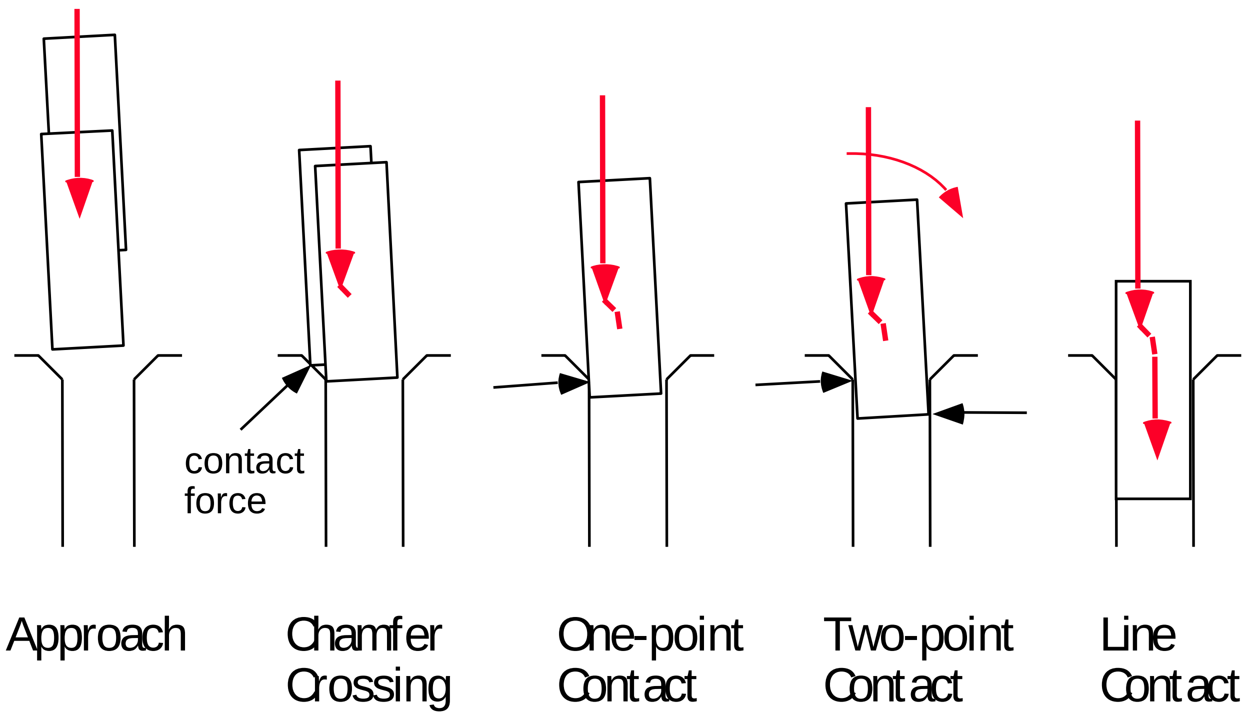
Rigid Part Mating

- Goals of this class
  - understand the phases of a typical part mate
  - determine the basic scaling laws
  - understand basic physics of part mating for simple geometries
  - relate forces and motions arising from geometric errors
  - compare logic branching and direct error-feedback part mating strategies

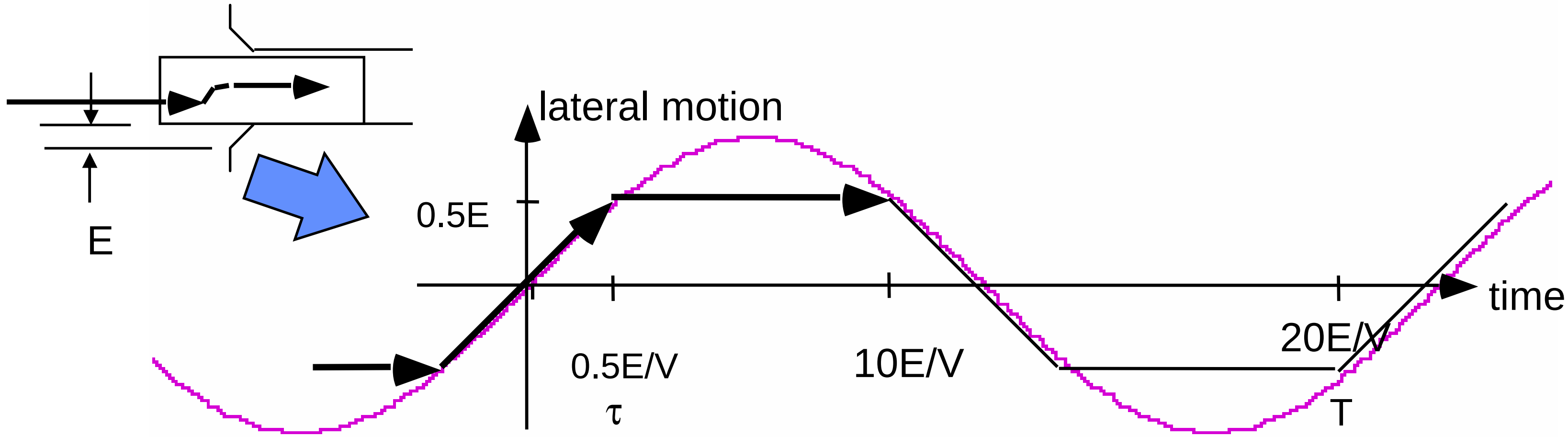
Basic Bandwidth Issues and Time-Mass-Distance Scaling Laws

- Torque required to move a mass M at the end of an arm of length L in time T is proportional to
  - $M L^2/T^2$
- This implies that really fast motions must be really small or use a small arm with small mass
- I estimated
  - my hand's mass = 250g, effective length = 10cm
  - my arm + hand's mass = 1700g, effective length = 35 cm
  - ratio arm:hand of $ML^2 = T^2 = 85$
- Don't forget: arm mass+payload mass=M

Main Phases of a Part Mating Event



Required Bandwidth for Chamfer Crossing



Fourier coefficient = $2 \pi T / (n^2 \pi^2 \tau) \sin (2 n \pi\tau /T)$

$T = 20 E / V$; $\tau = E / 2 V$; $T / \tau = 40$

Period = $2\pi = \omega T = \omega 20E/V$

$\omega = \pi V /10 E$

$f = V / 20 E$

If V = 10 in/s and E = 0.05", f = 10 Hz

If 5th harmonic must be adhered to, bandwidth needed = 50Hz

Trapezoidal Wave Harmonics

Source:

Figure 9-7 in [Whitney 2004] Whitney, D. E. *Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development*. New York, NY: Oxford University Press, 2004. ISBN: 0195157826.

Conclusions

- Gross motions can be (must be) done by large arms that necessarily will move slowly
- No robot arm with practical reach can make fine motion error removal adjustments at 50 Hz
- Fine motions can be fast if they are done by small arms, and must be fast to absorb typical errors at economical speeds
- Big tasks with big parts will take a long time compared to small tasks with small parts
- What we see: small parts cycle times are ~5s while big parts cycle times are ~ 60s.

Essentials of Part Mating Theory for Fine Motions

- Quasi-static assumption
- Geometry of pegs and holes
- Applied forces
- Normal reaction forces and friction reaction forces
- Entry geometry limits
- Wedging conditions
- Jamming conditions
- Alternate strategies for accomplishing fine motion

The Basic Idea

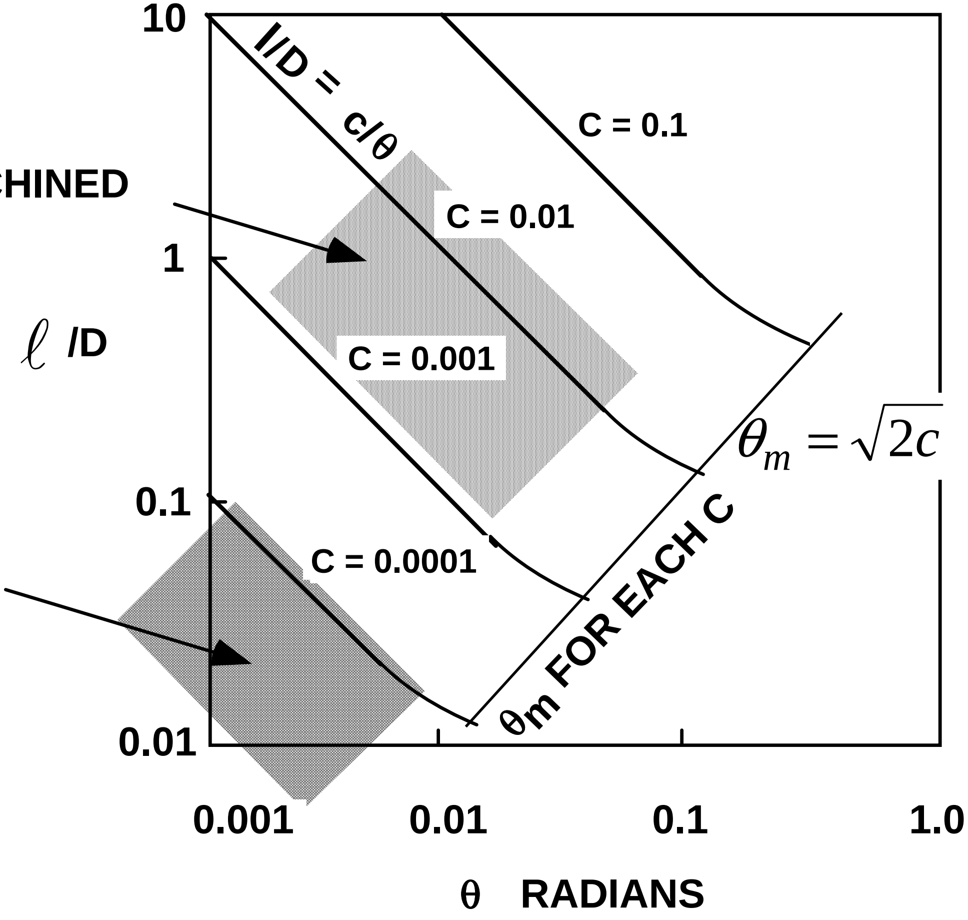
- In gross motions, it pays to pre-plan to prevent errors
- In fine motion, it does not pay to try to prevent errors
- So the principle is to anticipate errors and figure out how to make assembly happen anyway
- This requires us to understand three factors:
  - Geometry
  - Compliance
  - Friction

