Taking a Math Test

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Characters

First Voice
Second Voice
Student

Stage has a chair down center. Student is sitting in the chair taking a test. Open notes test. Voices are upstage arguing, student reacting. From offstage, a professor says "You may begin."

S - Number 1. Let lambda 1 be the largest eigenvalue of the adjacency matrix of a graph with maximum degree D. Show that lambda 1 is less than D.

2 - Any ideas?

1 - Not yet.

2 - Ok, move on.

1 - Number 2.

2 - Wait, Stanley said that 2 would be hard.

1 - That's true.

2 - Move on to 3.

1 - There's a chance we'll get all four.

2 - That's not how the test is designed. We should aim for a solid three. Four would be a bonus.

1 - [*disgruntled*] Fine. Number 3.

S - Number 3. Find a group G such that this diagram is the Hasse diagram of B_7 mod G. Otherwise, prove no such group exists.

1 - This looks easy. If we find the group, it's the proof.

2 - Why would he ask the question that way if the group exists?

1 - I don't know. He's a good writer.

2 - Yes, but there's something about the way he wrote it, you know. The "otherwise" part. It's more of an addendum.

1 - That could be to not have it sound like an existence proof. Here, I think a construction would be easy. G must be generated by at least one cycle. Remember the G_5 example? Maybe it's something like that. What if rank 2 is generated by two cycles?

2 - I don't see an easy reason that it works.

1 - But you don't see why it doesn't, either.

2 - It just doesn't feel right. [*beat*] Crap, Josh just walked in.

1 - Ignore him.

2 - Are you kidding? That kid is like a god. He skipped Artin's class because he decided it was too easy. Now we don't stand a chance.

1 - Could we focus on the test, please?

2 - Oh god. He sat next to us. Wow, he smells. [*beat*] So smelly and yet so smart.

1 - Ok, we can come back to this. Number 4.

S - Number 4. In Strangetown, there are k people and n clubs. Any club has an even number of members and any two clubs share an even number of members. The only exception is club i and club k+1-i, which share an odd number of members.

1 - Great! This is just Oddtown. We got this.

2 - Agreed. Let's start with the vector form.

1 - Not this again...

2 - What?

1 - The vector form isn't general enough. We need to use the matrix rank trick.

2 - Ugh. The vector trick is so much easier.

1 - And also too weak. [*beat*] Here, I'm writing up the matrix solution.

2 - Great. We're done. Let's move on.

1 - Wait. Do you think I should write the conclusion explicitly on the next line, too?

2 - Oh my god. No. Move on. [*beat*] I bet Josh is done, already.

1 - Ok, I wrote out the conclusion explicitly. Which problem should we do next?

2 - Problem 3. Definitely problem 3.

1 - Why not 1?

2 - It sounds like a matrix theorem we don't know.

1 - We might be able to re-derive it.

2 - I doubt it.

1 - It might even just be something about the limiting case. There's something asymptotic here.

2 - Stanley said it would be easy. And it is. It's easy for Josh because he knows the theorem. We don't. Focus on 3, space cadet!

1 - [*sarcastic*] Whatever you say, slave-driver.

2 - So, for number 3, any new ideas?

1 - I don't know. The construction's ugly, but I don't have any better ideas. You?

2 - [*beat*] No. [*beat*] Let's look in the notes. Chapter 5.

S - Chapter 5. Main Theorem. If S is isomorphic to B_n mod G, then S is rank-symmetric, rank-unimodal-

1 - -and Sperner. [*beat*] How did we not think of this sooner? Look, there is a four node anti-chain! That was so simple.

2 - I told you it was a negative result.

1 - Yes, fine, you were right. I'm just glad we got it.

2 - Same. [*beat*] Oh, no. Josh is standing. Did he finish? If he finished, we're screwed. That means Justin will finish, too, and that weird Korean guy that always talks during lecture. The curve is going to wreck us.

1 - Calm down. He's just asking a question. We're sure on 3 and 4. We just have 1 and 2, now.

