22.02 – Introduction to Applied Nuclear Physics

Problem set # 6

Issued on Saturday Apr7 31, 2012. Due on Friday Apr. 13, 2012

Problem 1: Hamiltonian in the Center of Mass coordinates

a) Show that the kinetic energy of a two particle system in the center of mass coordinates

$$\begin{cases} \vec{R} &= \frac{m_1\vec{x}_1+m_2\vec{x}_2}{m_1+m_2} \\ \vec{r} &= \vec{x}_1-\vec{x}_2 \end{cases} \qquad \begin{cases} \vec{P} &= \vec{p}_1+\vec{p}_2 \\ \vec{p} &= \frac{m_2\vec{p}_1-m_1\vec{p}_2}{M} \end{cases}$$

is given by

$$E_{kin} = \frac{P^2}{2M} + \frac{p^2}{2\mu}$$

where μ is the reduced mass

$$\mu = \frac{m_1 m_2}{m_1+m_2}$$

and $M = m_1 + m_2$.

b) Show that for the Hamiltonian

$$H = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + V(|\vec{x}_1 - \vec{x}_2|)$$

you can use separation of variables to find two differential equations. What is the center of mass kinetic energy, $\frac{\hat{P}^2}{2M}$?

Problem 2: Deuteron excited states

Show that the deuteron has no bound excited states by estimating the excited state energy and comparing it to the potential well. Consider two cases for the excited state:

a) higher order radial eigenfunctions,

b) non-zero angular momentum.

c) If the nuclear force had a longer range $R^*{>}R_0$ we could possibly have excited states. What would be the minimum and maximum values of R^* so that there could be an excited state with $l = 1$ but no excited state with higher orders of the radial eigenfunction?

Problem 3: Deuteron eigenfunction

a) Use the boundary continuity condition and normalization condition to determine the coefficients of the deuteron eigenfunction:

$$u(r) = A\sin(kr) + B\cos(kr), \qquad 0 < r < R_d$$
$$u(r) = Ce^{-\kappa r} + De^{\kappa r}, \qquad r > R_d$$

[Hint: use the known values for the nuclear interaction strength $V_0 = 35$MeV and range $R_d = 2.1$fm. Also assume that we solved numerically the problem to find the wavenumbers k and κ giving a corresponding binding energy $E = -2.2$MeV.]

b) Evaluate the root-mean-square radius of the deuteron from the wavefunction you found in a).

c) What is the probability of finding the deuteron outside the range of the nuclear interaction between neutron and proton (i.e. $P(r > R_d)$)?

Problem 4: Di-neutron

Prove that the di-neutron (a nucleus composed of two neutrons) cannot exist. The following question should help you in making your proof:

a) What angular momentum eigenvalue l would give a low enough energy to allow a bound state?

b) What is the behavior of the associated angular momentum eigenfunction under particle exchange?

c) Given your answer in b) and the symmetry properties of fermions, what should then be the spin state of the di-neutron? What does that imply for the total potential energy, given the spin dependence of the nuclear force ?

22.02 Introduction to Applied Nuclear Physics
Spring 2012