Geometry of Peg-Hole Mates

TYPICAL MACHINED PARTS

$$\ell / d = c / \theta$$

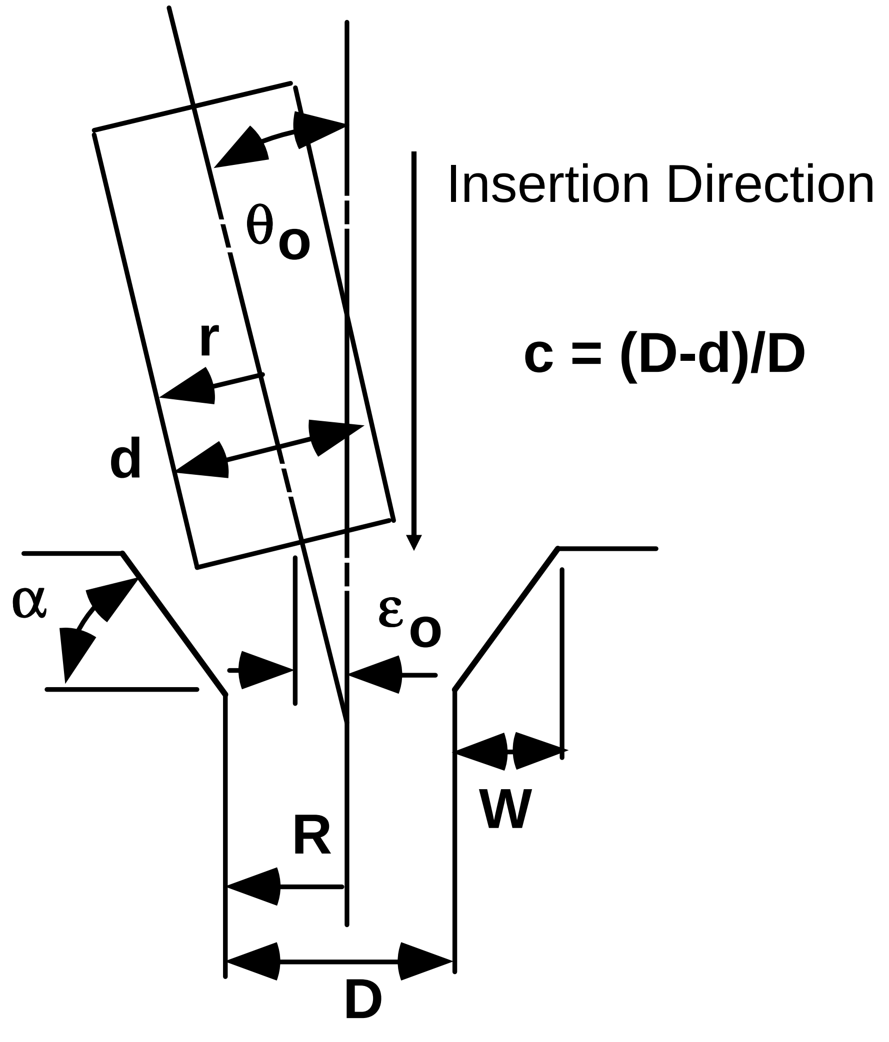
PRECISION PARTS



Dimensioning Practice

Image removed for copyright reasons.
Source:
Figure 10-16 in [Whitney 2004] Whitney, D. E. *Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development*. New York, NY: Oxford University Press, 2004. ISBN: 0195157826.

Geometry Definitions



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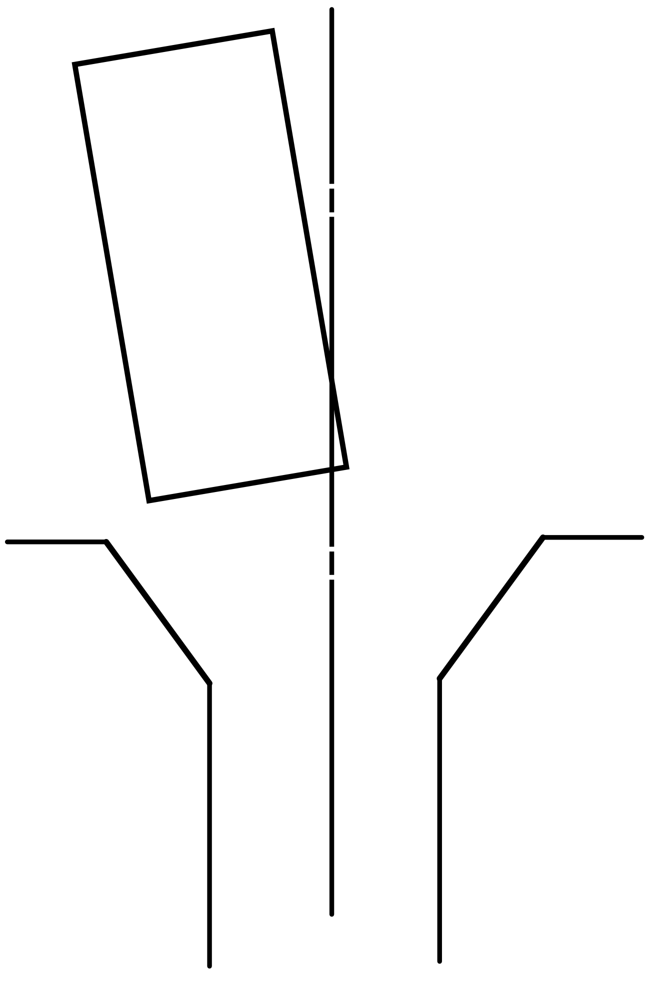

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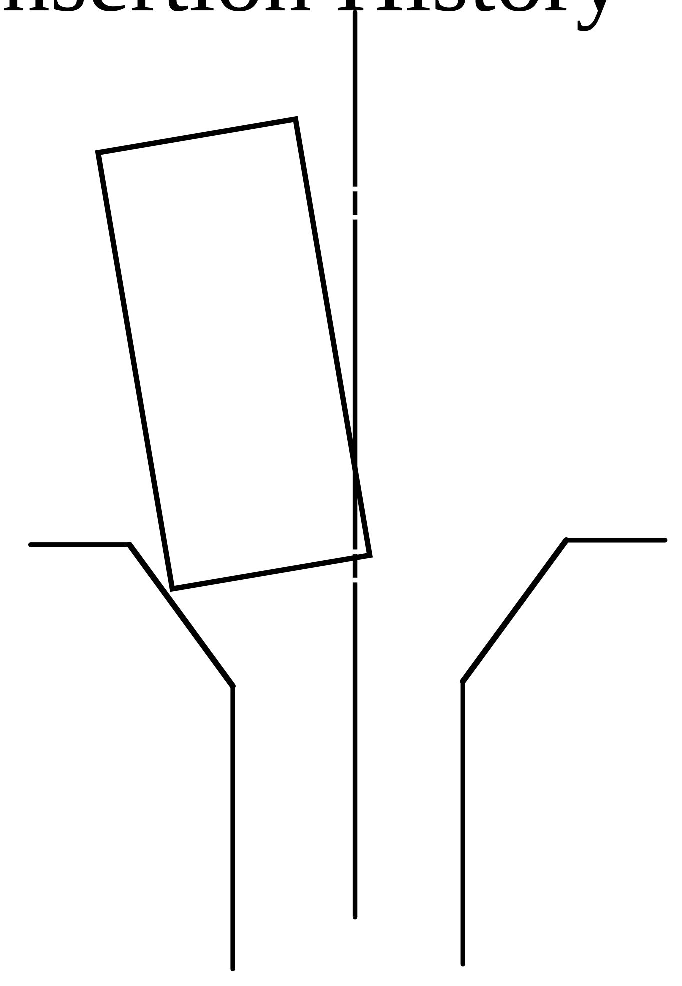

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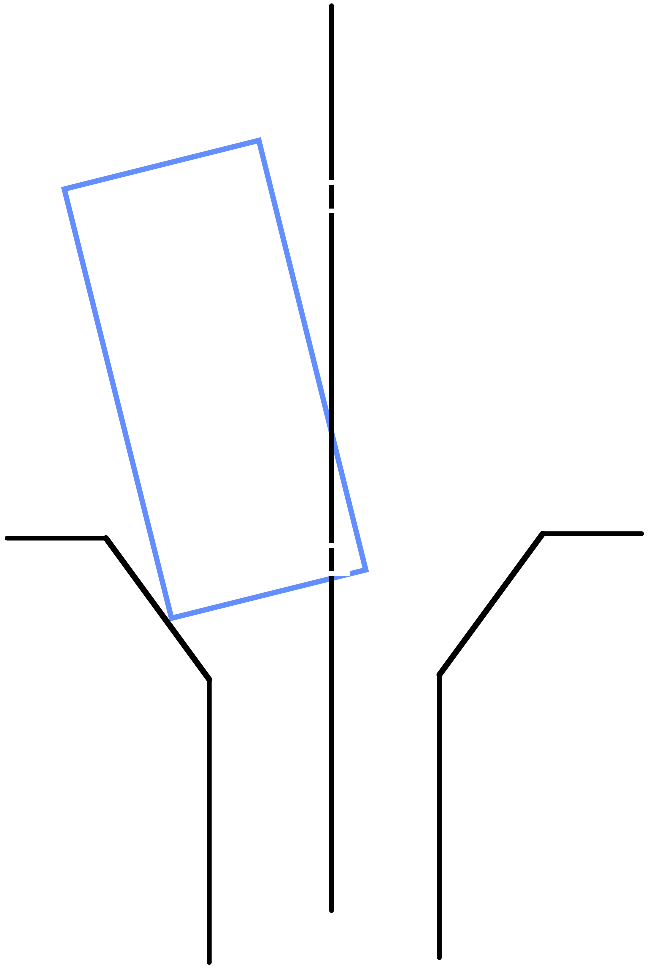

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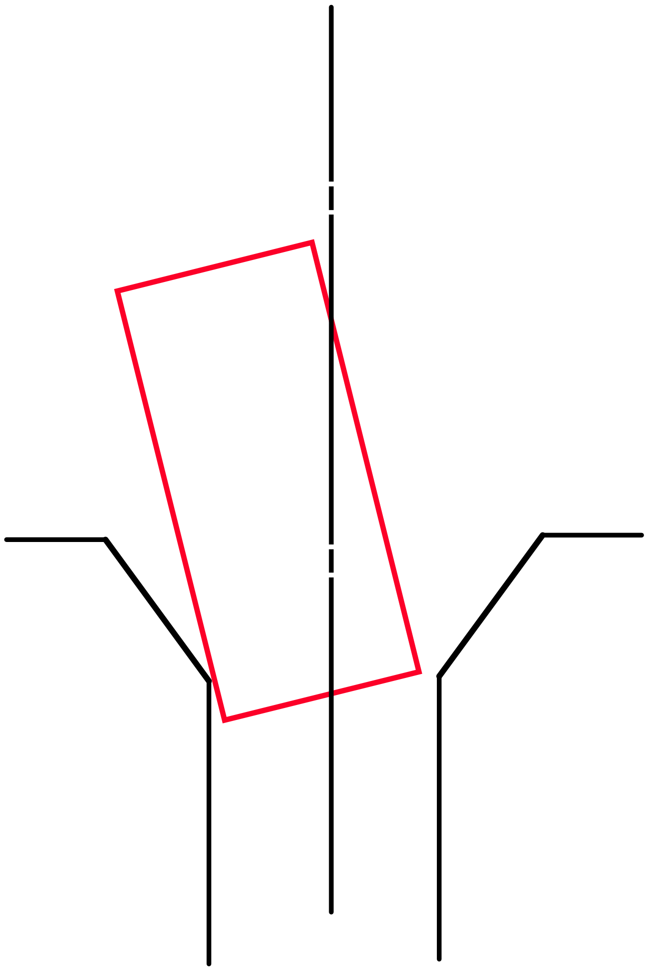

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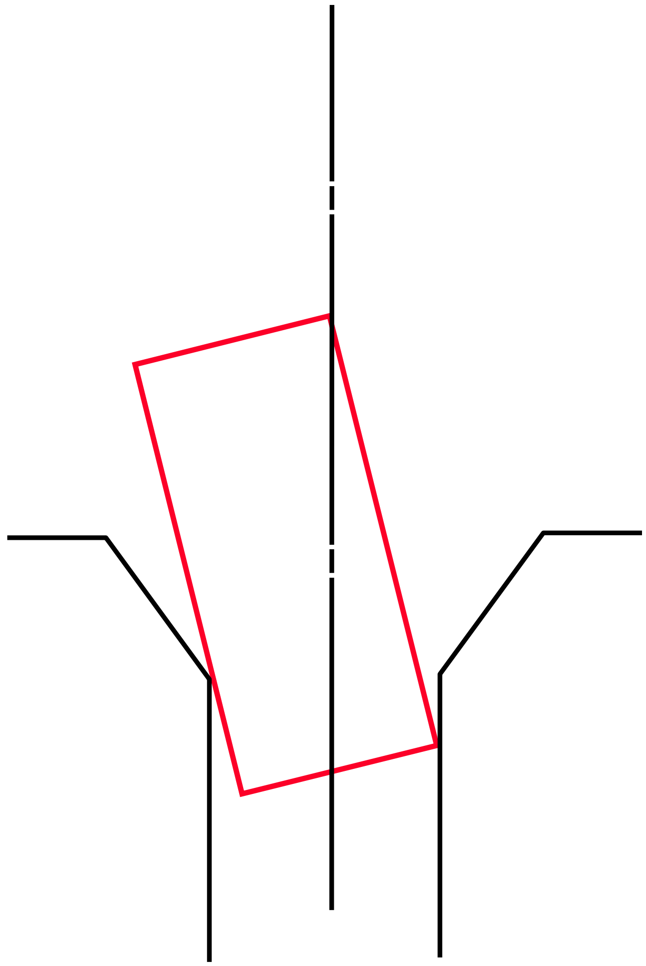

