MIT 2.852

Manufacturing Systems Analysis

Lectures 18–19

Loops

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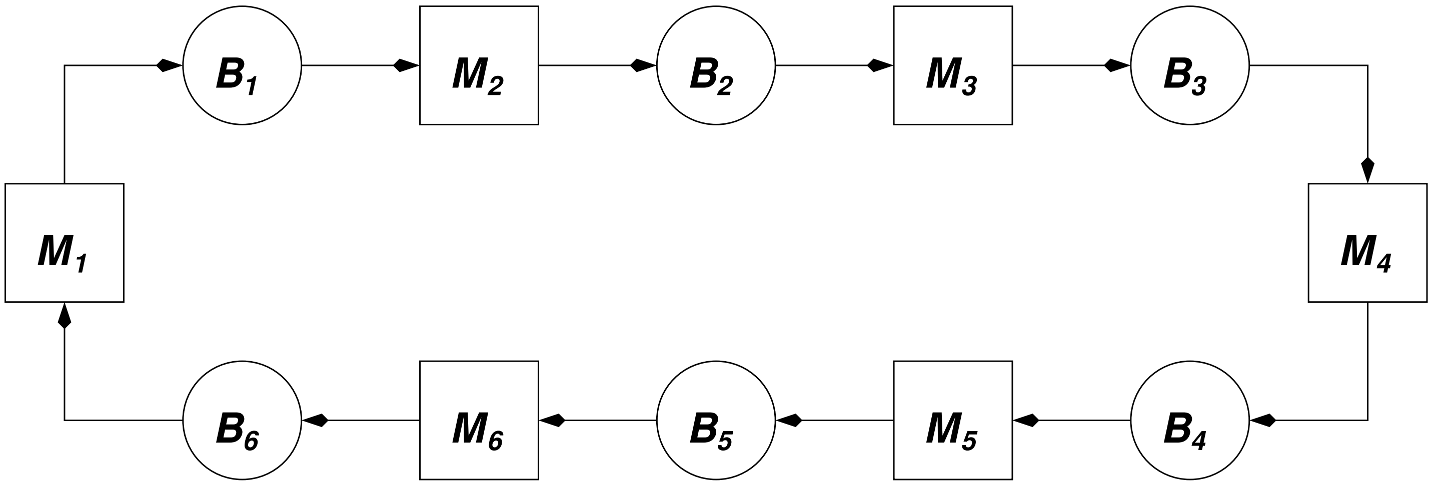
Problem Statement



- Finite buffers $(0 \leq n_i(t) \leq N_i)$.
- Single closed loop – fixed population $(\sum_i n_i(t) = N)$.
- Focus is on the Buzacott model (deterministic processing time; geometric up and down times). Repair probability = r_i; failure probability = p_i. Many results are true for more general loops.
- *Goal:* calculate production rate and inventory distribution.

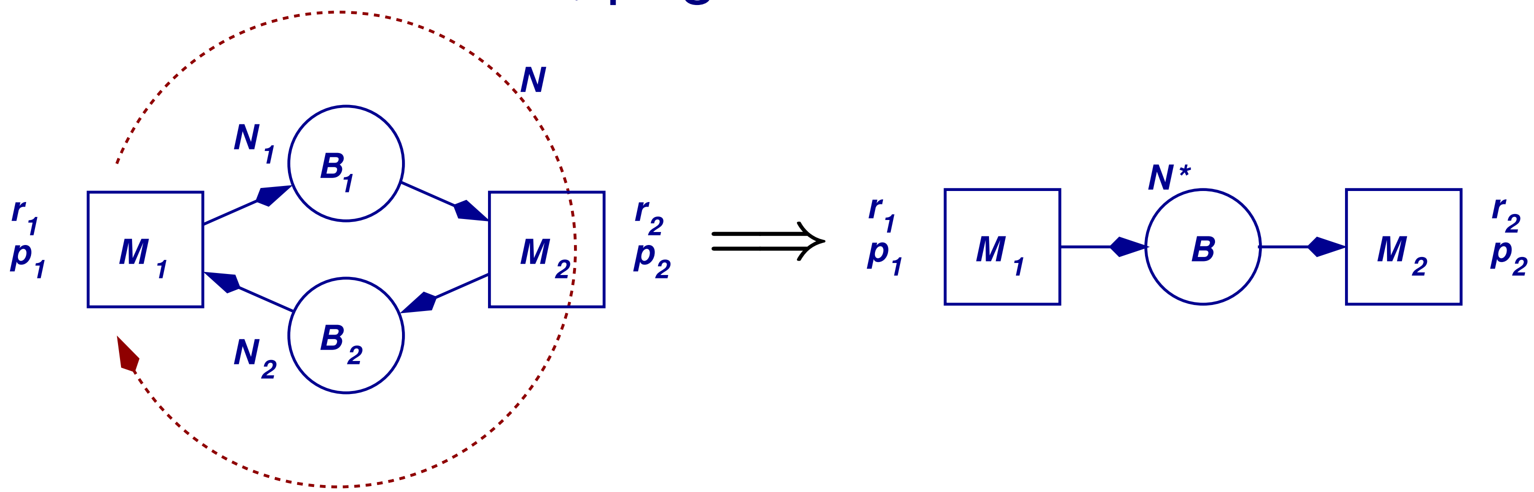
Problem Statement

Motivation

- Limited pallets/fixtures.
- CONWIP (or hybrid) control systems.
- Extension to more complex systems and policies.

Special Case

Refer to *MSE* Section 5.6, page 205.



