22.02 – Introduction to Applied Nuclear Physics

Problem set # 4

Issued on Tuesday March 13, 2012. Due on Monday March 19, 2018

Problem 1: Solved Problem

This problem will be solved during recitation on 03/13. You don't need to show your work here.

Consider a particle in a well: the potential is zero between a and $a + L$ and infinite everywhere else.

a) We measure the particle and we find that it is at the position $a + L/13$. What is the state of the particle immediately after the measurement? Do you expect the particle to be in the same state 10μs after the measurement?

b) We now measure the momentum of the particle. What are the possible outcomes? What is the state after the measurement? Will it evolve in time?

c) Given the wavefunction you found in b), what is the expectation value of a position measurement?

d) Given the wavefunction you found in b), what is the probability that an energy measurement outcome will give $E' = \frac{3\hbar^2\pi^2}{2mL^2}$? What is the probability for $E'' = \frac{9\hbar^2\pi^2}{2mL^2}$? What is the expectation value of an energy measurement?

Problem 2: Short Questions

a) What is an observable in quantum mechanics and by which mathematical object is it represented?

b) A system has Hamiltonian $\mathcal{H}$ with a discrete set of eigenfunctions $\{\psi_n(x)\}$, $n = 1, 2, \ldots$.
What is the system wavefunction?

c) Consider a particle described by the wavefunction $\varphi(x)$ and an Hamiltonian with eigenvalues $\{E_n\}$ and corresponding eigenfunctions $\{w_n(x)\}$. Is the probability of obtaining the energy E_4 given by $p(E_4) = \int_{-\infty}^{\infty} dx\, w_4^*(x)\varphi(x)$?

Problem 3: Particle in 1D box

Consider a free particle moving in a box with an infinite potential bounding the box at $x = -L/2$ and $x = L/2$.

a) Solve the energy eigenvalue problem, $\mathcal{H}[w_n(x)] = E_n w_n(x)$, and give a complete listing of both the normalized eigenfunctions, $\{w_n(x)\}$, and the energy spectrum, $\{E_n\}$.

b) Now suppose the system is in a *superposition* quantum state:

$$\psi(x) = a_2 w_2(x) + a_4 w_4(x)$$

where w_1, w_4 are the energy eigenfunctions found earlier. What are the possible outcomes of an energy measurement in this case?

c) Determine a_2 and a_4 if the probability of finding $E = E_2$ is $P_2 = 3/7$, and the probability of measuring $E = E_4$ is $P_4 = 4/7$.

d) What is the probability of finding the particle at $x = \frac{1}{3}L$? What is the expectation value of the position?

e) What are the possible outcomes of a momentum measurement and their probabilities?

Problem 4: Particle in a 3D box

Consider now a particle in the usual 3D space, moving in a box of size L_x, L_y, L_z. This corresponds to a potential that is zero in the region: $0 \le x \le L_x$, $0 \le y \le L_y$, $0 \le z \le L_z$ and infinite otherwise.

a) Solve the 3D Hamiltonian eigenvalue equation:

$$\mathcal{H}\psi(x, y, z) = E\psi(x, y, z)$$

applying the boundary conditions to find the eigenvalues and eigenvectors.

b) In the 1D case we found that the energy eigenvalues could be labeled by the *quantum number* n ($E_n = n^2 E_1$). How many quantum numbers do you need to define the energy eigenvalues in the 3D case?

Problem 5: Double Well

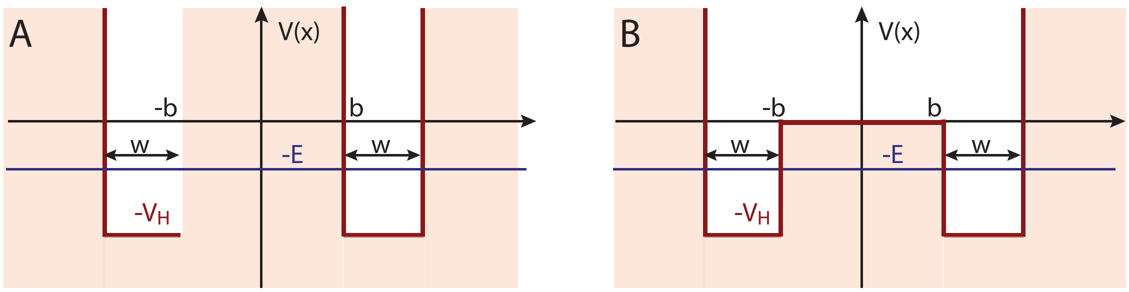

Figure 1: A) Infinite double well. B) Infinite well with a finite barrier.

a) Consider the double well in Fig. 1 A. What are the allowed stationary wavefunctions for a particle of mass m? What are the corresponding energy levels?

b) Consider now the double well in Fig. 1 B. Sketch the eigenfunctions (e.g. their real part) in each region defined by Fig. 1B. What are the limits of these eigenfunctions for $b \to 0$ and $w \to 0$?

c) What conditions do you need to impose on the allowed eigenfunctions because of the symmetry of the problem?

d) Write the even solution for the double well in Fig. 1 B and find the equation that will set the allowed eigenvalues (you do not need to solve the equation).

Problem 6: Bound states in a Well

Consider the finite square well potential of width $2a$, as seen in class, and assume that the odd-parity ground state is just barely bound, meaning that its wavelength is $\lambda = 4a$.

a) Compute the kinetic energy associated with this wave, $E_{kin} = \frac{\hbar^2 k^2}{2m}$, and add this to the well depth, $-V_H$ to obtain the ground state energy (negative of binding energy). Draw a picture showing the potential well, the kinetic energy and the binding energy and compute the binding energy value for the case of a deuteron in a well with

$$a = 6\ fm \qquad V_H = 5\ MeV \qquad mc^2 = 1876\ MeV$$

b) What happens when the well is a quarter as wide ($a = 1.5$fm)?

22.02 Introduction to Applied Nuclear Physics
Spring 2012