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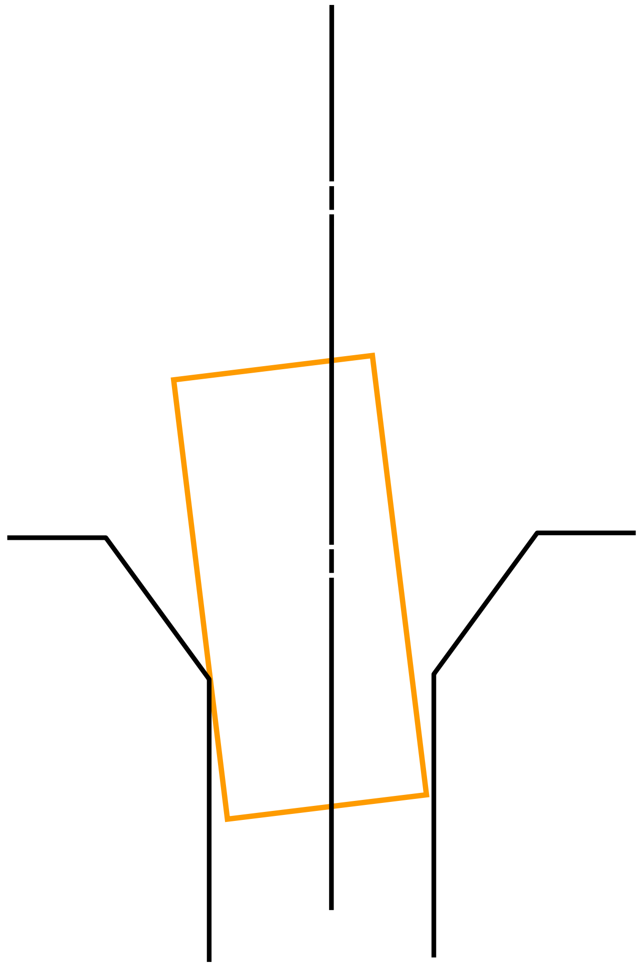

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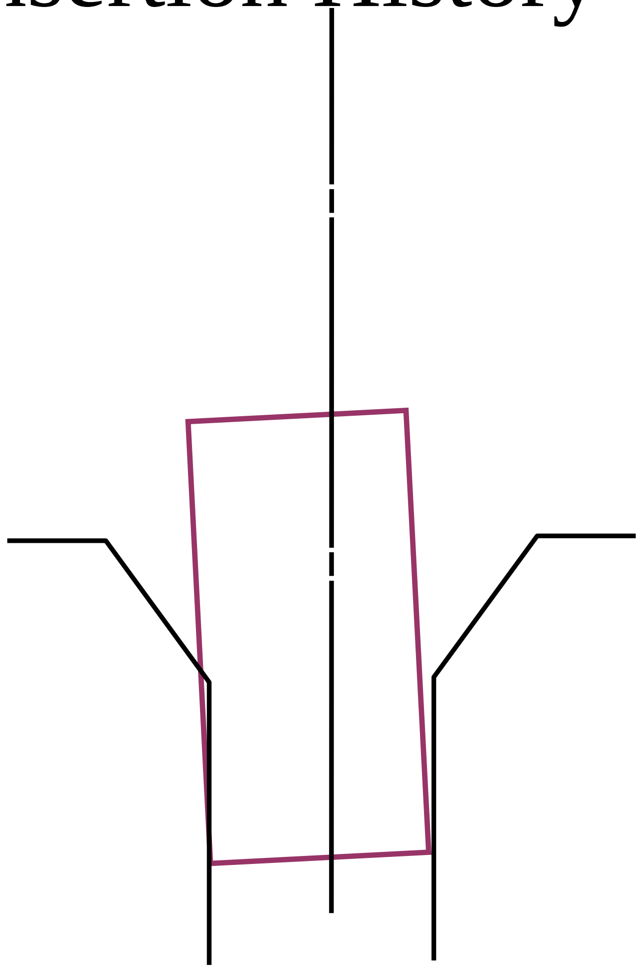
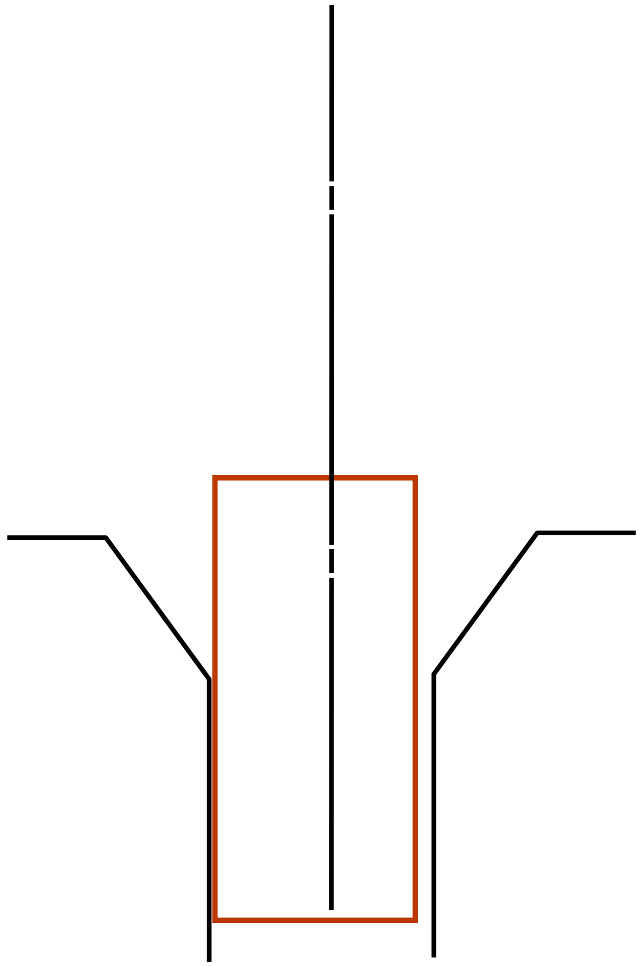


Insertion History



Life Cycle of a Part Mate

Image removed for copyright reasons.
Source:
Figure 10-12 in [Whitney 2004] Whitney, D. E. *Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development*. New York, NY: Oxford University Press, 2004. ISBN: 0195157826.

Model of a Compliant Support

Images removed for copyright reasons.
Source:
Figure 10-10 in [Whitney 2004] Whitney, D. E.
Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development.
New York, NY: Oxford University Press, 2004. ISBN: 0195157826.

All support is assumed concentrated at one point and consists of one lateral stiffness and one angular stiffness

How Compliance Center Reacts to Force

Images removed for copyright reasons.
Source:
Figure 10-11 in [Whitney 2004] Whitney, D. E. *Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development*. New York, NY: Oxford University Press, 2004. ISBN: 0195157826.

Force away from C. C. causes rotation and translation

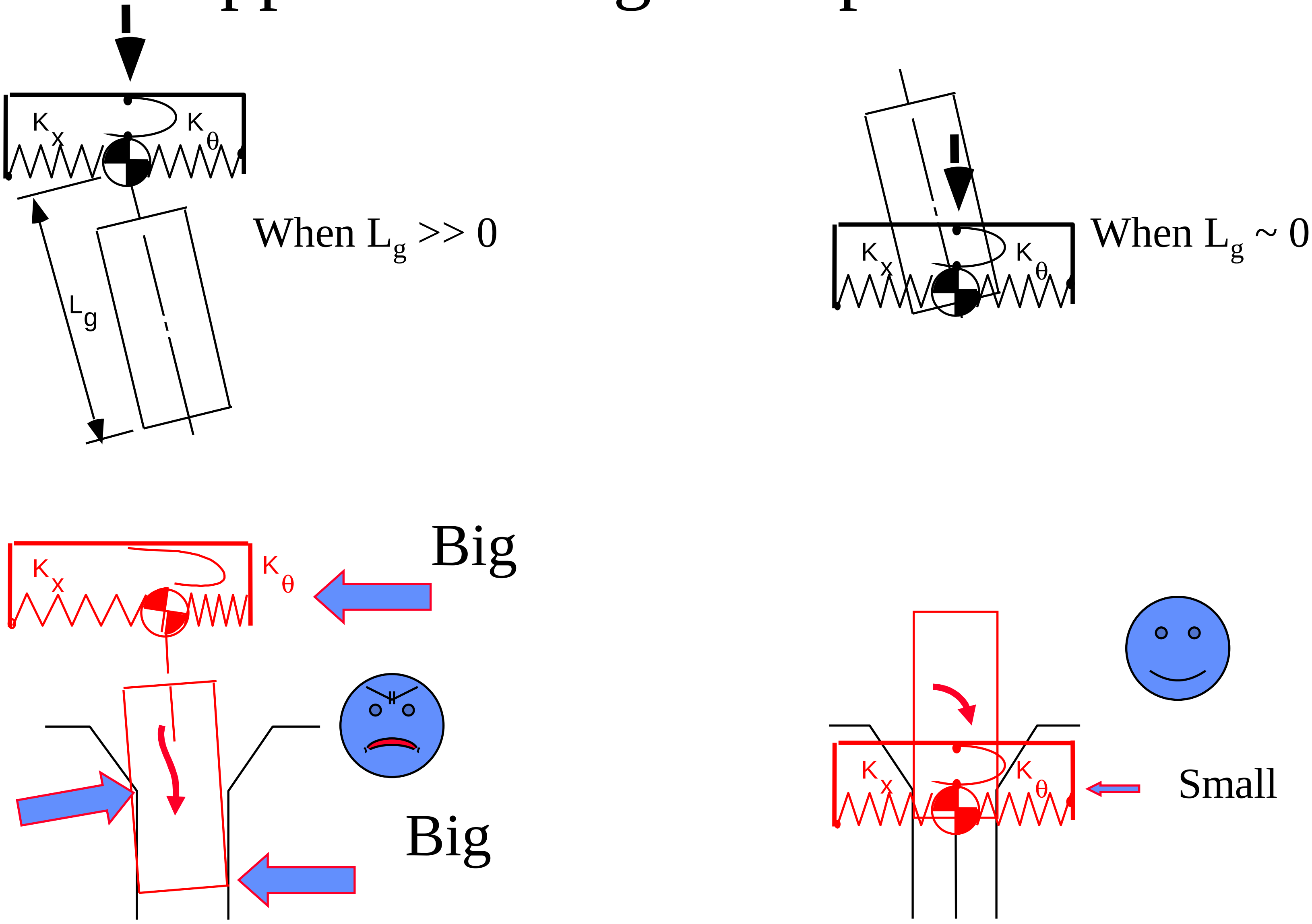
Force on C. C. causes only translation

Forces and Moments - Two Point Contact Case

Images removed for copyright reasons.
Source:
Figure 10-18 in [Whitney 2004] Whitney, D. E. *Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development*. New York, NY: Oxford University Press, 2004. ISBN: 0195157826.