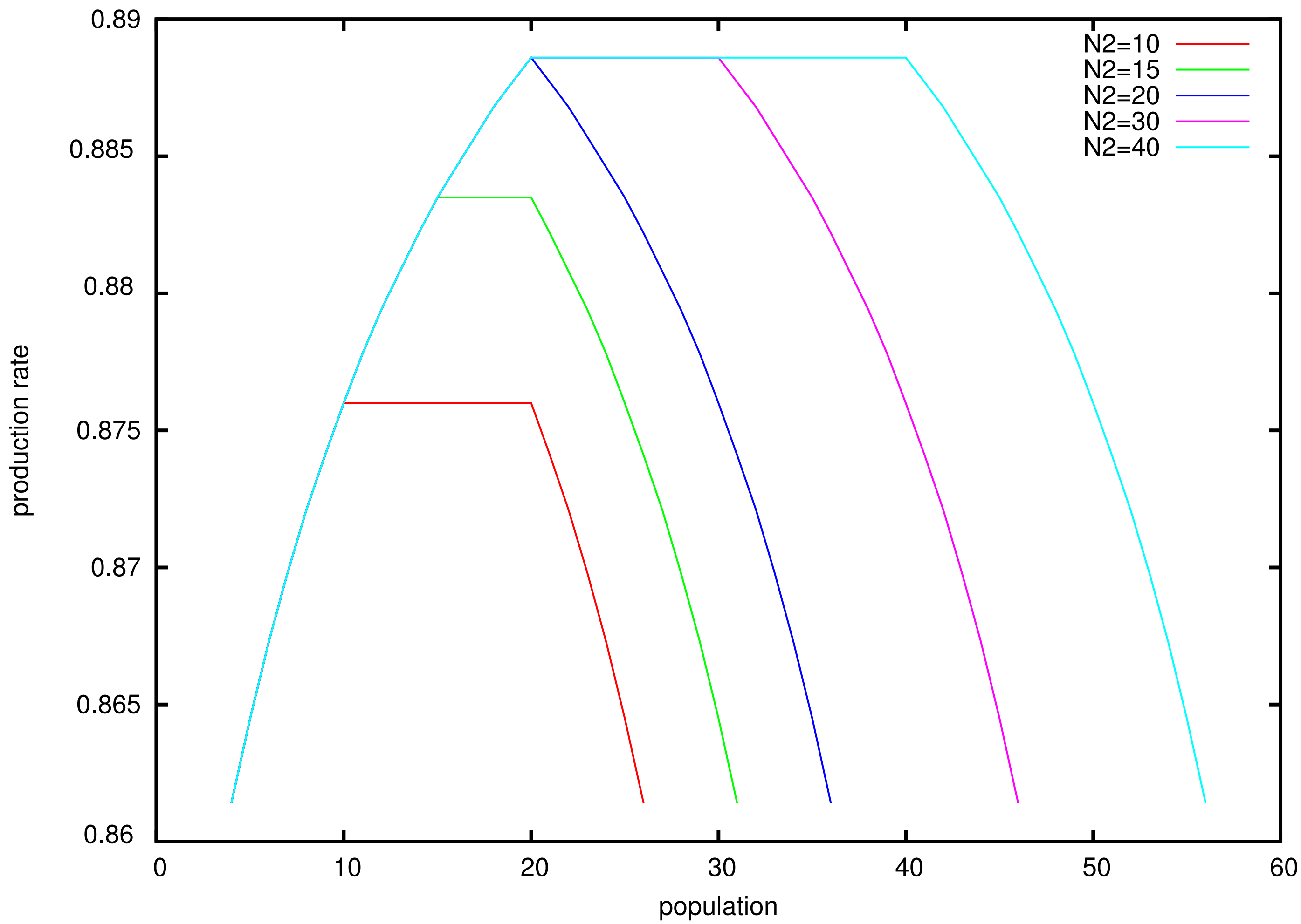
$$P_{\text{loop}}(r_1, p_1, r_2, p_2, N_1, N_2) = P_{\text{line}}(r_1, p_1, r_2, p_2, N^\star)$$

where

$$N^\star = \min(n, N_1) - \max(0, n - N_2)$$

Special Case

r_1	.14
p_1	.01
r_2	.1
p_2	.01
N_1	20



Expected population method

- Treat the loop as a line in which the first machine and the last are the same.
- In the resulting decomposition, one equation is missing.
- The missing equation is replaced by the *expected* population constraint $(\sum_i \bar{n}_i = N)$.

Expected population method

Evaluate i, $i-1$, $i+1$ modulo k (ie, replace 0 by k and replace $k+1$ by 1).

$$r_u(i) = r_u(i-1)X(i) + r_i(1 - X(i)); \qquad X(i) = \frac{p_s(i-1)r_u(i)}{p_u(i)E(i-1)}$$

$$p_u(i) = r_u(i)\left(\frac{1}{E(i-1)} + \frac{1}{e_i} - 2 - \frac{p_d(i-1)}{r_d(i-1)}\right), \boxed{i = 1, ..., k}$$

$$r_d(i) = r_d(i+1)Y(i+1) + r_{i+1}(1 - Y(i+1)); \; Y(i+1) = \frac{p_b(i+1)r_d(i)}{p_d(i)E(i+1)}.$$

$$p_d(i) = r_d(i)\left(\frac{1}{E(i+1)} + \frac{1}{e_{i+1}} - 2 - \frac{p_u(i+1)}{r_u(i+1)}\right), \boxed{i = k, ..., 1}$$

This is $4k$ equations in $4k$ unknowns. But only $4k-1$ of them are independent because the derivation uses $E(i) = E(i+1)$ for $i = 1, ..., k$. The first $k-1$ are $E(1) = E(2)$, $E(2) = E(3)$, ..., $E(k-1) = E(k)$. But this implies $E(k) = E(1)$, which is the same as the kth equation.

Expected population method

Therefore, we need one more equation. We can use

$$\sum_i \bar{n}_i = N$$

If we guess $p_u(1)$ (say), we can evaluate $\bar{n}^{\text{TOTAL}} = \sum_i \bar{n}_i$ as a function of $p_u(1)$. We search for the value of $p_u(1)$ such that $\bar{n}^{\text{TOTAL}} = N$.

Expected population method

Behavior:

- Accuracy good for large systems, not so good for small systems.
- Accuracy good for intermediate-size populations; not so good for very small or very large populations.

Expected population method

- *Hypothesis:* The reason for the accuracy behavior of the population constraint method is the correlation in the buffers.
  - ⋆ The number of parts in the system is actually *constant* .
  - ⋆ The expected population method treats the population as *random,* with a specified mean.
  - ⋆ If we know that a buffer is almost full, we know that there are fewer parts in the rest of the network, so probabilities of blockage are reduced and probabilities of starvation are increased. (Similarly if it is almost empty.)
  - ⋆ Suppose the population is smaller than the smallest buffer. *Then there will be no blockage.* The expected population method does not take this into account.

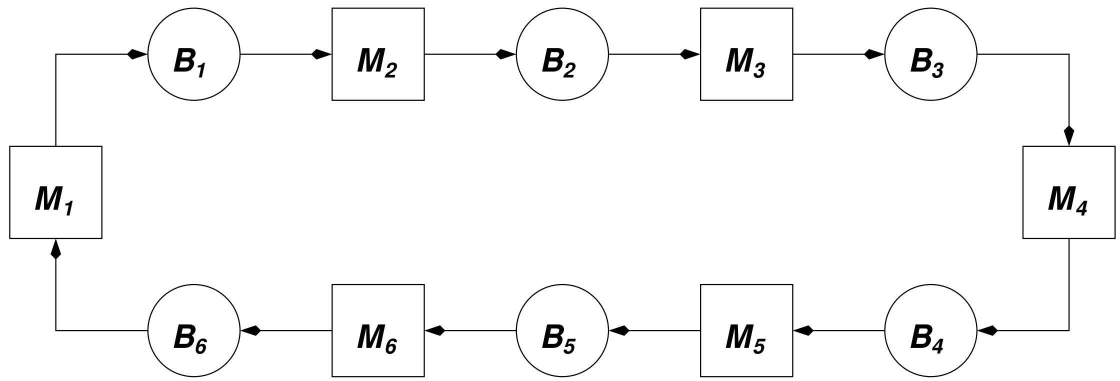
Loop Behavior

To construct a method that deals with the invariant (rather than the expected value of the invariant), we investigate how buffer levels are related to one another and to the starvation and blocking of machines.

