22.01 Fall 2016, Quiz 2

November 22, 2016

<u>Quiz Instructions:</u> Answers can be given symbolically or graphically, no calculation is necessary. ***No calculators, devices, or anything else allowed, except one double-sided, 8.5 x 11 inch sheet of paper.*** Define any intermediate variables which you need to complete the problems. Generous partial credit will be given for showing correct methodology, even if the solution is not given.

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1 (48 points) Short Answers, 6 points each

Each of these problems can be solved with one sentence, one equation, and/or one graph/picture.

1.1 Why does ionization stopping power *decrease* at energies below $\mathrm{E} = 500\bar{\mathrm{I}}$?

1.2 What is the ratio of the ionization stopping power of an electron to that of a positron, and why? Neglect the Barkas effect, if you know what that is.

1.3 Why can you *not* just use the integral of ionization stopping power to accurately determine the range of protons in soft tissue?

1.4 Find three things wrong with this neutron transport equation, and explain physically (not just mathematically) why they are wrong. Integrating variables (*dStuff*) have been omitted for ease of reading:

$$\frac{1}{v}\frac{d\phi\left(\mathbf{r},E,\Omega,t\right)}{dt}=\frac{\nu\chi\left(E\right)}{2\pi}\int\limits_{V}\int\limits_{E}\int\limits_{\Omega'}\Sigma_f\left(E'\right)\phi\left(\mathbf{r},E',\mathbf{\Omega},t\right)+S_0\left(\mathbf{r},E,\mathbf{\Omega},t\right) \tag{1}$$

$$+\int\limits_{V}\int\limits_{E}\int\limits_{\Omega}\Sigma_s\left(E\right)\phi\left(\mathbf{r},E,\Omega,t\right)F\left(E\rightarrow E',\Omega\rightarrow\Omega'\right)-\int\limits_{V}\Sigma_a\left(E'\right)\phi\left(\mathbf{r},E',\mathbf{\Omega},t\right)-\int\limits_{S}\hat{\Omega}\cdot\nabla\phi\left(\mathbf{r},E,\mathbf{\Omega},t\right)$$

Extra credit (2 points): Find the fourth one.
More extra credit (3 points): Find the fifth one!

1.5 Use the Q-Equation to explain why hydrogen moderator is best moderator.

1.6 What, physically, is a Compton wavelength shift, and why, mathematically, does the Compton energy shift decrease as photon energy decreases?

1.7 What *is* the Klein-Nishina cross section physically speaking, and why does it reach a minimum at $\theta = \frac{\pi}{2}$ at very low incoming photon energies?

1.8 Why is an attenuation coefficient *the same thing* as a macroscopic cross section? Explain both physically and mathematically.

2 (26 points) Photon Spectral Identification

Reminder: Answers can be given symbolically or graphically, no calculation is necessary.

Answer the following questions about the two observed photon spectra from ^{60}Co, which emits two gamma rays at 1,173 keV and 1,332 keV. The spectra were obtained using a standard, small HPGe detector, and a new fiber-optic based detector (FORS), which uses plastic/glass as the active material. [1]

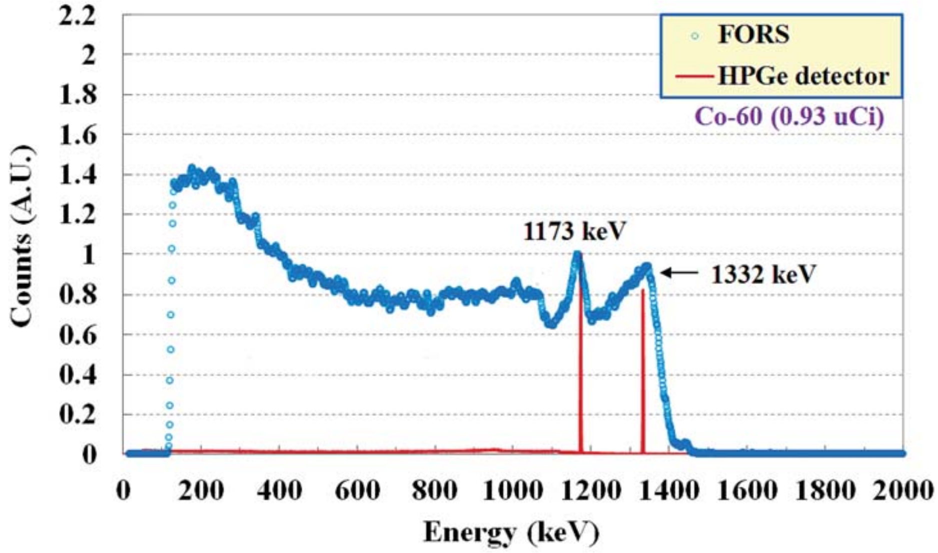

Image courtesy of *Sensors*—Open Access Journal.