All applied and reaction forces are expressed in coordinates at peg's tip

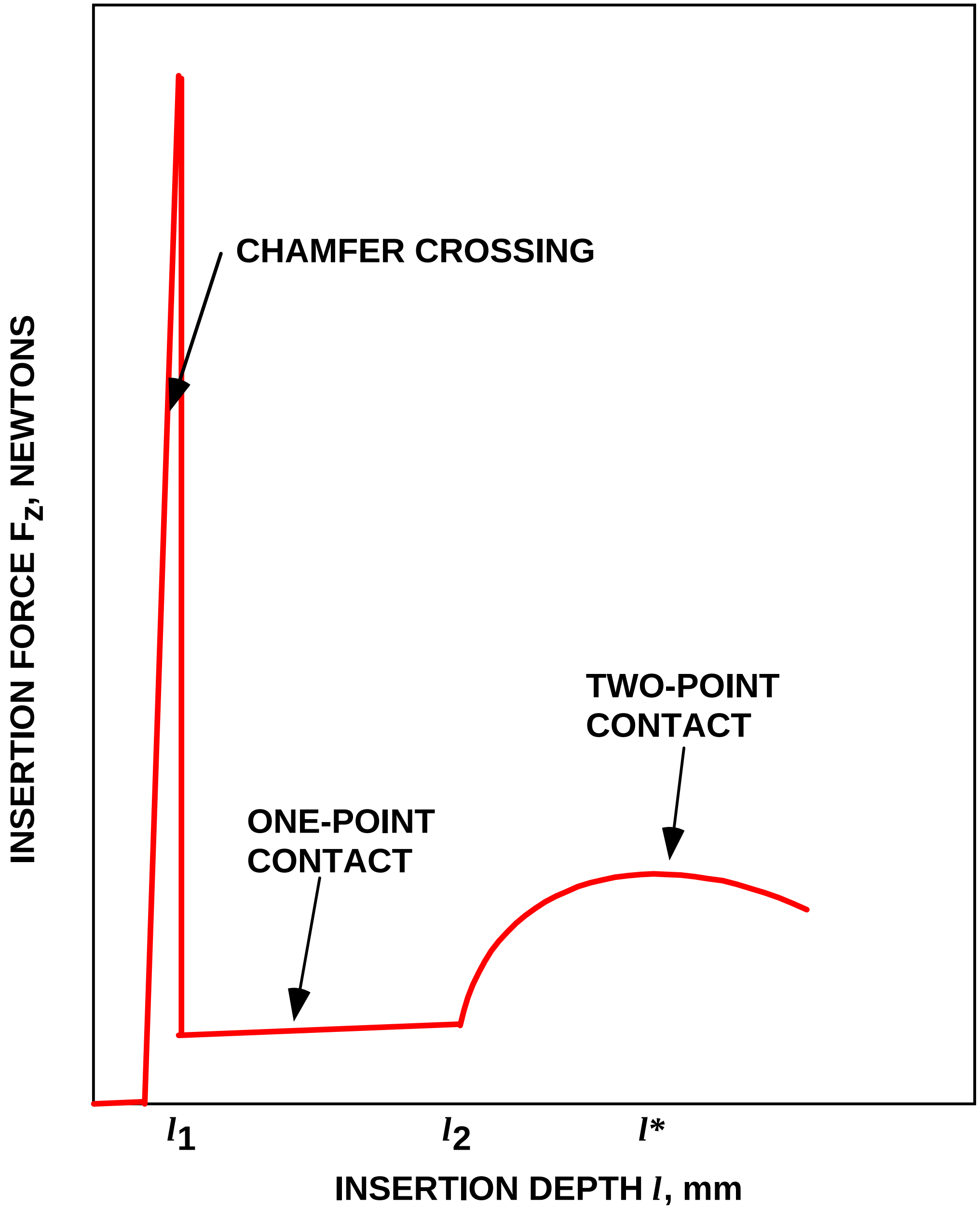
Forces Applied During Two-point Contact



Making L_g Small is Good

- How to do it?
- Active Robot Force Feedback
  - Costly
  - Slow
- Some way that acts by itself
- It was invented almost 30 years ago
- Called Remote Center Compliance
- Reduces assembly force
- Avoids one of two main failure modes

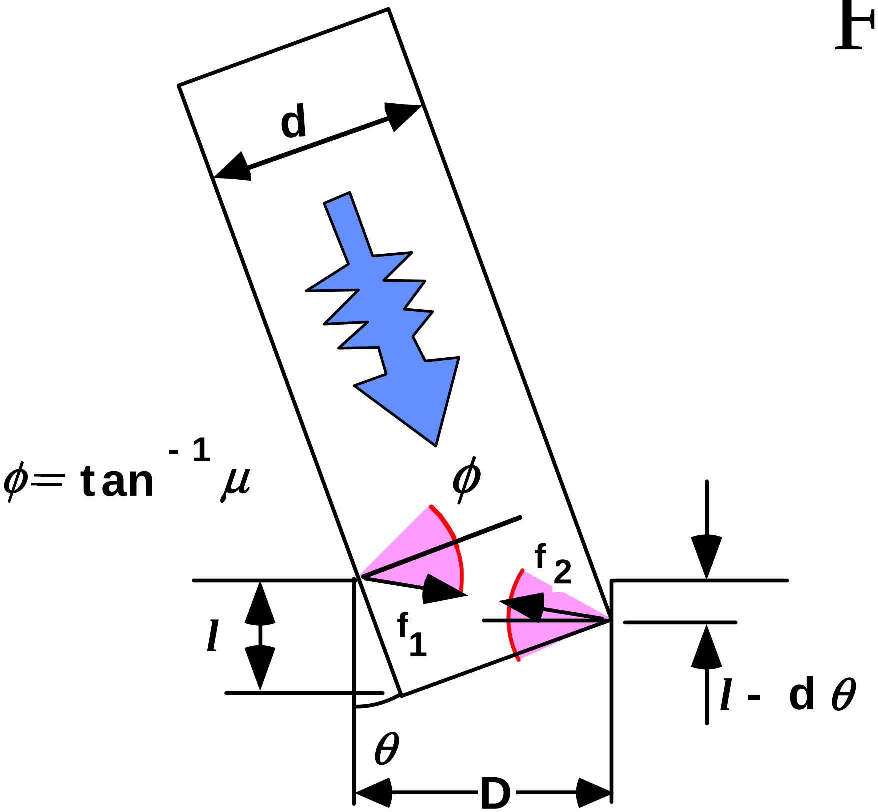
Insertion Force History

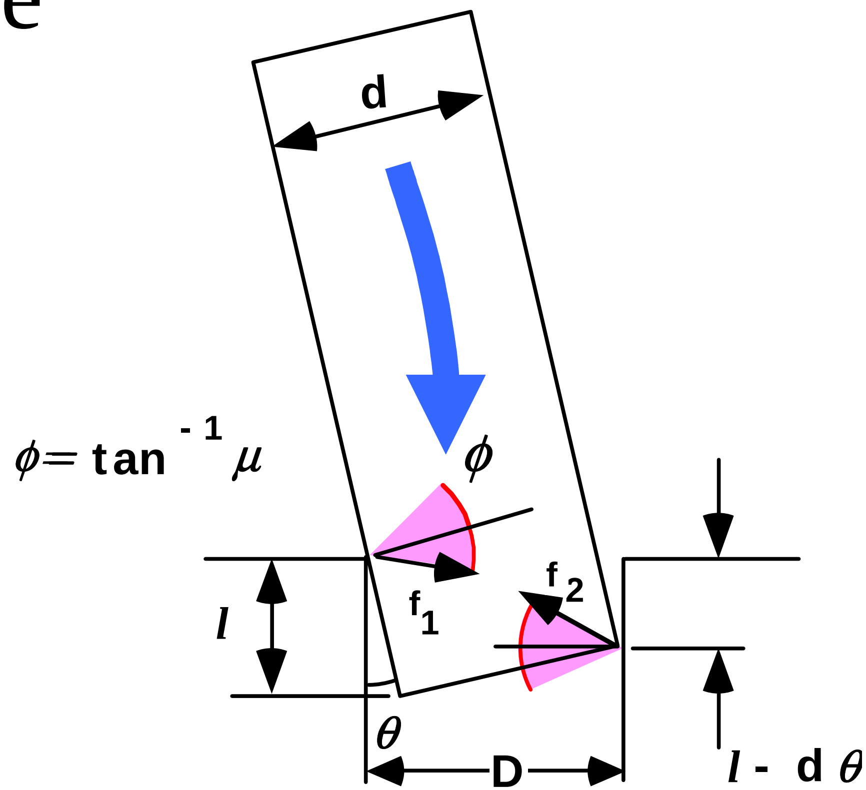


Assembly Failure Modes

- Both occur during two-point contact
- Wedging sets up compressive forces inside the parts
- Jamming results from incorrect insertion forces
- We can derive the requirements to avoid both of these failure modes

Wedging: Compressive Friction Forces Prevent Insertion Regardless of Insertion Force

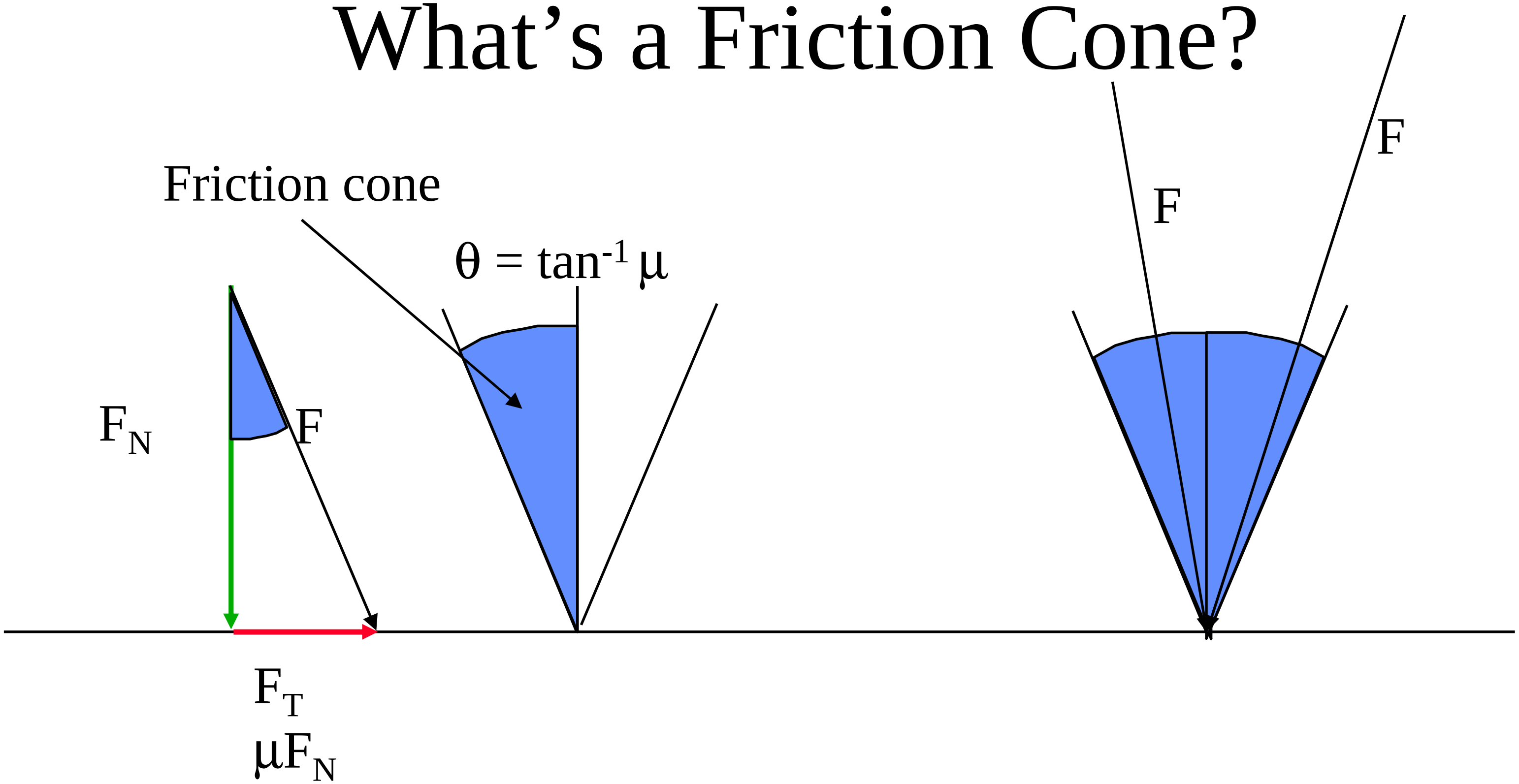




Wedging can happen if θ > c/μ when two-point contact occurs

Wedging can be avoided if μ is small enough or if two-point contact occurs deep enough in the hole

What's a Friction Cone?