In a line, every downstream machine could block a given machine, and every upstream machine could starve it. In a loop, blocking and starvation are more complicated.

Loop Behavior

Ranges

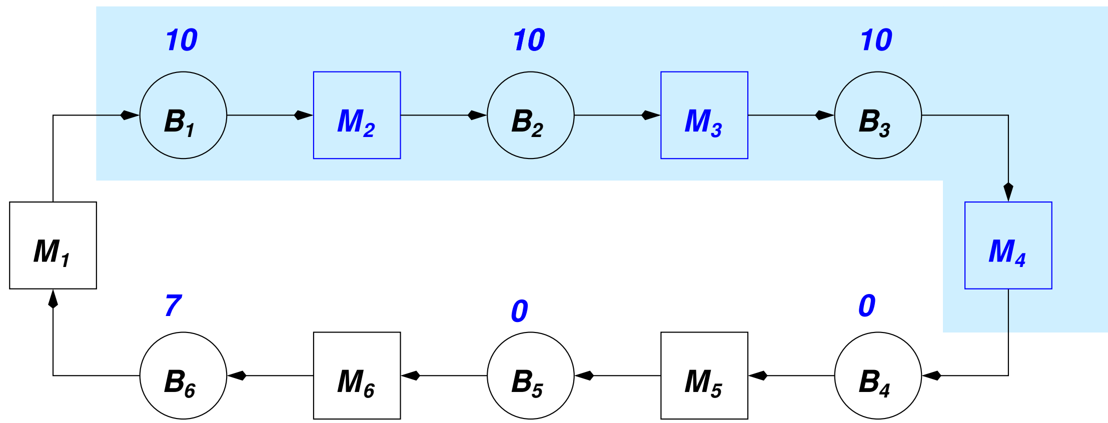


- The *range of blocking of a machine* is the set of all machines that could block it if they stayed down for a long enough time.
- The *range of starvation of a machine* is the set of all machines that could starve it if they stayed down for a long enough time.

Loop Behavior

Ranges

Range of Blocking

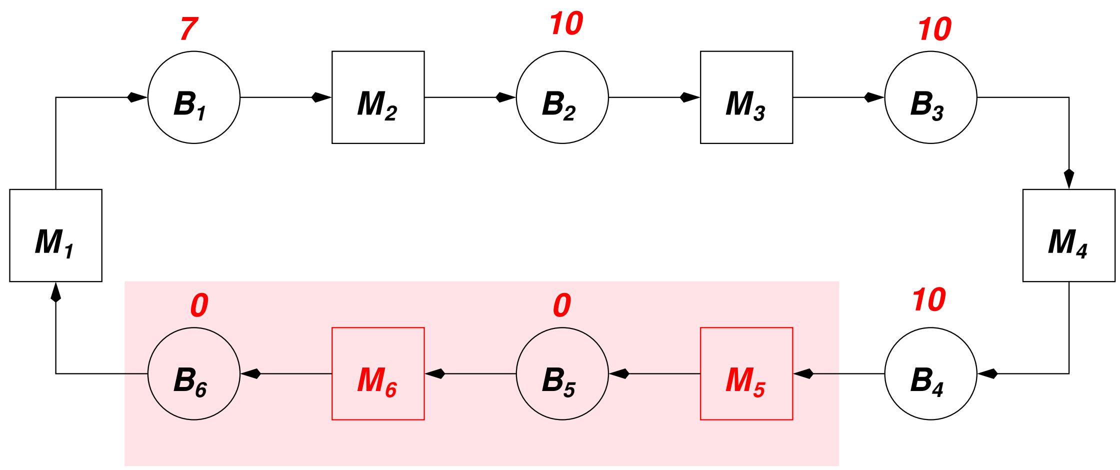


- All buffer sizes are 10.
- Population is 37.
- If M_4 stays down for a long time, it will block M_1.
- Therefore M_4 is in the range of blocking of M_1.
- Similarly, M_2 and M_3 are in the range of blocking of M_1.

Loop Behavior

Ranges

Range of starvation

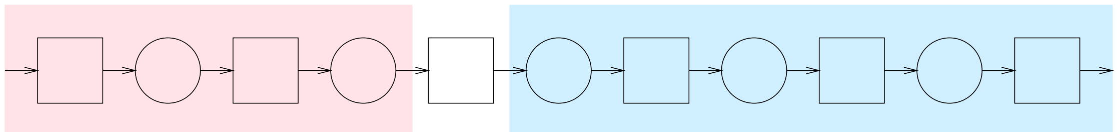


- If M_5 stays down for a long time, it will starve M_1.
- Therefore M_5 is in the range of starvation of M_1.
- Similarly, M_6 is in the range of starvation of M_1.

