binding energy B, setting $B \propto A$ (with A the mass number). What does this tell us about the nuclear force keeping nucleons together?
[Hint: your answer – and the word! – would be much different if the volume term were $B \propto A(A-1)$]

b) True or False?:
i) The Q-value of a fusion reaction is *always* $Q > 0$ (and very large!).
ii) Still it is very difficult to obtain fusion because of the Coulomb repulsion V_C between the fusing nuclei.
ii) Thus we *always* need to give the fusing nuclei enough energy that their energy is $E > V_C$, by going to very high temperatures.

c) What is an observable in quantum mechanics? What mathematical object represents it? (Give also an example).

d) What is a complete set of commuting observables and why do we need it?
Are $\{L_{xy},\ L_z\}$ a complete set of commuting observables? (Here $L_{xy} = L_x^2 + L_y^2$)

e) If we measure the potential energy $\hat{V} = V(\hat{x})$ of a quantum particle and *immediately* after we measure its position $\hat{x}$, is the result of the second measurement random?

f) Which one of the following statements (if any) is correct, based on the properties of the angular momentum and its eigenfunctions?
1) A particle is in the angular momentum eigenstate: $\psi_{l,m_z}(\vartheta, \varphi) = |l\text{=-}1, m_z\text{=0}\rangle$.
2) A particle is in the angular momentum eigenstate: $\psi_{l,m_z}(\vartheta, \varphi) = |l\text{=0}, m_z\text{=-1}\rangle$.
3) A particle is in the angular momentum eigenstate: $\psi_{l,m_z}(\vartheta, \varphi) = |l\text{=1}, m_z\text{=0}\rangle$.
4) A particle is in the angular momentum eigenstate: $\psi_{l,m_y}(\vartheta, \varphi) = |l\text{=1}, m_y\text{=-1}\rangle$.

g) Consider a finite quantum well of depth V_w and width $2a$ (between $-a$ and $+a$, while $V = 0$ outside the well) and a particle in an even energy eigenfunction with energy E. Is the particle bound?

h) A system is subject to a potential $V(x)$ and is found in two possible states described by two different wavefunctions. The first wavefunction is an eigenfunction of the momentum while the second wavefunction is an eigenfunction of the Hamiltonian. Which wavefunction describes a stationary state? Why?

Problem 2:   Temperature   *13 points*

a) In classical thermodynamics, the temperature of an ideal gas (in 1D) is given by $T = \frac{m}{k_B} v^2$ (where k_B is the Boltzmann constant, m the mass and v the velocity). Define the corresponding quantum mechanical observable in terms of observables seen in class.

b) Consider a "gas of particles" inside an infinite well ($V(x) = 0$ for $0 < x < L$ and infinite otherwise). Each particle is in the state

$$\psi(x) = \sqrt{\frac{1}{2L}} \sin\left(\frac{3\pi x}{L}\right) + \sqrt{\frac{1}{L}} \sin\left(\frac{10\pi x}{L}\right) + \sqrt{\frac{1}{2L}} \sin\left(\frac{11\pi x}{L}\right)$$

What is the expectation value (average) of the temperature?
[Note: you should be able to solve this problem without doing any integral!]

Problem 3: Scattering   *12 points*

A source of neutrons produces a flux of neutrons of intensity Φ_{inc} and energy E. To protect the worker, in front of the source a wall of thickness L has been built, which we can model with a potential barrier $V > E$. To monitor her health, the worker wears a detector that measure the neutron flux.

a) Sketch a simple drawing showing how you model this problem (assume we leave in a 1D world!). Show in the sketch the characteristics of the wavefunction describing the neutron.

b) What is the flux of neutrons measured by the detector? Assume a simple model for the tunneling, in the limit where the tunneling probability is small: this means that you do not need to solve completely the problem to find the –approximate– answer.

Problem 4: Spontaneous Fission   *30 points*

Useful quantities: $\hbar c = 197 MeV fm$; $\frac{e^2}{\hbar c} = \frac{1}{137}$; $c = 3 \times 10^8 m/s$; $R_0 = 1.25 fm$.

A note about this problem: Because of the small and large quantities involved, small rounding errors give very different results. This is ok in the Midterm, but if you want to calculate something like this in real life, make sure you pay attention to numerical errors.

Also, although the final answer does give you an idea about the fact that Cm can indeed undergo spontaneous fission, estimating the fission rate in this way is a bad approximation, since we expect Cm to fission not in just these two fragments but to lead to a distribution of possible pairs of fragment, all accompanied by the release of some neutrons. This would be a 3 (or larger) body problem, which complicates estimates.

a) Consider the isotope Curium-250 ($^{250}_{96}$Cm), with mass 232.938 GeV. Given its A and Z numbers, do you expect this isotope to be stable?