Sliding will occur if $F_T > \mu F_N$

$F_T / F_N = \tan\theta$

So, sliding will occur if $\tan\theta > \mu$ and F will lie on the boundary of the cone

If F is inside the cone then sliding will not happen because $F_T < \mu F_N$ and F can be *any* value

Conditions for Avoiding Wedging

$$S = \frac{L_g}{L_g^2 + K_\theta / K_x}$$

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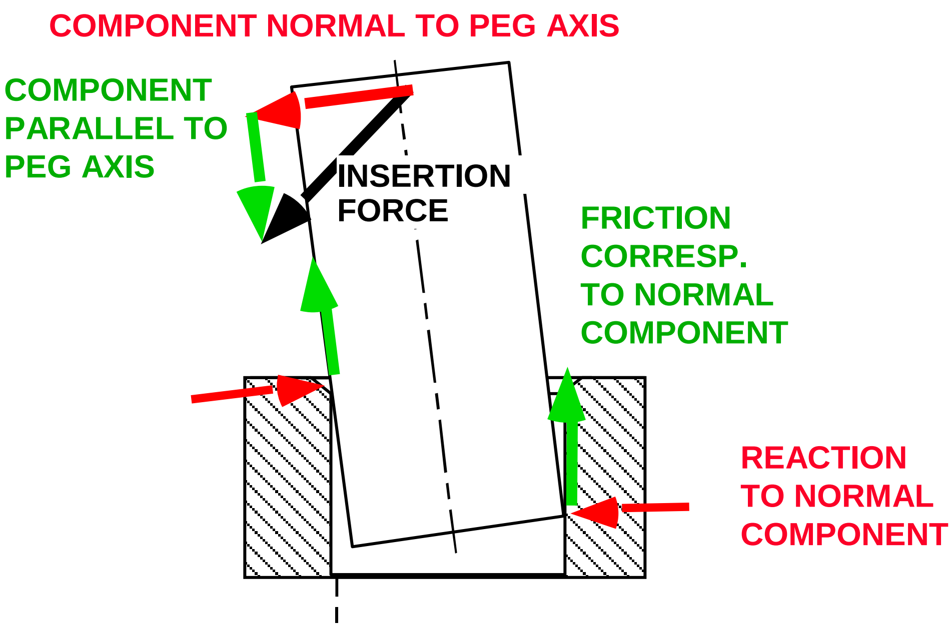
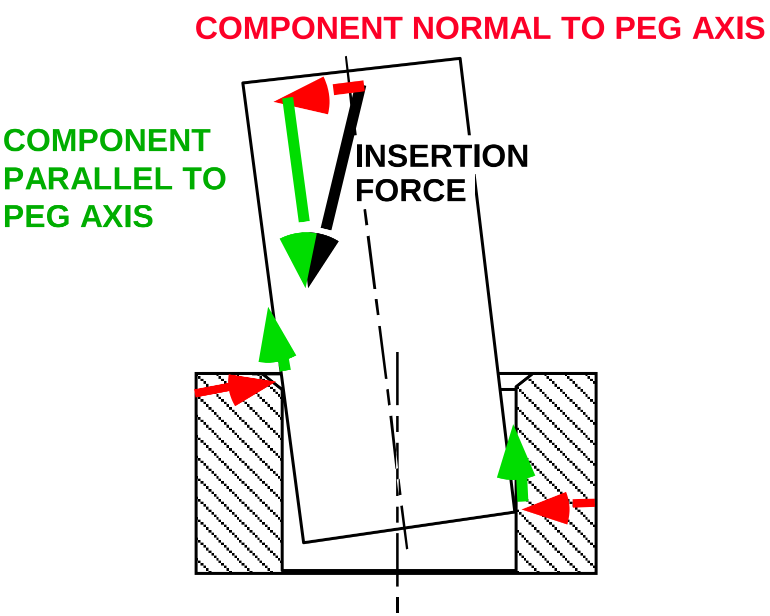
Source:
Figure 10-20 in [Whitney 2004] Whitney, D. E. *Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development*. New York, NY: Oxford University Press, 2004. ISBN: 0195157826.

Jamming: Insertion Force Directed the Wrong Way - Can't Overcome Friction





Component of
Insertion force
Along insertion direction
Not big enough:
Peg Is Jammed

Component of
Insertion force
Along insertion direction
Is big enough:
Peg Goes In

Conditions for Avoiding Jamming

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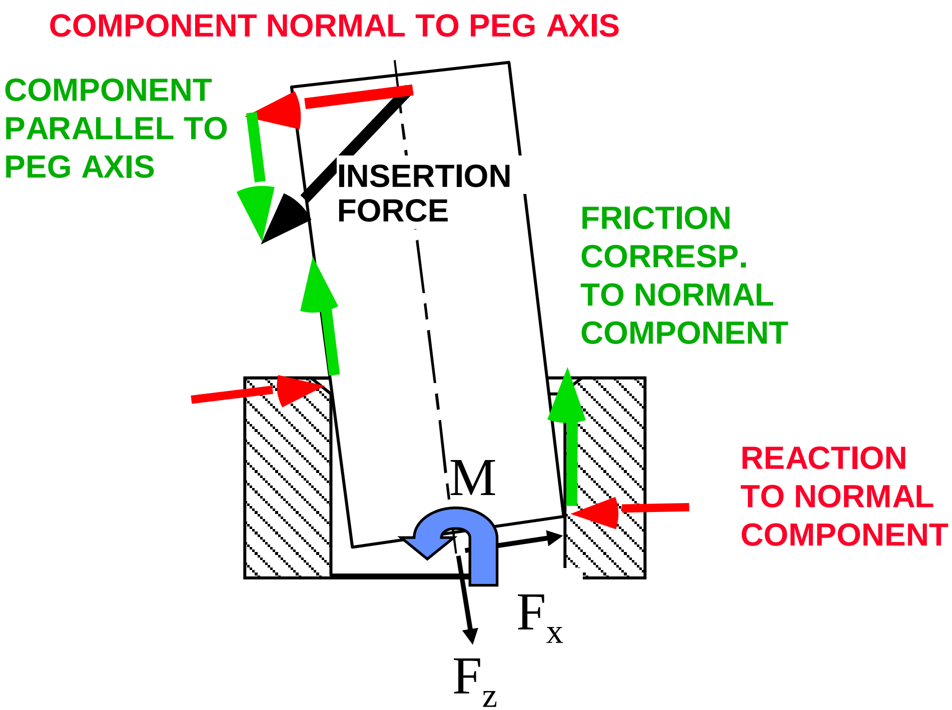
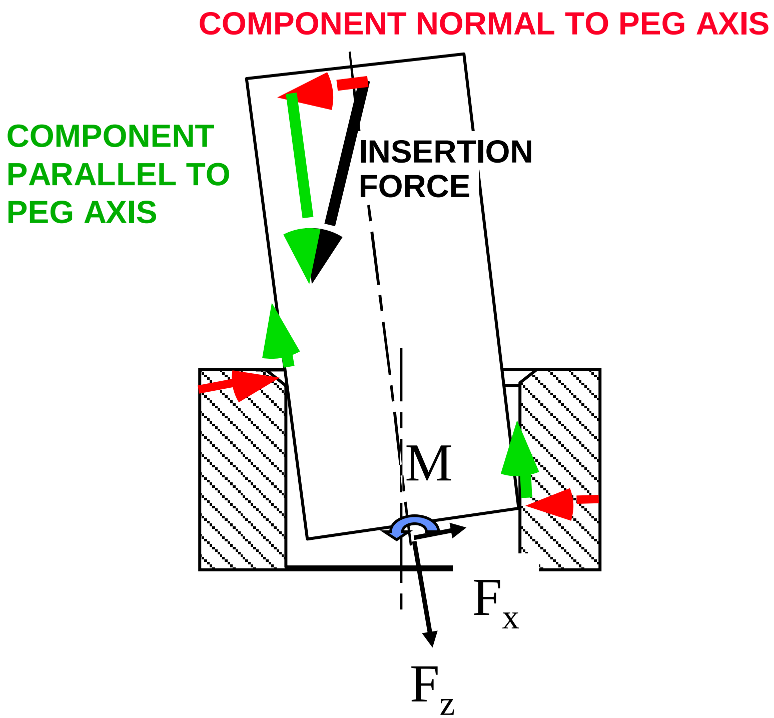
Source:
Figure 10-21 in [Whitney 2004] Whitney, D. E. *Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development*. New York, NY: Oxford University Press, 2004. ISBN: 0195157826.

Jamming Examples

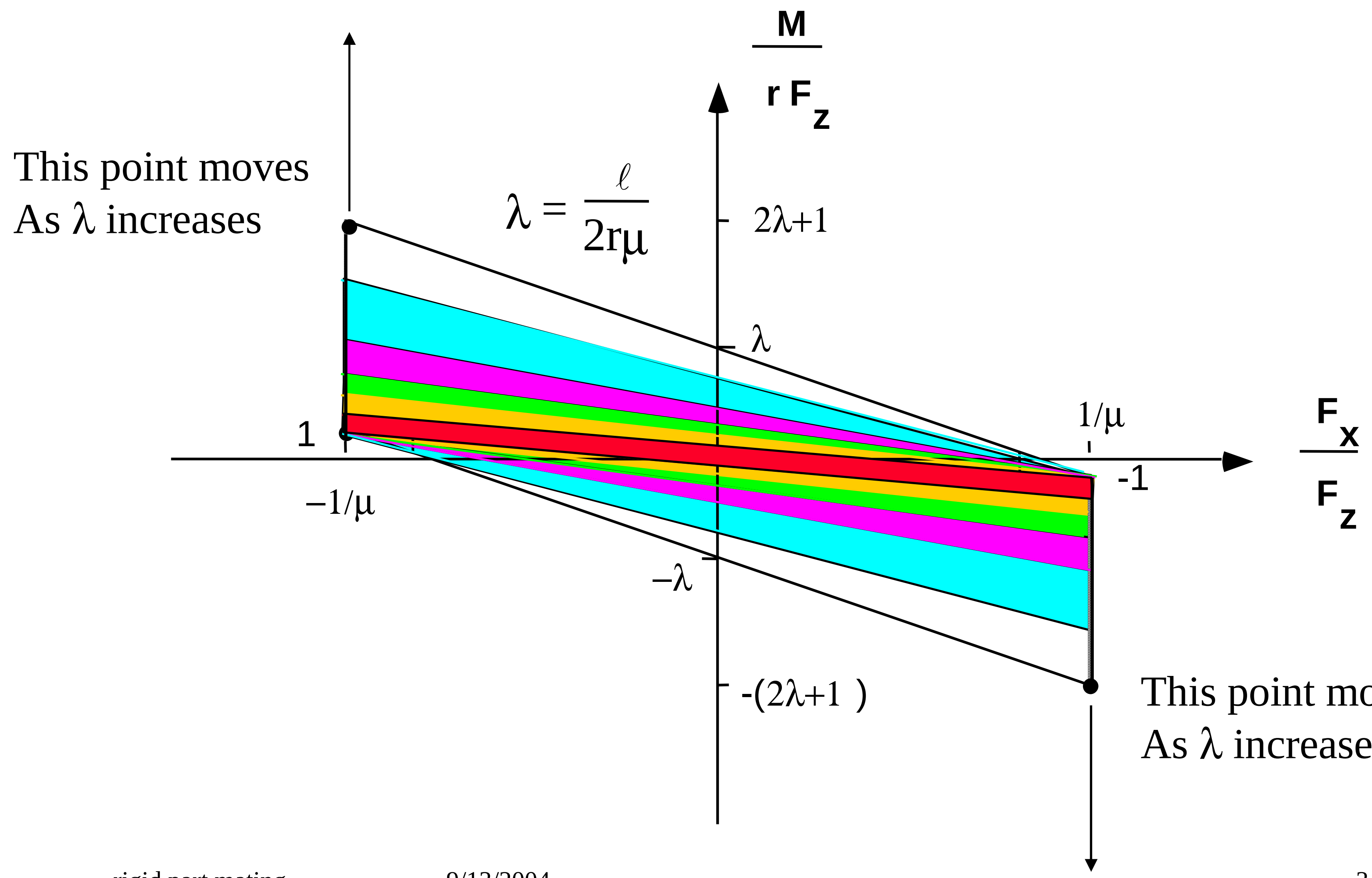


F_x/F_z is big.
M/rF_z is big.