Loop Behavior

Ranges

Line



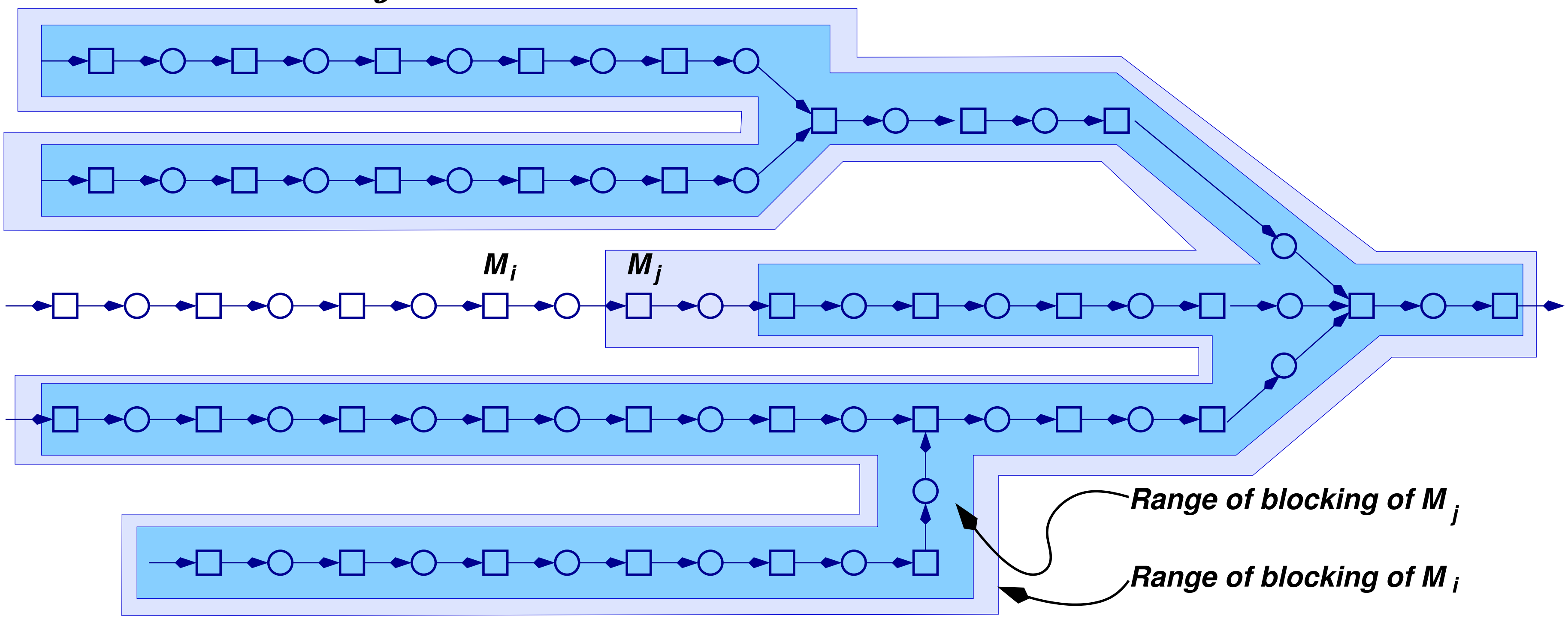
- The range of blocking of a machine in a line is the entire downstream part of the line.
- The range of starvation of a machine in a line is the entire upstream part of the line.

Loop Behavior

Ranges

Line

In an acyclic network, if M_j is downstream of M_i, then the range of blocking of M_j is a *subset* of the range of blocking of M_i.



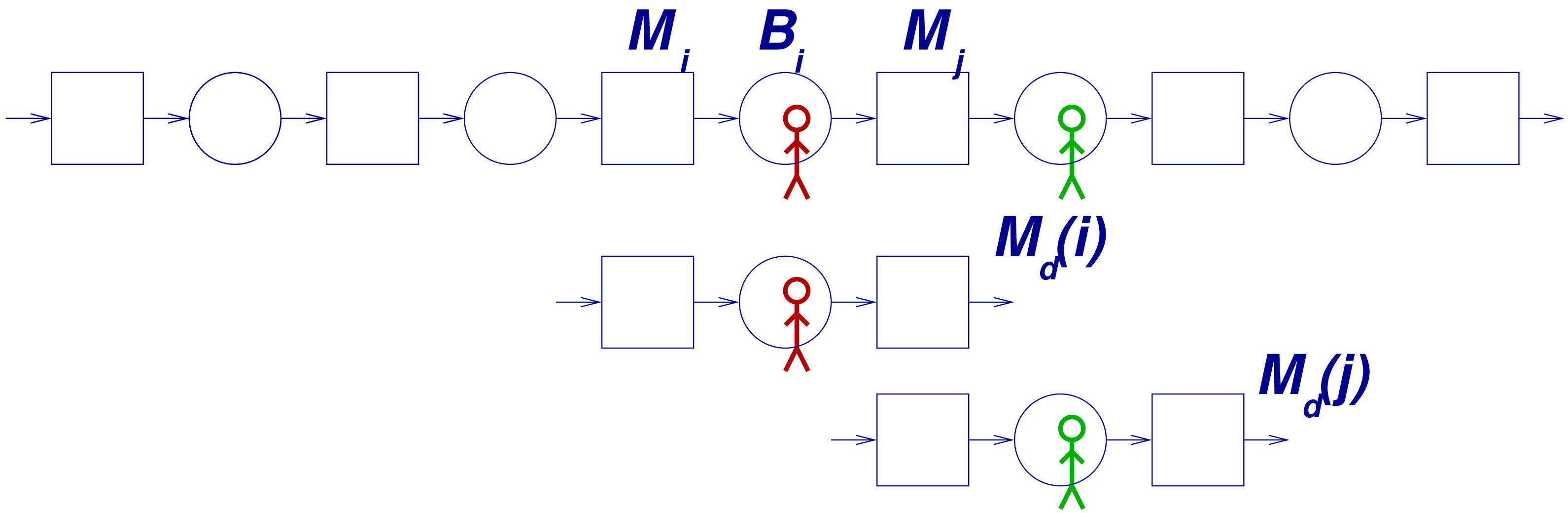
Similarly for the range of starvation.

Loop Behavior

Ranges

Line

In an acyclic network, if M_j is downstream of M_i, any real machine whose failure could cause $M_d(j)$ to appear to be down could also cause $M_d(i)$ to appear to be down.



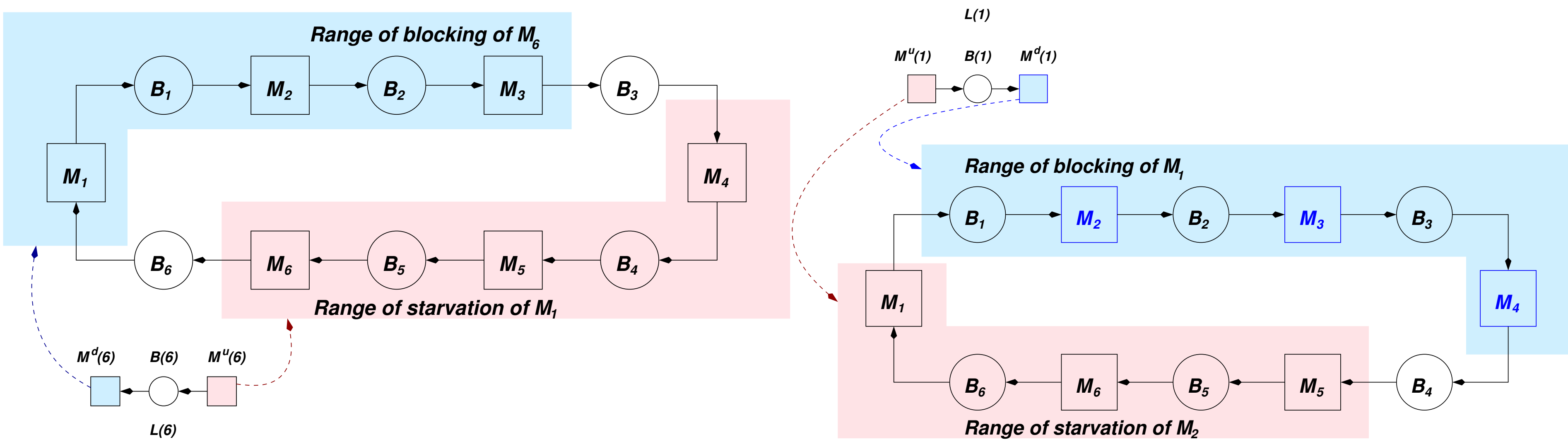
Consequently, we can express $r_d(i)$ as a function of the parameters of $L(j)$.

