2.882 System Design and Analysis

February 28

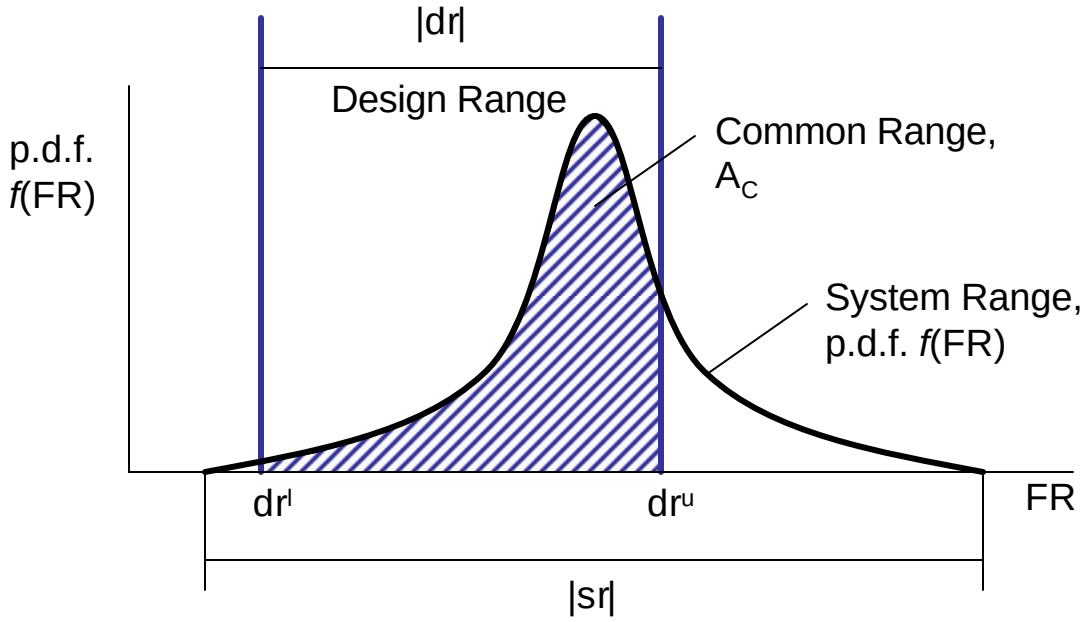
What we'll do today

- Information content for multi-FR
  - Basic statistics/probability
- Allowable tolerance (linear tolerancing) vs. statistical tolerancing