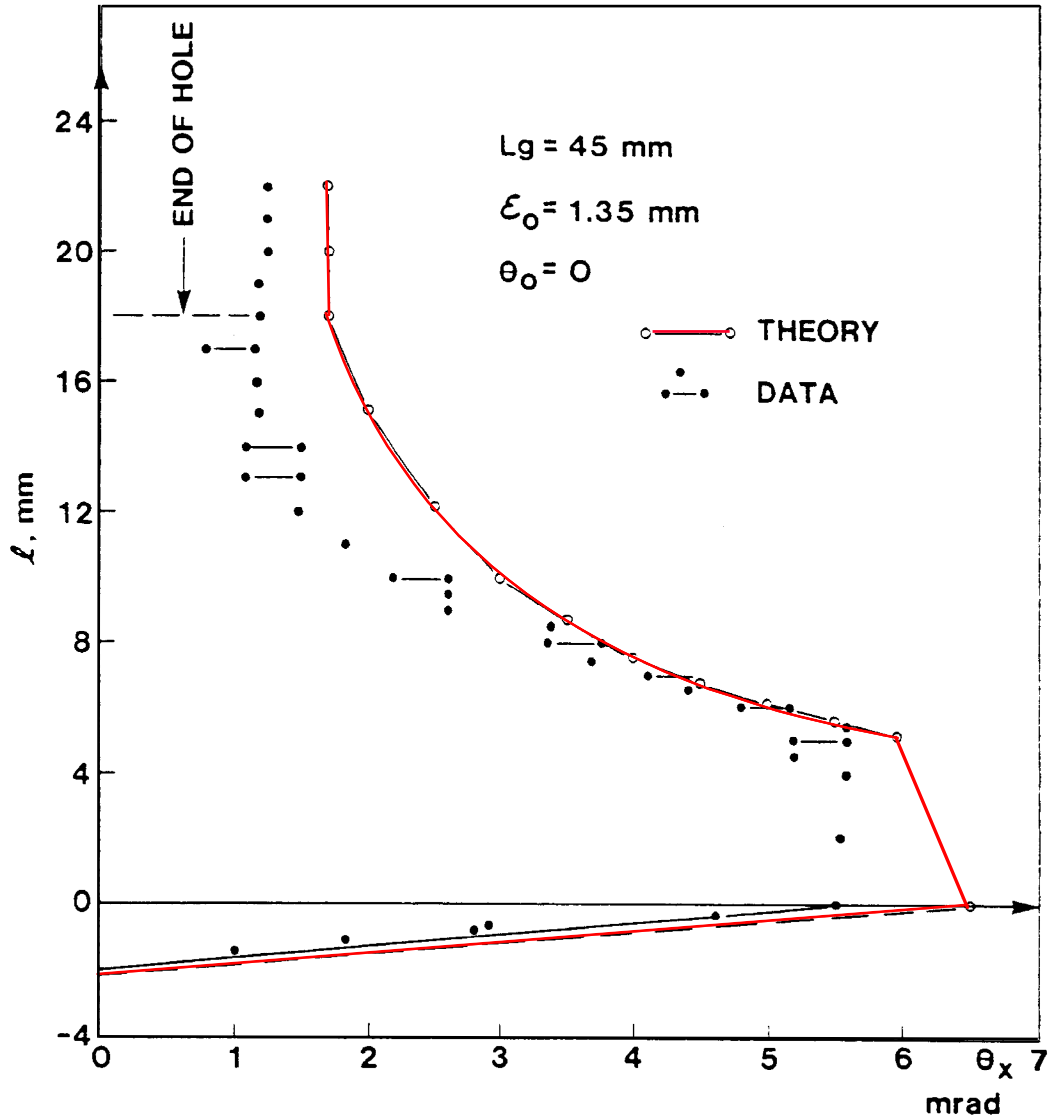


F_x/F_z is small.
M/rF_z is small.

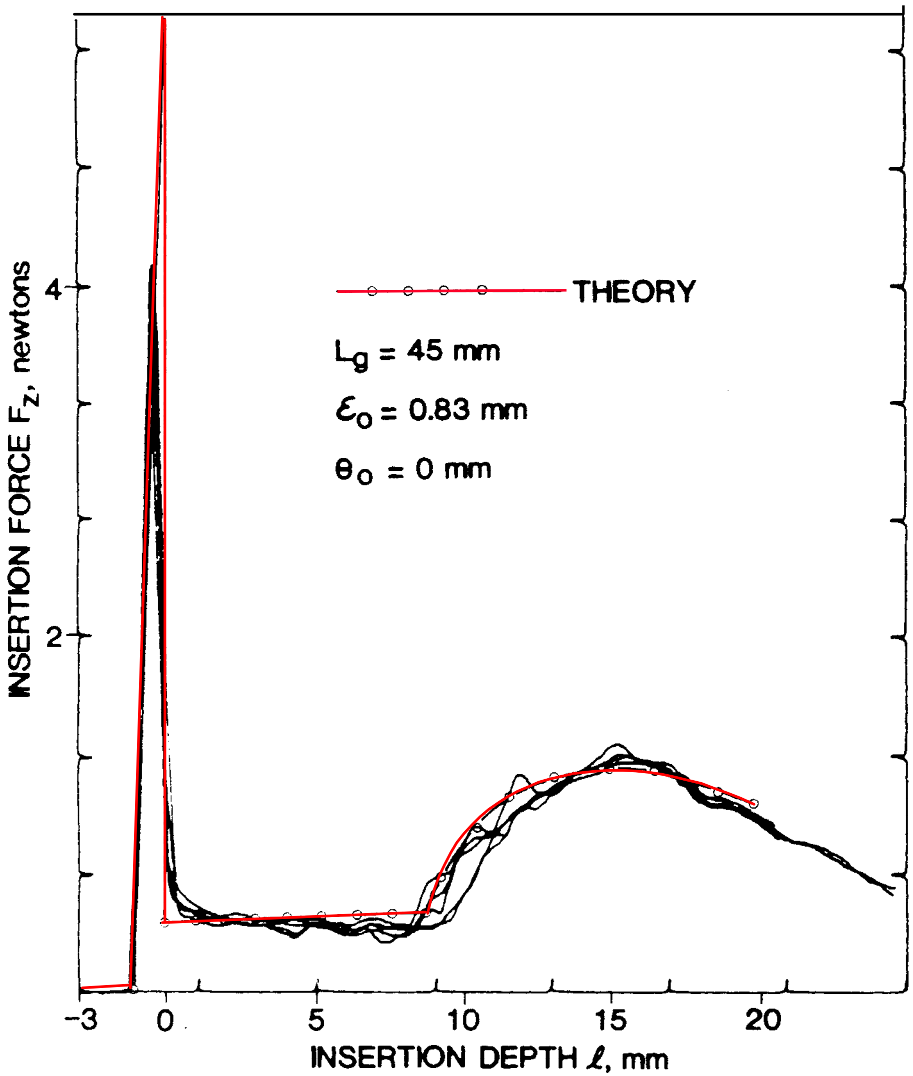
Target Expands as Depth Increases



Experimental Data



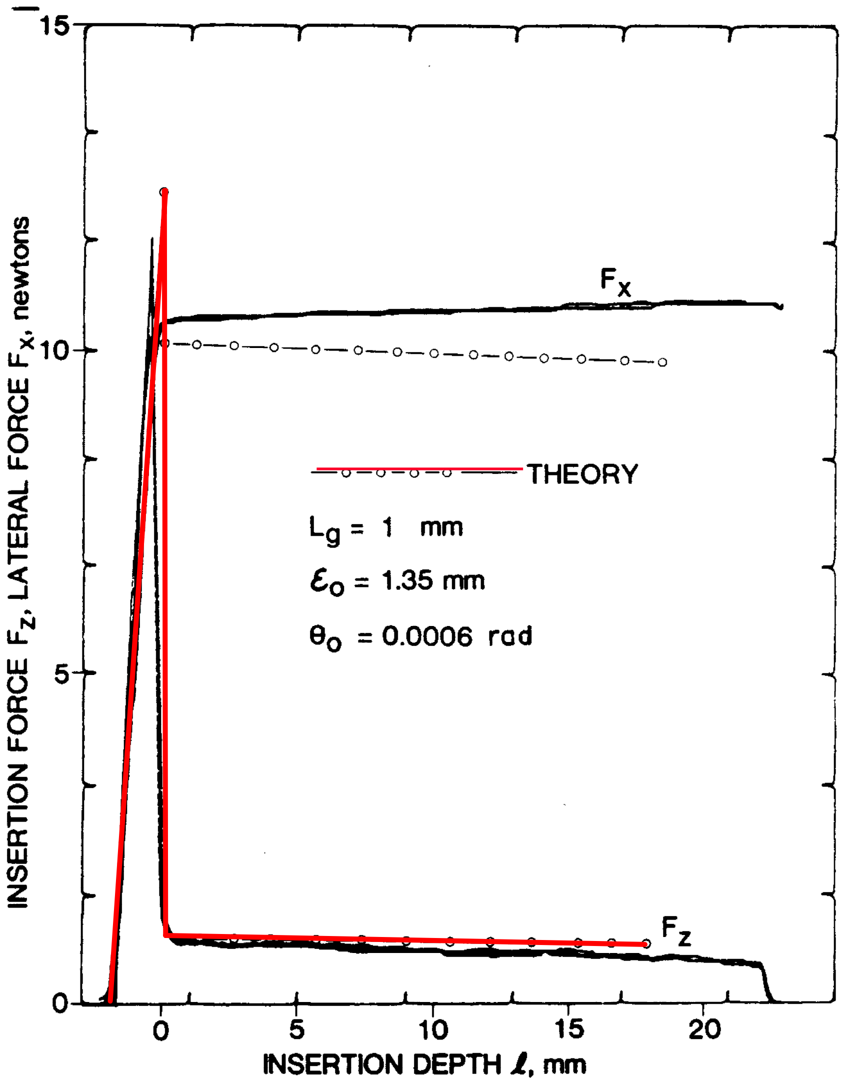
Experimental Data -2



Test Your Understanding

- Why does insertion force rise and then fall during two-point contact?

Experimental Data - 3



When $L_g = 0$ there is barely any insertion force. All that's left is chamfer crossing force.

Test Your Understanding Again

- Why does the insertion force not rise after chamfer-crossing is finished?