This is *not* possible in a network with a loop because some machine that blocks M_j does not block M_i.

Loop Behavior

Ranges

Difficulty for decomposition

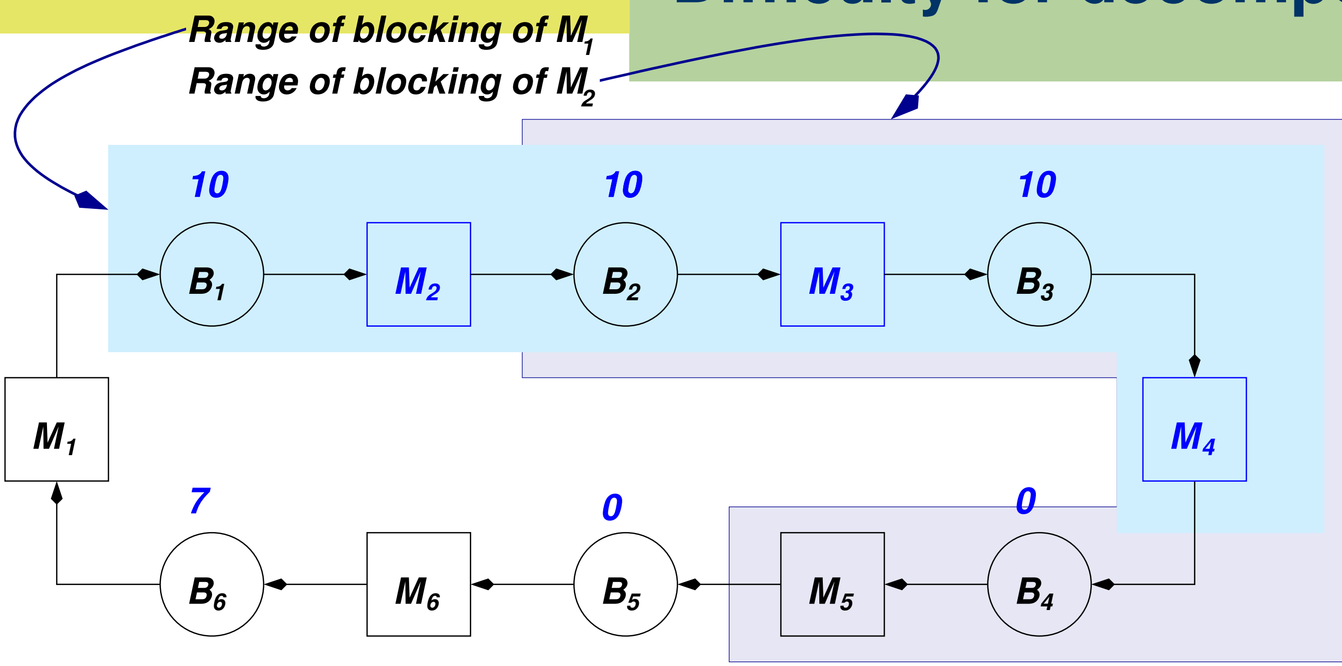


Ranges of blocking and starvation of adjacent machines are *not* subsets or supersets of one another in a loop.

Loop Behavior

Ranges

Difficulty for decomposition

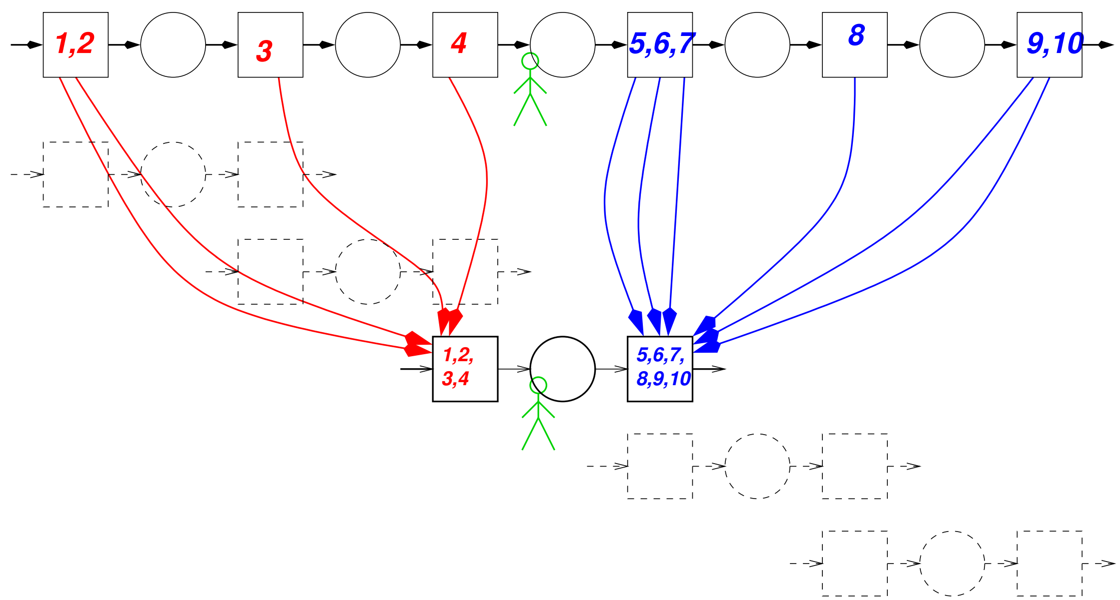


M_5 can block M_2. Therefore the parameters of M_5 should directly affect the parameters of $M_d(1)$ in a decomposition. However, M_5 cannot block M_1 so the parameters of M_5 should not directly affect the parameters of $M_d(6)$. Therefore, the parameters of $M_d(6)$ cannot be functions of the parameters of $M_d(1)$.