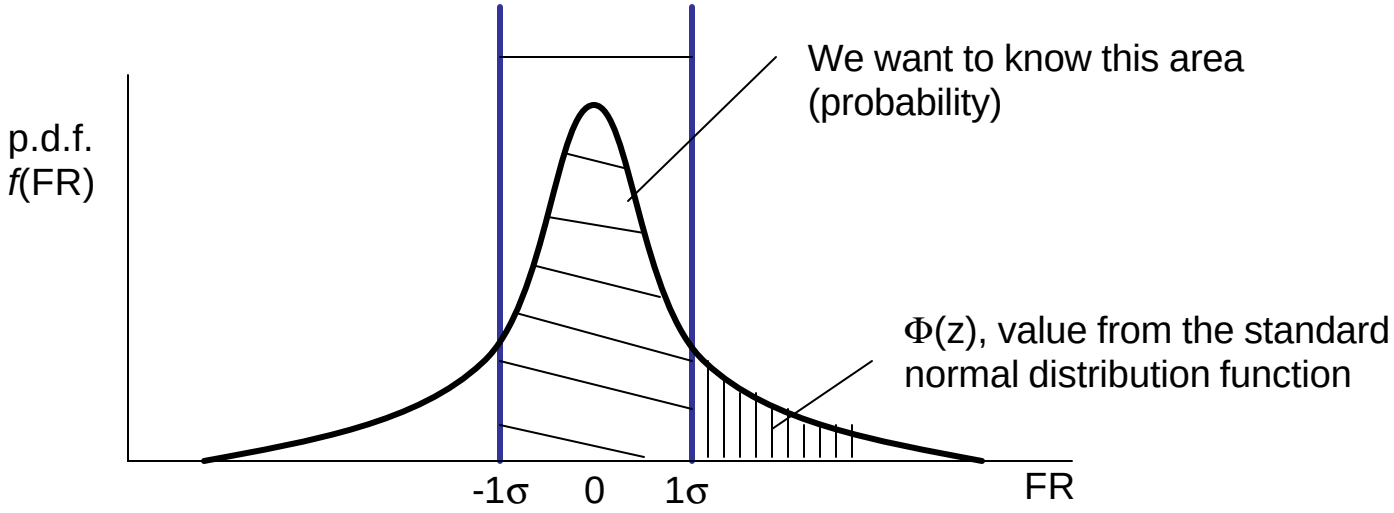
Information content

$$P(FR) = \int_{dr^l}^{dr^u} f(FR)\,dFR$$

$$I = -\log_2 P = -\log_2 P(FR) = -\log_2 \int_{dr^l}^{dr^u} f(FR)\,dFR$$



Normal Distribution



$X \sim N(\mu, \sigma^2)$

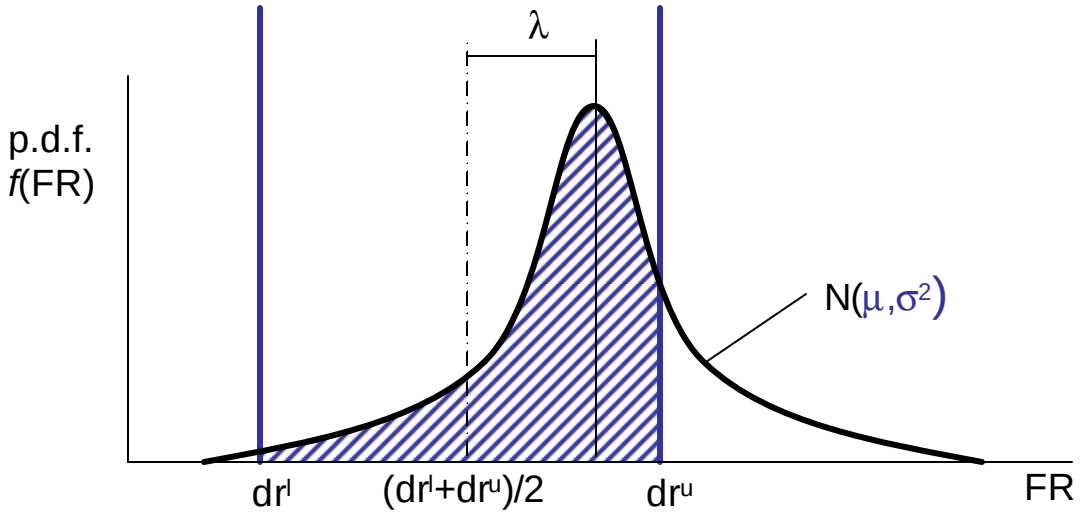
Then, $Z = (X-\mu)/\sigma \sim N(0, 1)$

z	phi(z)	1-phi(z)	prob between +z, -z	ppm	ppb
1	0.84134474	0.15865526	0.68268948	317310.5	
2	0.977249938	0.022750062	0.954499876	45500.12	
3	0.998650033	0.001349967	0.997300066	2699.934	
4	0.999968314	3.1686E-05	0.999936628	63.37207	
5	0.999999713	2.87105E-07	0.999999426	0.57421	574.21
6	0.999999999	9.90122E-10	0.999999998	0.00198	1.980244

HW1, #5

- System range, FR1 ~ $N(\mu,\sigma^2)$
- Design range dr_l =< FR1 =< dr_u
- $\lambda = (dr_u + dr_l)/2 - \mu$

Q: Information Content ?