2.1 **(5 points) Label every observed feature of physical significance on the two spectra (separately for FORS and HPGe). Redraw the spectra separately in your test booklet for ease of reading.**

2.2 **(10 points) Denote the location of every physically significant peak which should be present from each spectrum, but was not observed. Why was each not observed?**

2.3 **(11 points) Develop simplified expressions for the locations in keV for physically significant features that are or should be present on the spectra, as functions of the observed photopeak energies E_1 (1,173 keV) and E_2 (1,332 keV). You do not need to calculate the actual locations, just make expressions that would evaluate to the correct answer.**

3 (26 points) Teetering with Criticality - The Demon Core

The Demon Core was a barely sub-critical sphere of plutonium that claimed two lives in two different experimental mistakes. One time, a tungsten carbide reflector brick fell on top of the barely sub-critical Demon Core, reflecting the neutrons back in and causing the core to briefly go supercritical.

3.1 (6 points) Develop an expression for the critical radius of a bare sphere of plutonium, in terms of its material properties and nuclear data.

3.2 (20 points) Explain what happened to the criticality of the sphere during each of the following events in the demon core. Using the one-group criticality relation, explain physically what happened to the parameters in each step, and answer each question. Follow this example:

$$Control\, rods\, lowered\, into\, reactor \qquad k_{eff} \downarrow\downarrow = \frac{A + B \downarrow}{C \uparrow + D \downarrow} \tag{2}$$

3.2.1 The Demon Core was at rest, uncovered. Write the *criticality inequality*, and state physically why the sphere was sub-critical.

3.2.2 The tungsten carbide brick was dropped onto the Demon Core. (Write an expression for the total macroscopic absorption cross section of tungsten carbide (WC) in terms of its constituent elements and microscopic cross sections)

3.2.3 The Demon Core went prompt-supercritical, releasing a huge amount of power. (*What does prompt-supercritical mean?*)

3.2.4 Fission products built up in the Demon Core. (This doesn't mean that it went sub-critical just yet...)

3.2.5 The tungsten carbide brick was knocked off the Demon Core, ending the criticality excursion.

References

[1] W. J. Yoo et al. "Development of a Small-Sized, Flexible, and Insertable Fiber-Optic Radiation Sensor for Gamma-Ray Spectroscopy." *Sensors*, 15(9):21265 (2015).

Useful Formulas

$$T = E\frac{\alpha\left(1-cos\left(\theta\right)\right)}{1+\alpha\left(1-cos\left(\theta\right)\right)} \qquad \frac{\omega}{\omega'} = 1+\alpha\left(1-cos\left(\theta\right)\right) \qquad \alpha = \frac{E}{m_e c^2} \tag{3}$$

$$\frac{d\sigma}{d\theta} = \frac{k_0^2 e^4}{2m_e^2 c^4}\left(\frac{\nu'}{\nu}\right)^2\left(\frac{\nu}{\nu'}+\frac{\nu'}{\nu}-sin^2\theta\right) \tag{4}$$

$$\Delta\lambda = \frac{h}{m_e c}\left(1-cos\left(\theta\right)\right) \qquad cot\left(\phi\right) = \left(1+\alpha\right)tan\left(\frac{\theta}{2}\right) \tag{5}$$

$$\sigma_C \propto \frac{Z}{E} \qquad \sigma_\tau \propto \frac{Z^5}{E^{7/2}} \qquad \sigma_\kappa \propto Z^2 ln\left(\frac{2E}{m_e c^2}\right) \tag{6}$$

$$Q = T_3\left(1+\frac{M_3}{M_4}\right) - T_1\left(1-\frac{M_1}{M_4}\right) - \frac{2}{M_4}\sqrt{T_1 T_3 M_1 M_3}cos\left(\theta\right) \tag{7}$$