Multiple Failure Mode Line Decomposition

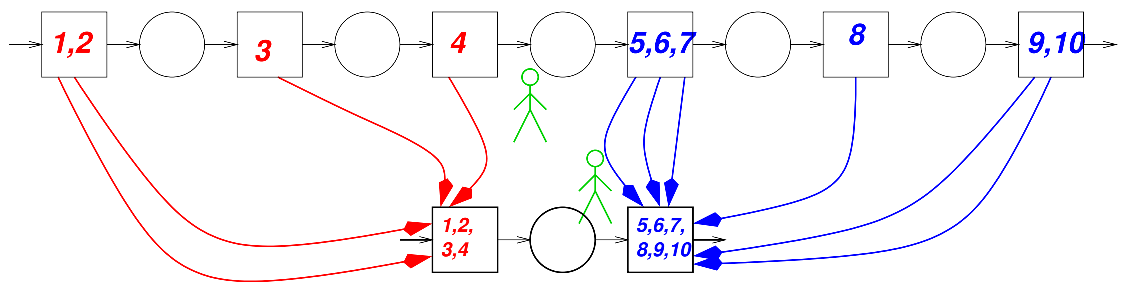
- To deal with this issue, we introduce a new decomposition.
- In this decomposition, we do not create failures of virtual machines that are mixtures of failures of real machines.
- Instead, we allow the virtual machines to have distinct failure modes, each one corresponding to the failure mode of a real machine.

Multiple Failure Mode Line Decomposition



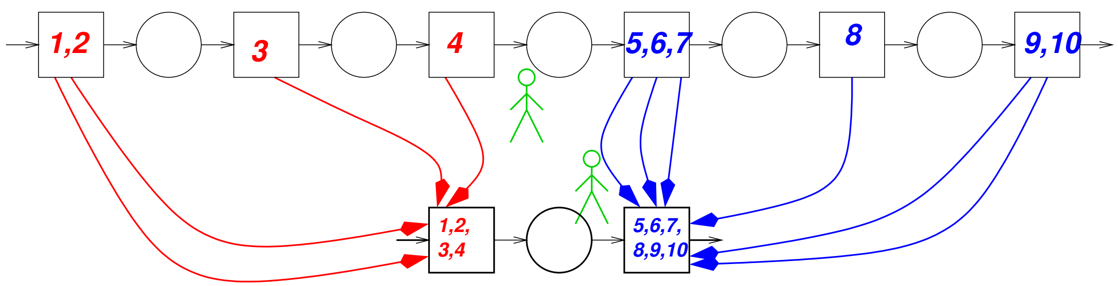
- There is an observer in each buffer who is told that he is actually in the buffer of a two-machine line.

Multiple Failure Mode Line Decomposition



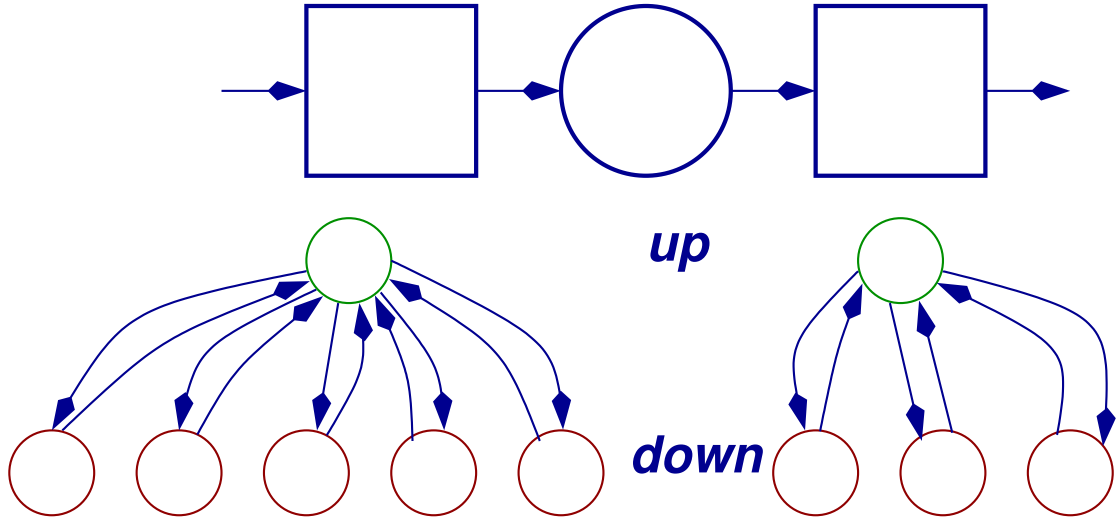
- Each machine in the original line *may* and in the two-machine lines *must* have multiple failure modes.
- For each failure mode downstream of a given buffer, there is a corresponding mode in the downstream machine of its two-machine line.
- Similarly for upstream modes.

Multiple Failure Mode Line Decomposition



- The downstream failure modes appear to the observer after propagation through *blockage* .
- The upstream failure modes appear to the observer after propagation through *starvation* .
- The two-machine lines are more complex that in earlier decompositions but the decomposition equations are simpler.

Two-Machine Line



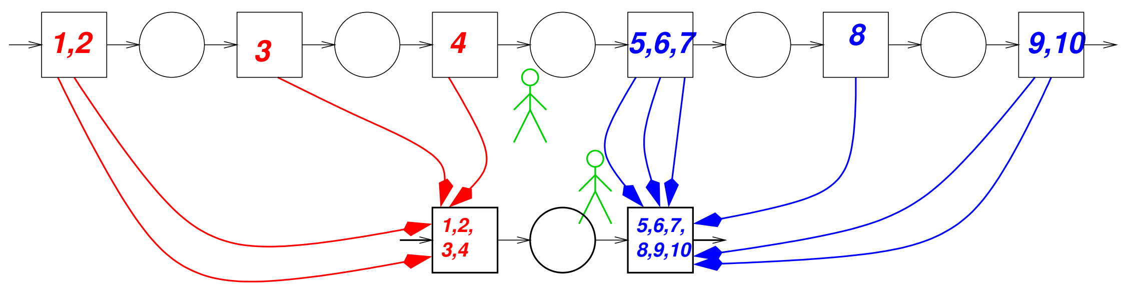
Form a Markov chain and find the steady-state probability distribution. The solution technique is very similar to that of the two-machine-state model. Determine the production rate, probability of starvation and probability of blocking in each down mode, average inventory.

Multiple Failure Mode Line Decomposition

- A set of decomposition equations are formulated.
- They are solved by a Dallery-David-Xie-like algorithm.
- The results are a little more accurate than earlier methods, especially for machines with very different failures.