$dr^l - (dr^l+dr^u)/2 + \lambda$ $dr^u - (dr^l+dr^u)/2 + \lambda$

In terms of σ multiple: $\{dr^l - (dr^l+dr^u)/2 + \lambda\}/\sigma$ $\{dr^u - (dr^l+dr^u)/2 + \lambda\}/\sigma$

Review

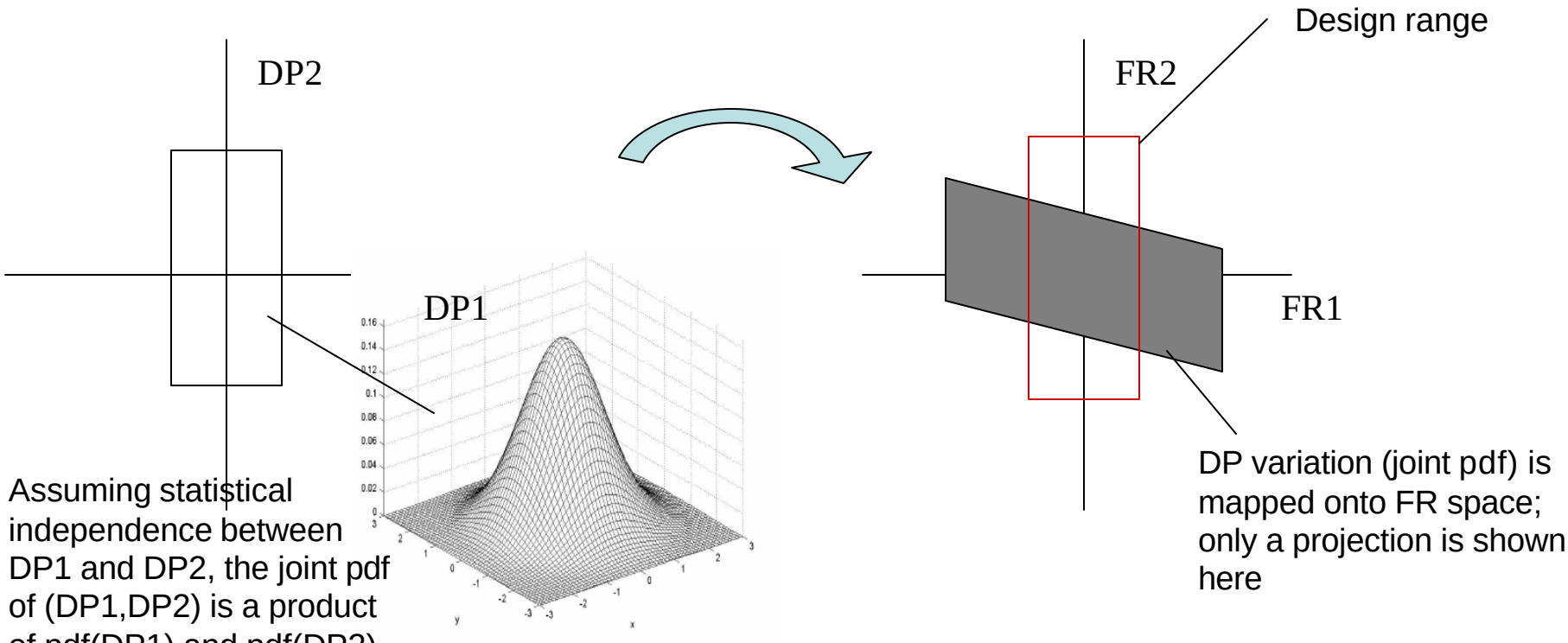
Multiple FR system range

Example

$$\begin{Bmatrix} FR1 \\ FR2 \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{Bmatrix} DP1 \\ DP2 \end{Bmatrix}$$

Design range

FR1: [-0.5 , 0.5]
FR2: [-2.0 , 2.0]



Detecting change in system range

"Monitoring marginal probability of each FR is not only inaccurate but potentially misleading"