2 - And less than 20 minutes. What are you doing?

1 - Hang on. I'm just circling the nodes to show it's not Sperner.

2 - Oh my god. Did you really just recopy the entire thing? Just say it's not Sperner and be done.

1 - [*beat*] Ok, problem 1.

2 - This should be easy. It's problem 1 for christ's sake.

1 - We must be over-thinking it. It feels like an infinite walk, but there's something about it...

2 - Start writing.

1 - But I want to think more.

2 - Start writing so we can get to 2!

1 - I feel like we're on the cusp of something. If we just think a little longer we'll connect the dots.

2 - If we start now, we'll waste less time.

1 - How could we somehow bound the total number of walks?

2 - You're still introducing a new variable. Too complicated.

1 - What if I could show it's inside some K_D by adding edges?

2 - That sounds ugly. Too much computation.

1 - I suppose Perron-Frobenius is too strong for this?

2 - No kidding.

1 - What if we do something with eigenvectors? So let's say we scale the eigenvector so it's max entry is 1. [*beat*] This still sounds messy.

2 - But it works.

1 - It's not very enlightening...

2 - I don't care. Write!

1 - Fine, fine.

2 - Five more minutes before solutions. Let's go. Number 2. Now!

1 - I'm tired.

2 - Hang in there. Let's just write up some progress for 2.

1 - Could we just go? I want to see if we got number 1 right.

2 - No. We need to stay.

1 - Why?

2 - Why? You're asking why? Because look at Josh. You see him? This is second nature to him. We want to go to grad school, right?

1 - Yes.

2 - Well then we need an A in this course. And if we're going to get an A in this course, then we're going to need to compete with people like him. We need to get number 2.

1 - Hey, look at that... the hands on the clock look so pretty.

2 - Focus! Question 2.

S - Question 2. If G is a graph such that G to the L has all odd entries for some L ...

1 - Ah, parity. We must be working over F_2.

2 - Good. Write that down.

1 - Ok, so we have a matrix with one eigenvalue.

2 - Do we compute it? Never mind. Read the rest of the question.

S - … prove that G has some even subset of vertices each adjacent to an even number of vertices.

1 - Does that mean anything to you?

2 - Just write something down.

1 - I can’t. How would I write that as a matrix? Maybe there’s an intermediate step.

2 - Unlikely. We have two minutes. Write.

1 - Wait, what if I’m wrong?

2 - It doesn’t matter. You can only gain points, you can’t lose them. Write it down.

1 - But look at us so far. So far we’re sure everything we wrote is true. Isn’t that a good feeling?

2 - You don’t know it doesn’t work. You just don’t know that it does work. There’s a difference.

1 - It doesn’t feel like a difference.

2 - You’re ridiculous.

1 - I kind of want to hand in the test early. If we hand in now, we can see the solutions. That’s enough for me.

2 - Won’t it feel great if we write something and it’s right?

1 - Won’t it feel bad if we write something and it’s wrong?

Pause. There is about a minute left in the test. Student goes to hand in test, picks up solutions.

2 - How did we do?

1 - Looks like we did pretty well.

2 - We did. Our proofs were solid. Almost identical to the solutions. [*beat*] Good work.

1 - You were right. We could have gotten an extra point on number 2. Sorry.

2 - One point won’t matter in the long run.

1 - Do you want to ask Josh how he did?

2 - No. [*beat*] I think I’m more interested in the solutions.

1 - Look, it’s so beautiful outside.

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2 - I know, right? It's not going to be this nice next week. Want to work on Functional outside?

1 - Sure. I think I came up with an elegant way of showing restricted function spaces are Hilbert. It's very pretty.

2 - Do you mean the proof or the sky?

1 - [*beat*] I'm not sure. Both, I think.

2 - The benches by the tennis court should be free.

1 - I kind of want to explore a little bit. There should be some good benches by Stata.

2 - Let's make sure to find a spot with good Wi-Fi.

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