Multiple Failure Mode Line Decomposition

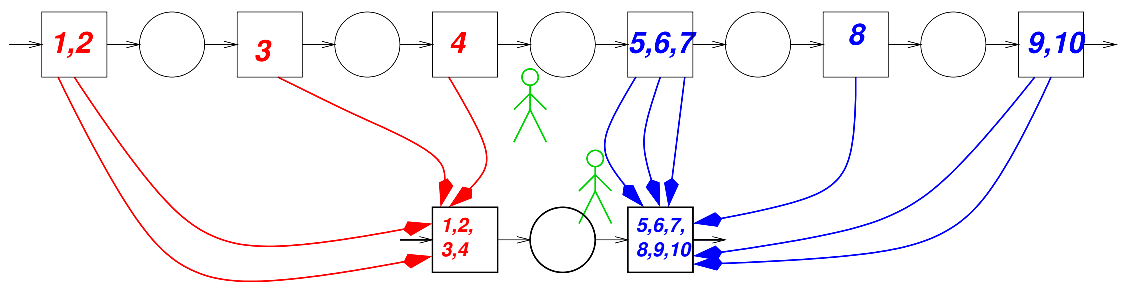
Line Decomposition



- In the upstream machine of the building block, failure mode 4 is a *local* mode; modes 1, 2, and 3 are *remote* modes. Modes 5, 6, and 7 are local modes of the downstream machine; 8, 9, and 10 are remote modes.
- For *every* mode, the repair probability is the same as the repair probability of the corresponding mode in the real line.
- *Local modes:* the probability of failure into a local mode is the same as the probability of failure in that mode of the real machine.

Multiple Failure Mode Line Decomposition

Line Decomposition



- *Remote modes:* i is the building block number; j and f are the machine number and mode number of a remote failure mode. Then

$$p^u_{jf}(i) = \frac{P_{s,jf}(i-1)}{E(i)} r_{jf}; \qquad p^d_{jf}(i-1) = \frac{P_{b,jf}(i)}{E(i-1)} r_{jf}$$

where $p^u_{jf}(i)$ is the probability of failure of the upstream machine into mode jf; $P_{s,jf}(i-1)$ is the probability of starvation of line $i-1$ due to mode jf; r_{jf} is the probability of repair of the upstream machine from mode jf; etc.

- Also, $E(i-1) = E(i)$.

- $p^u_{jf}(i)$, $p^d_{jf}(i)$ are used to evaluate $E(i)$, $P_{s,jf}(i)$,$P_{b,jf}(i)$ from two-machine line i in an iterative method.

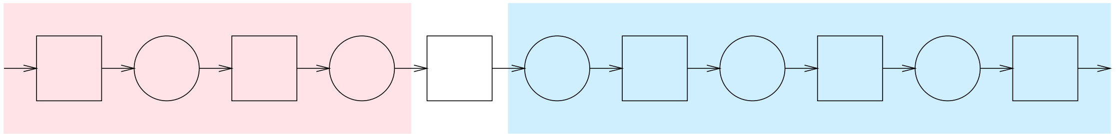
Multiple Failure Mode Line Decomposition

Consider

$$p^u_{jf}(i) = \frac{P_{s,jf}(i-1)}{E(i)} r_{jf}; \qquad p^d_{jf}(i-1) = \frac{P_{b,jf}(i)}{E(i-1)} r_{jf}$$

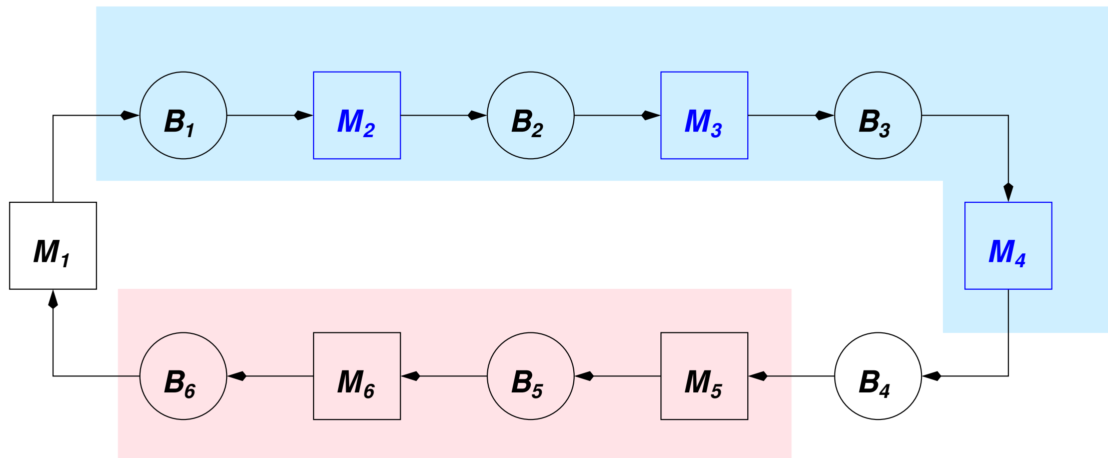
In a line, jf refers to all modes of *all* upstream machines in the first equation; and all modes of *all* downstream machines in the second equation.

We can interpret the upstream machines as the range of starvation and the downstream machines as the range of blockage of the line.



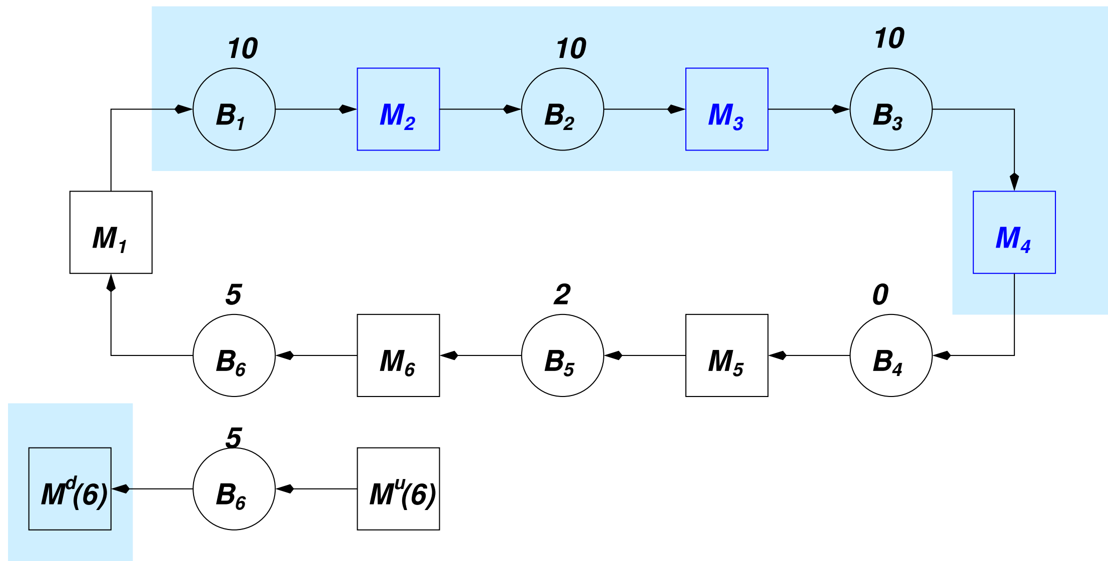
Multiple Failure Mode Line Decomposition