Example

$$\begin{Bmatrix} FR1 \\ FR2 \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{Bmatrix} DP1 \\ DP2 \end{Bmatrix}$$

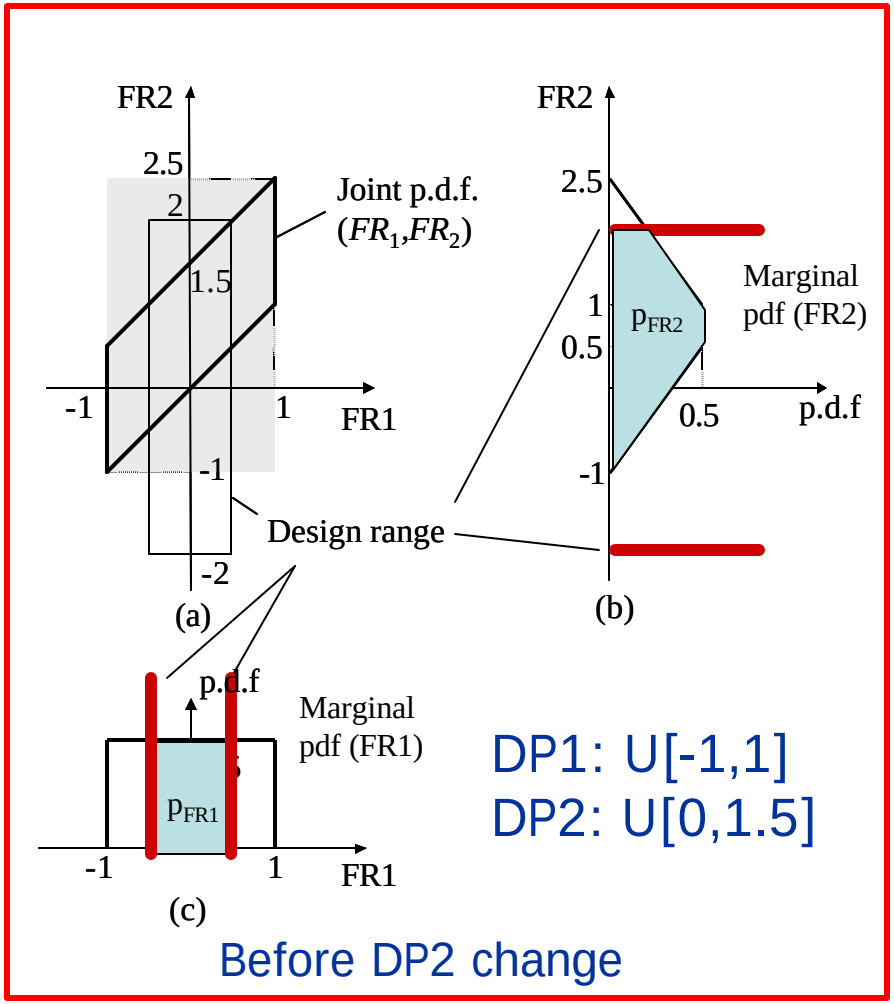
Design range