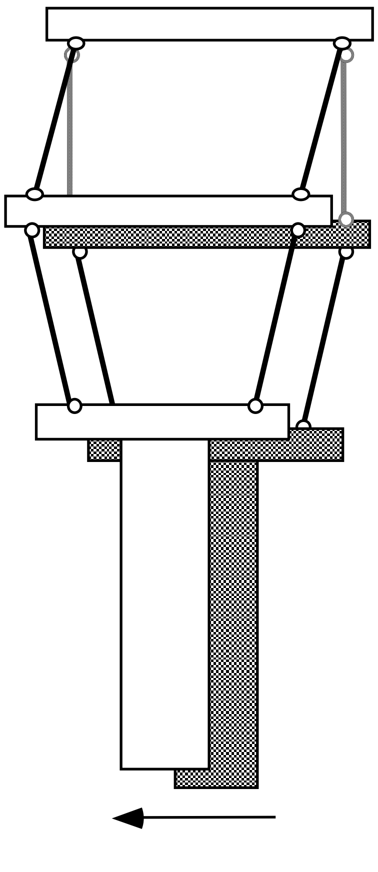
Review of Force Feedback Strategy

- Create a coordinate frame at the “working point” of the part or tool
- Separate lateral and angular sensing and response motions in that frame
- Devise a response strategy
- The Remote Center Compliance is a purely passive implementation of one such strategy

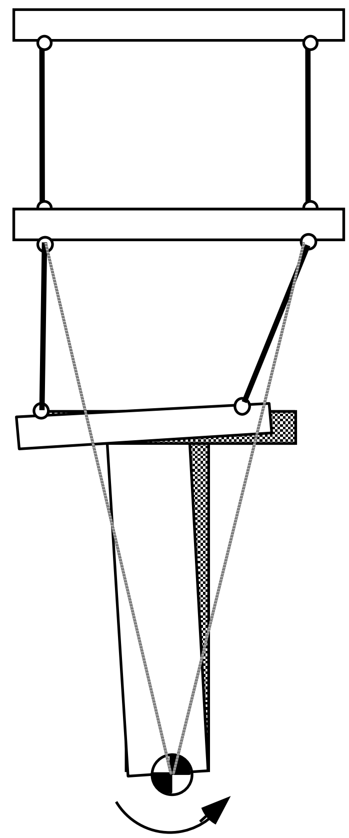
Simplified Explanation of the Remote Center Compliance (RCC)

Images removed for copyright reasons.
Source:
Figure 9-8 in [Whitney 2004] Whitney, D. E. *Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development*. New York, NY: Oxford University Press, 2004. ISBN: 0195157826.