Curium-250 is the lightest nuclide to undergo spontaneous fission as the main decay mode. We want to analyze this decay mode following the same theory we saw for alpha decay and in particular *estimate* the half-life of ^{250}Cm. The following questions will guide you through the estimation.

b) Assume that the spontaneous fission leads to the decay:

$$^{250}_{96}\text{Cm} \rightarrow {}^{130}_{52}\text{Te}_{78} + {}^{120}_{44}\text{Ru}_{76}$$
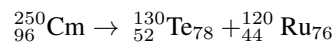
What is the Q-value for the reaction? How does this compare to the usual Q-value for typical alpha decay?
[The mass of $^{250}_{96}$Cm is 232.938GeV, the mass of $^{130}_{52}$Te$_{78}$ is 121.002 GeV and the mass of $^{120}_{44}$Ru$_{76}$ is 111.724GeV]

c) What is the Coulomb potential $V_C(R) = \frac{Q_1 Q_2}{R}$ at the distance $R = R_{Te} + R_{Ru}$, where R_X is the nuclear radius? What is the distance R_c at which the Coulomb potential is equal to the Q-value?

d) We want to estimate the probability of tunneling for an effective particle in the center of mass frame of the Tellurium and Ruthenium nuclei (with reduced mass μ).
To estimate the tunneling probability we replace the Coulomb barrier with a rectangular barrier of height $V_H = V_C(R)$ and length $L = (R_c - R)$. What is the tunneling probability?

e) Besides the tunneling probability, to calculate the spontaneous fission rate we need to calculate the frequency $f = \frac{v}{R}$ for the reduced effective particle to be at the edge of the Coulomb potential. Here v is the reduced particle speed inside the nuclear well (depth $V_0 = 35$MeV) when taking Q as the (classical) energy.

f) Finally, give the decay rate λ and the half-life for spontaneous fission of Curium 250.

Problem 5: Match the potential   *21 points*

A quantum system in a 1D geometry has energy as shown by the green line and is subjected to the potential energy as in the figures labeled A-C, with 3 or 5 regions of different potential height.

For each figure 1-7, state whether the curve plotted is a good energy eigenfunction for one (or more!) of the potentials and energies in Fig. A-C. Provide a **brief** explanation of the reasoning that lead you to your conclusion.

(Notice: here I plot the real part of the eigenfunction).

A

1

2

3

B

C

4

5

6

7
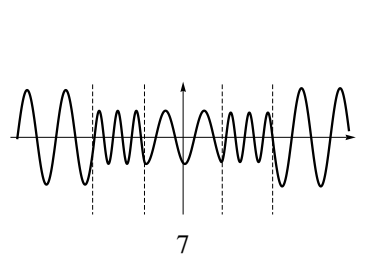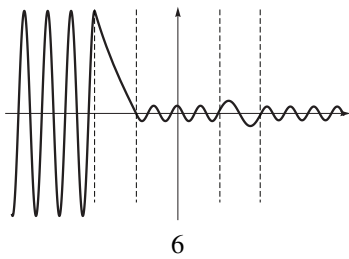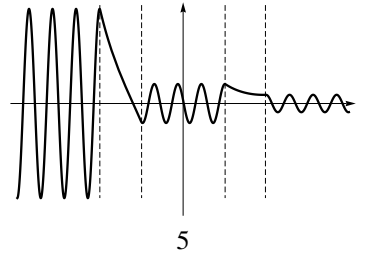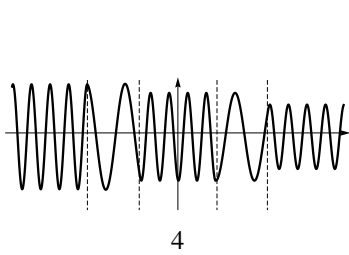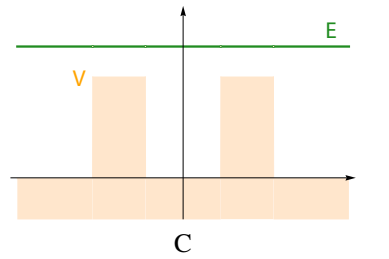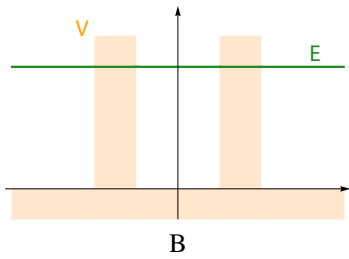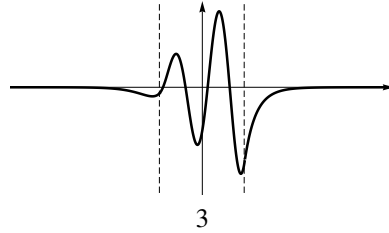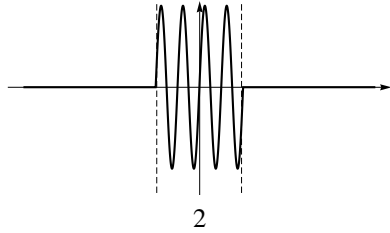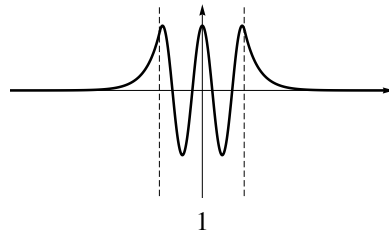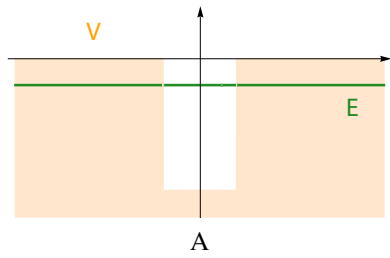
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