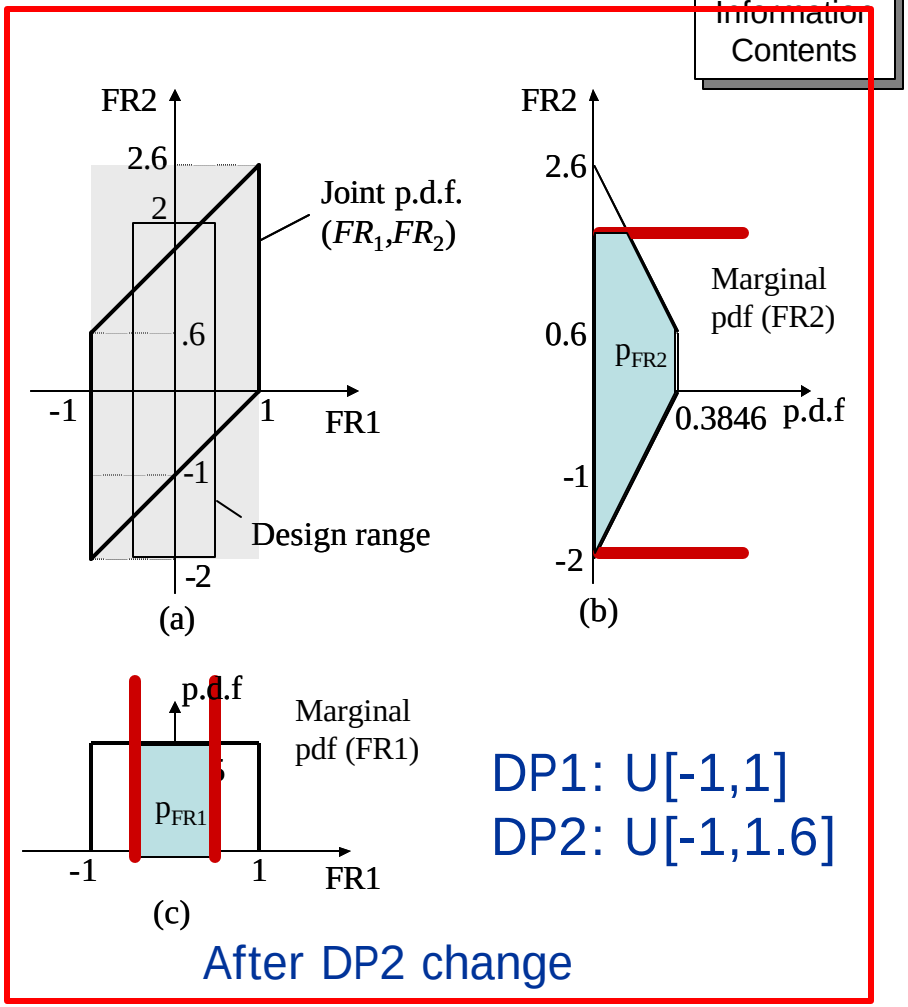
FR1: [-0.5,0.5]
FR2: [-2,2]

Design parameter variation

Initial
DP1: U[-1,1]
DP2: U[0,1.5]

After change
DP1: U[-1,1]
DP2: U[-1,1.6]