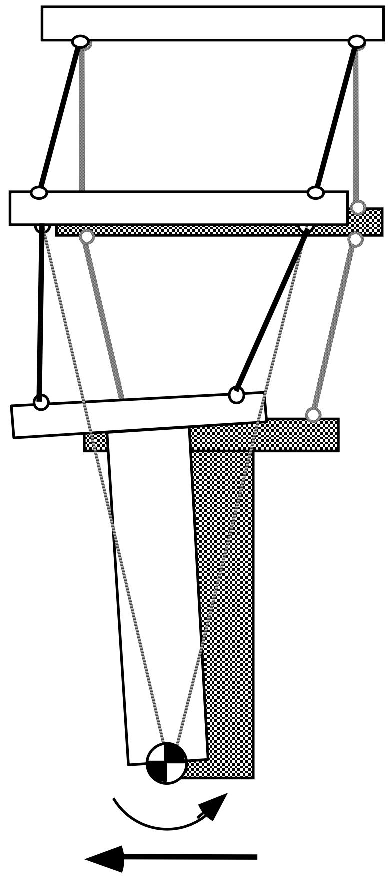
RCC Response to External Loads



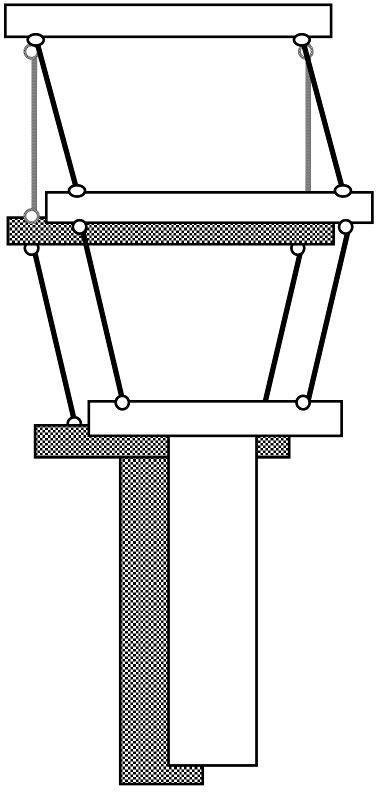
(d) RCC UNDER
LATERAL LOAD



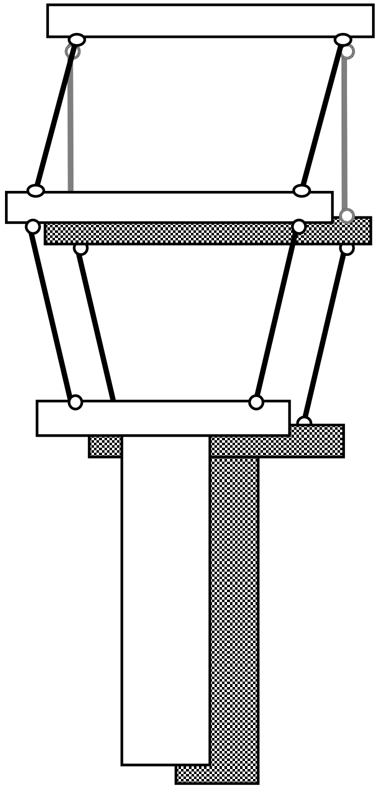
(e) RCC UNDER
ANGULAR LOAD



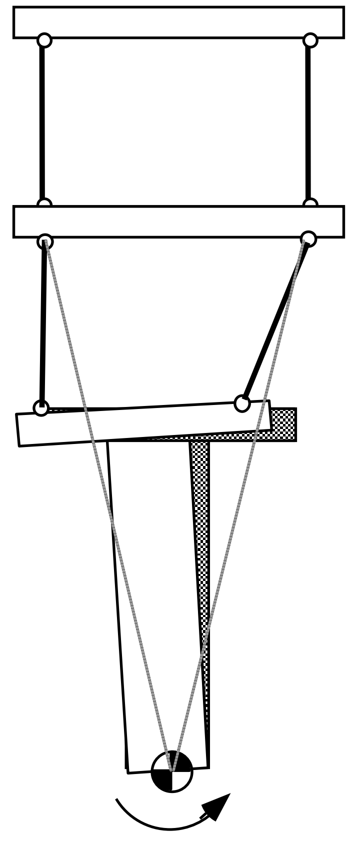
(f) LINKAGE
RCC UNDER LATERAL
AND ANGULAR
DEFORMATION



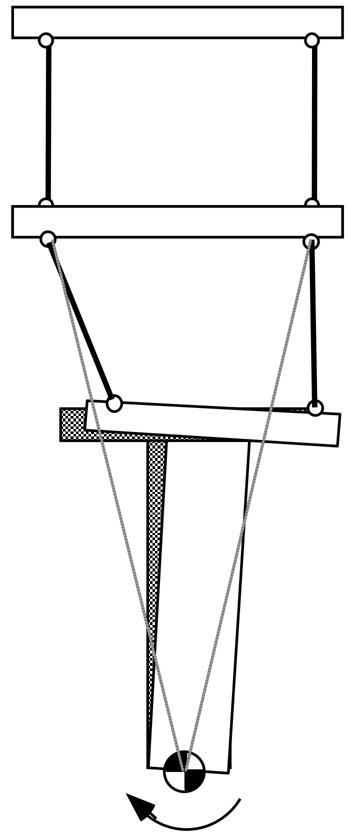
(d) RCC UNDER LATERAL LOAD



(d) RCC UNDER LATERAL LOAD

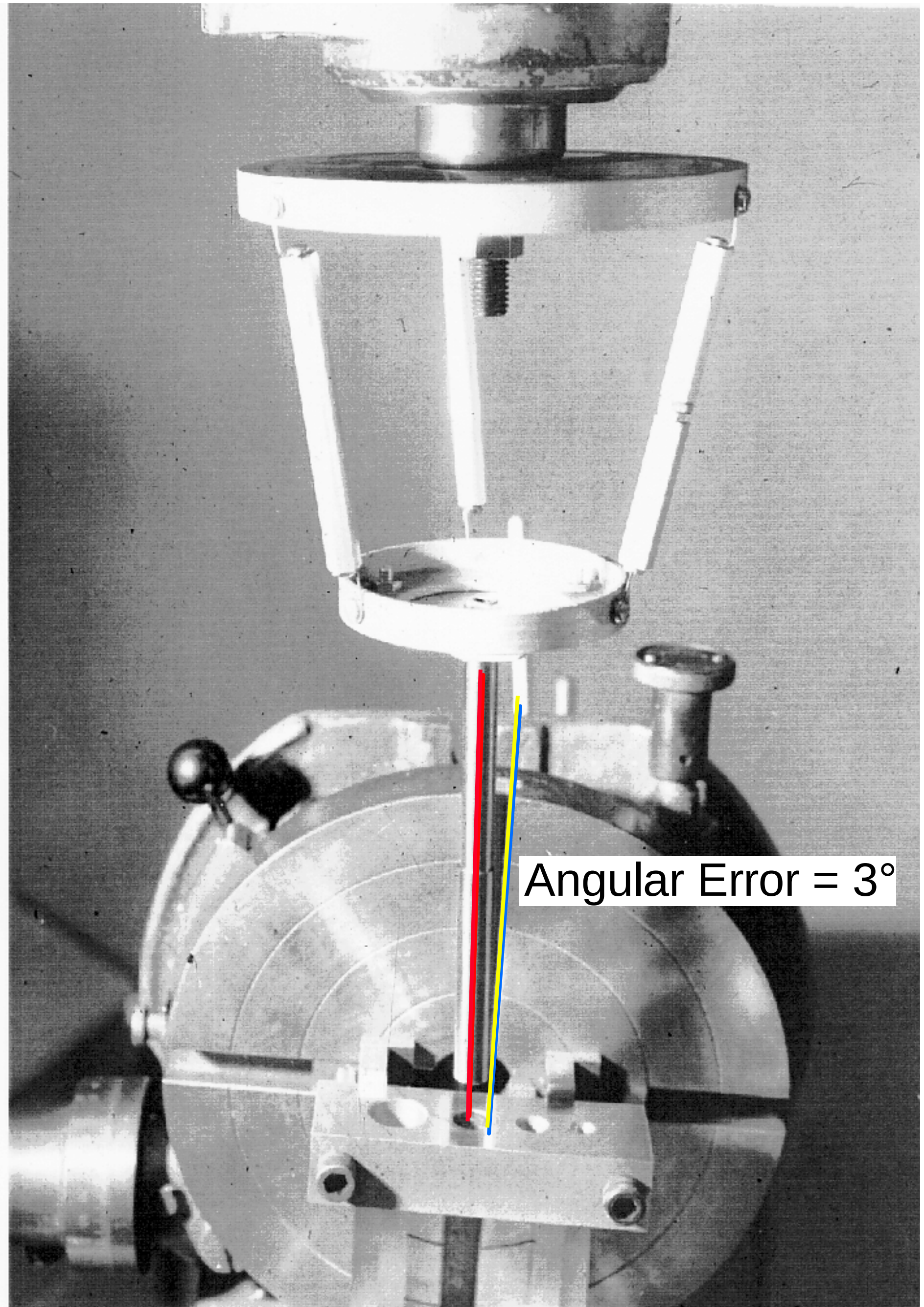


(e) RCC UNDER ANGULAR LOAD

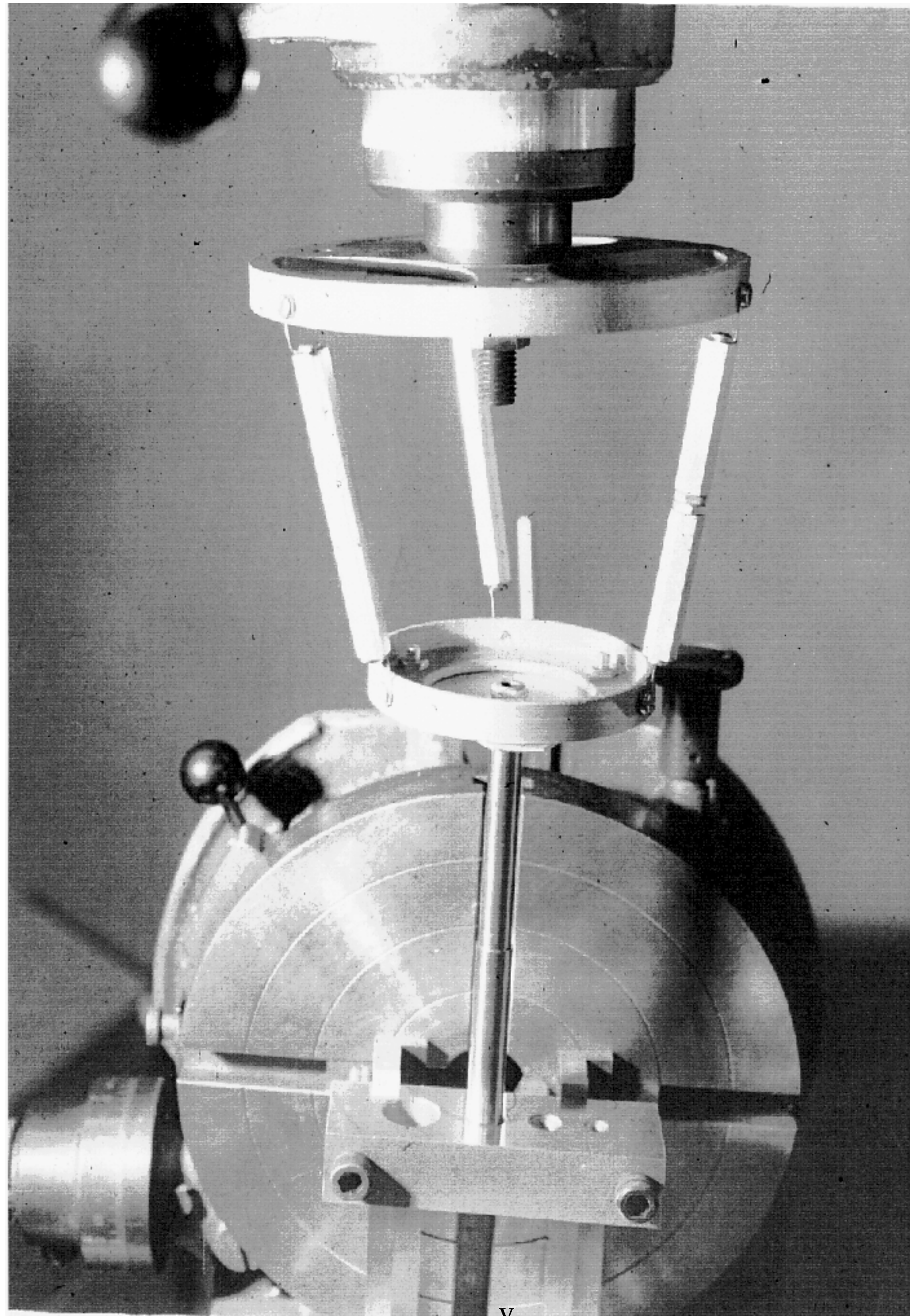


(e) RCC UNDER ANGULAR LOAD

First RCC Experiment - 1



First RCC Experiment - 2



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Commercial Remote Center Compliances

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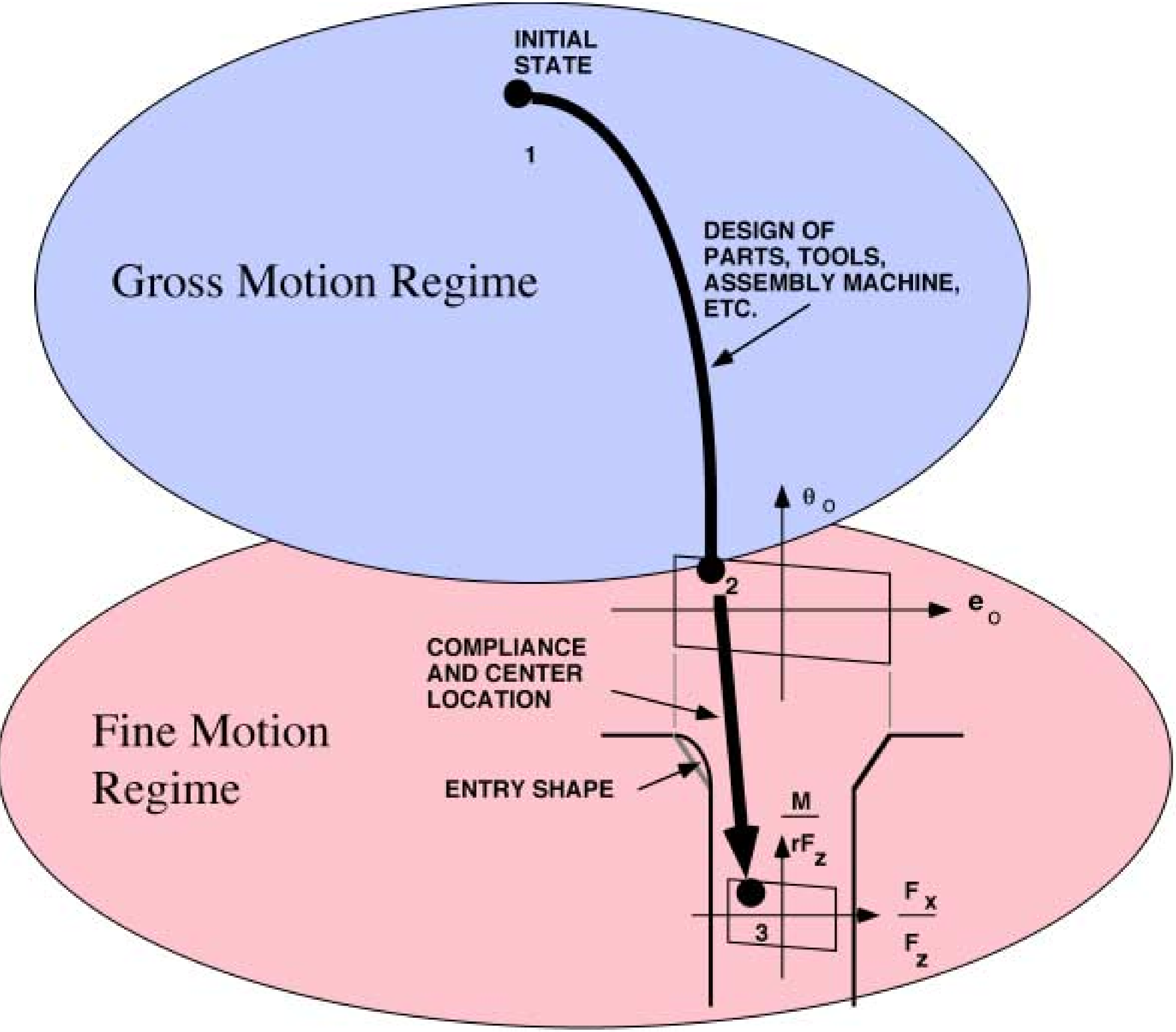
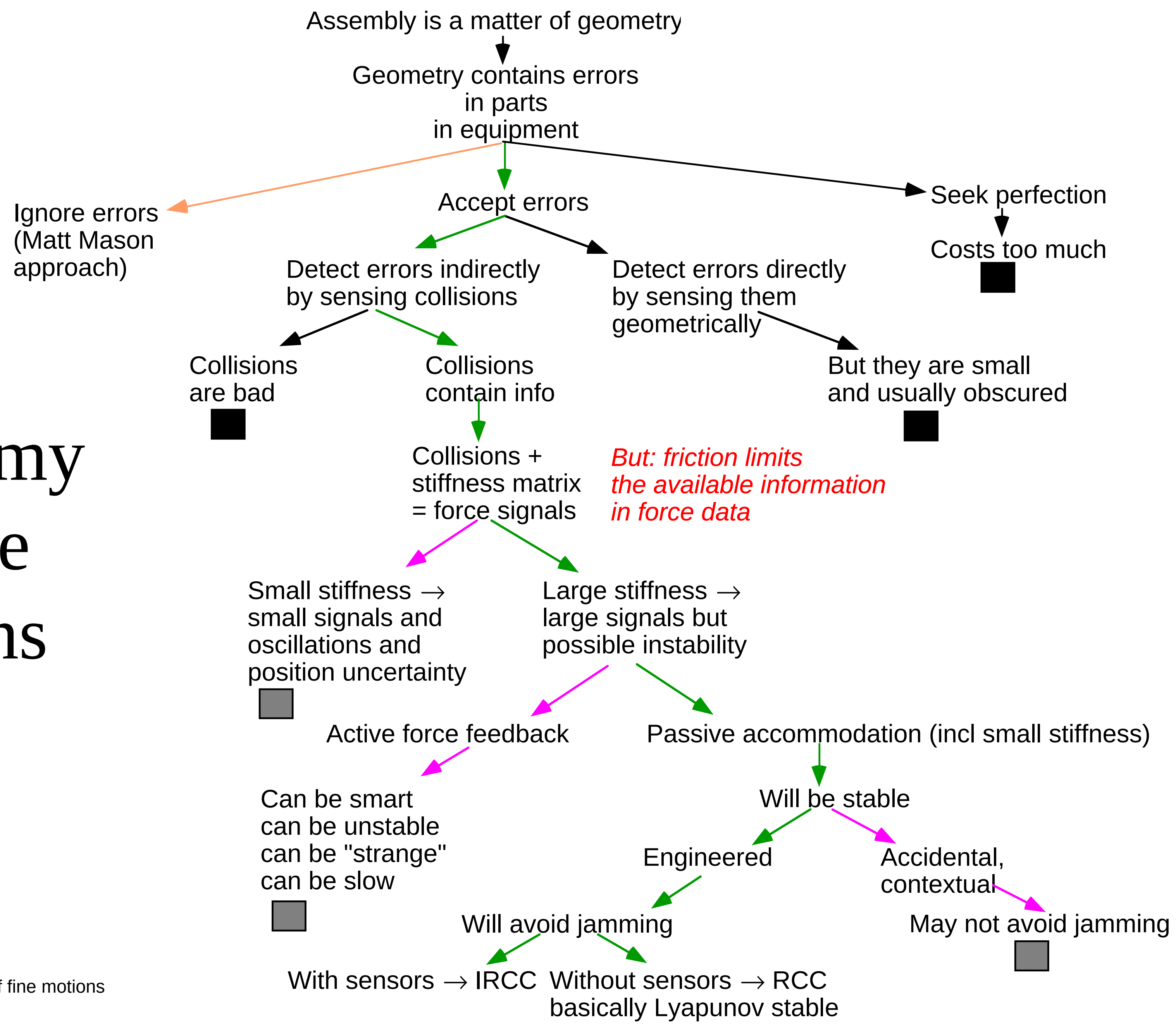
Source:
Figure 9-9 in [Whitney 2004] Whitney, D. E. *Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development*. New York, NY: Oxford University Press, 2004. ISBN: 0195157826.

Compliant Assembly Strategy:
Decouples Robot Accuracy from Task Precision



Taxonomy of Fine Motions



taxonomy of fine motions