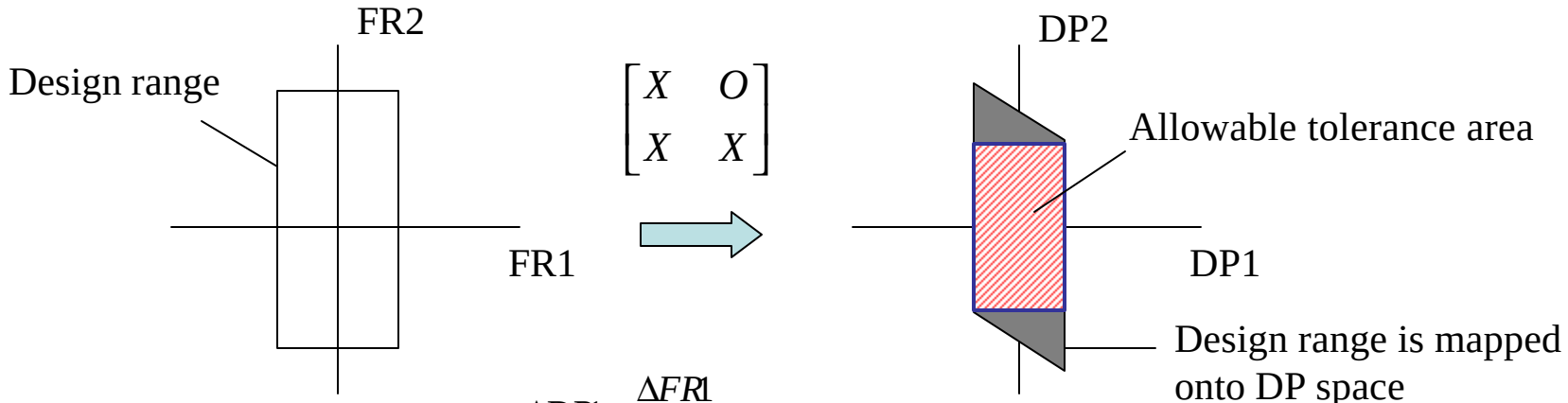


	Wrong			Correct
	p_{FR1}	p_{FR2}	$p_{FR1} \times p_{FR2}$	$p_{FR1,FR2}$
Before	0.5	0.9583	**0.4792**	**0.5**
After	0.5	0.9654	**0.4827**	**0.499**

Review

Allowable tolerance

- Defined for DP
- Tolerances that DPs can take while FRs still remaining completely inside design ranges
- Unconditional tolerance
- Conservative tolerancing



$$\Delta DP1 = \frac{\Delta FR1}{A11}$$

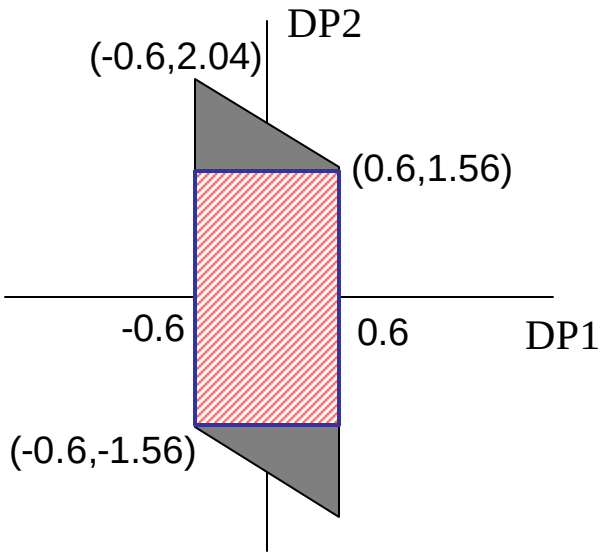
$$\Delta DP2 = \frac{\Delta FR2 - |A21 \cdot \Delta DP1|}{A22}$$

Linear tolerancing vs. Statistical tolerancing

$$\begin{Bmatrix} FR1 \\ FR2 \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 0.4 & 1 \end{bmatrix} \begin{Bmatrix} DP1 \\ DP2 \end{Bmatrix}$$

Design range FR1: [-0.6,0.6]
FR2: [-1.8,1.8]

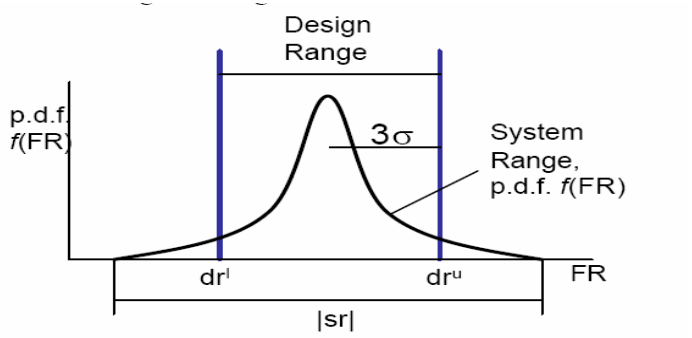
Linear tolerancing



Allowable tolerance

DP1: [-0.6,0.6]
DP2: [-1.56,1.56]

Statistical tolerancing



$3\sigma_{FR1} = 0.6 \rightarrow \sigma_{FR1} = 0.2$
Therefore, $\sigma_{DP1} = 0.2$

$Var(FR2) = 0.4^2 Var(DP1) + 1^2 Var(DP2)$
Thus, $\sigma_{DP2} = 0.5946$

$3\ \sigma_{DP1} = 0.6$
$3\ \sigma_{DP2} = 1.784$