Extension to Loops



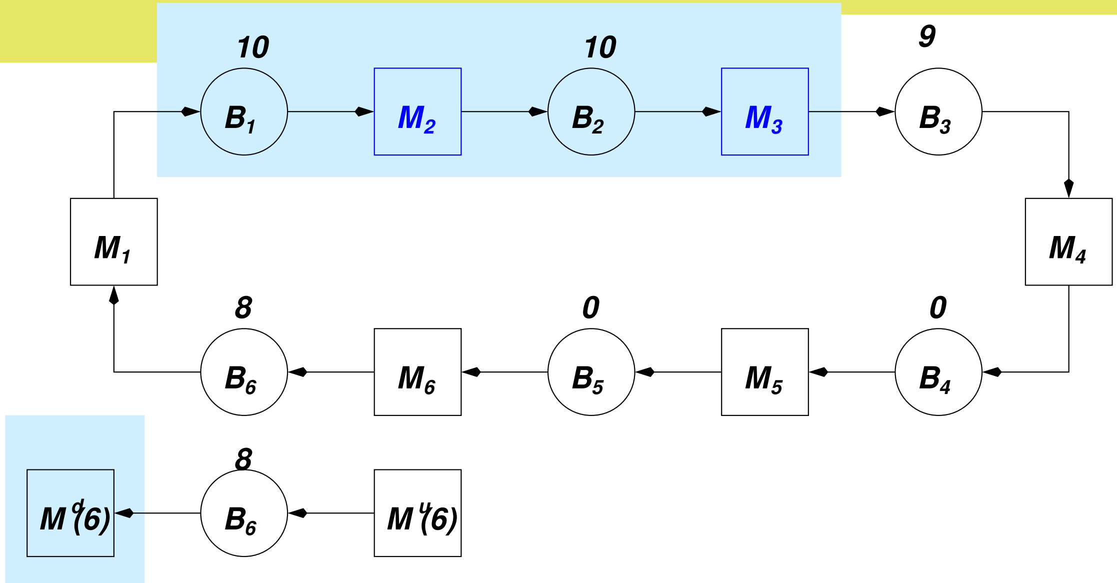
- *Use the multiple-mode decomposition, but adjust the ranges of blocking and starvation accordingly.*
- However, this does not take into account the local information that the observer has.

Thresholds



- The B_6 observer knows how many parts there are in his buffer.
- If there are 5, he knows that the modes he sees in $M^d(6)$ could be those corresponding to the modes of M_1, M_2, M_3, and M_4.

Thresholds



- However, if there are 8, he knows that the modes he sees in $M^d(6)$ could only be those corresponding to the modes of M_1, M_2, and M_3; and *not* those of M_4.
- *The transition probabilities of the two-machine line therefore depend on whether the buffer level is less than 7 or not.*

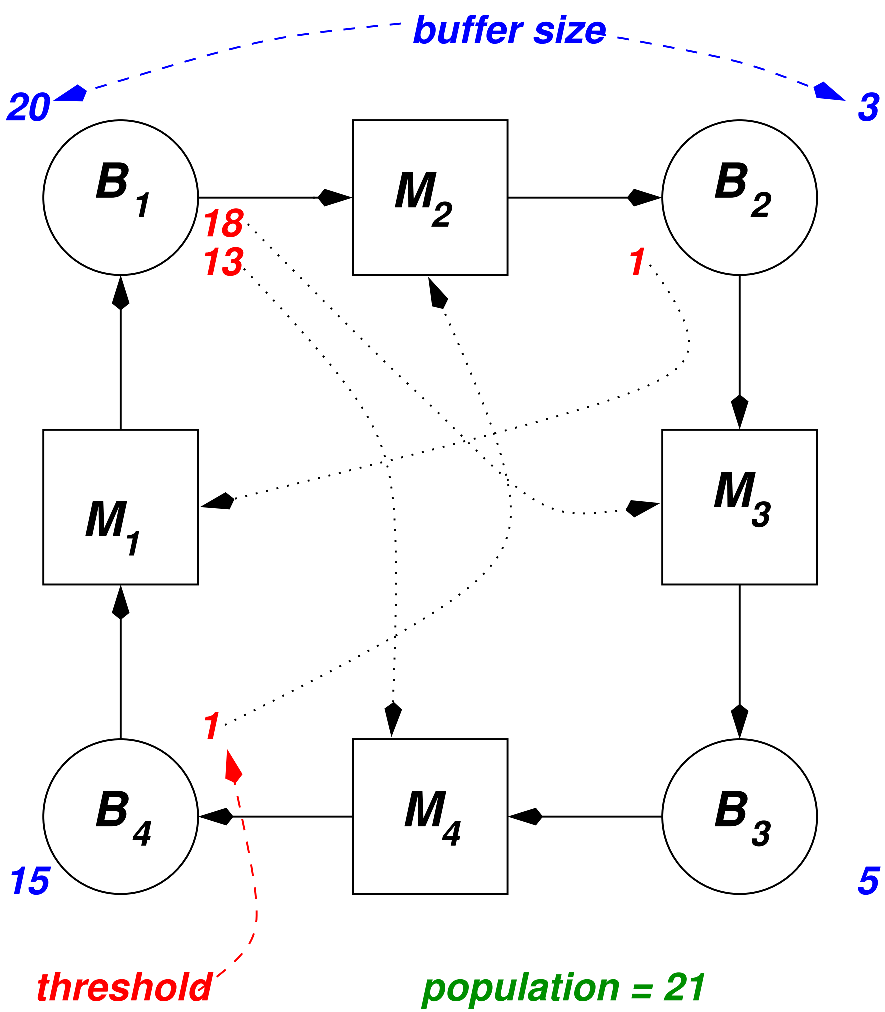
Thresholds

- This would require a new model of a two-machine line.
- The same issue arises for starvation.
- In general, there can be more than one threshold in a buffer.
- Consequently, this makes the two-machine line *very* complicated.

Transformation

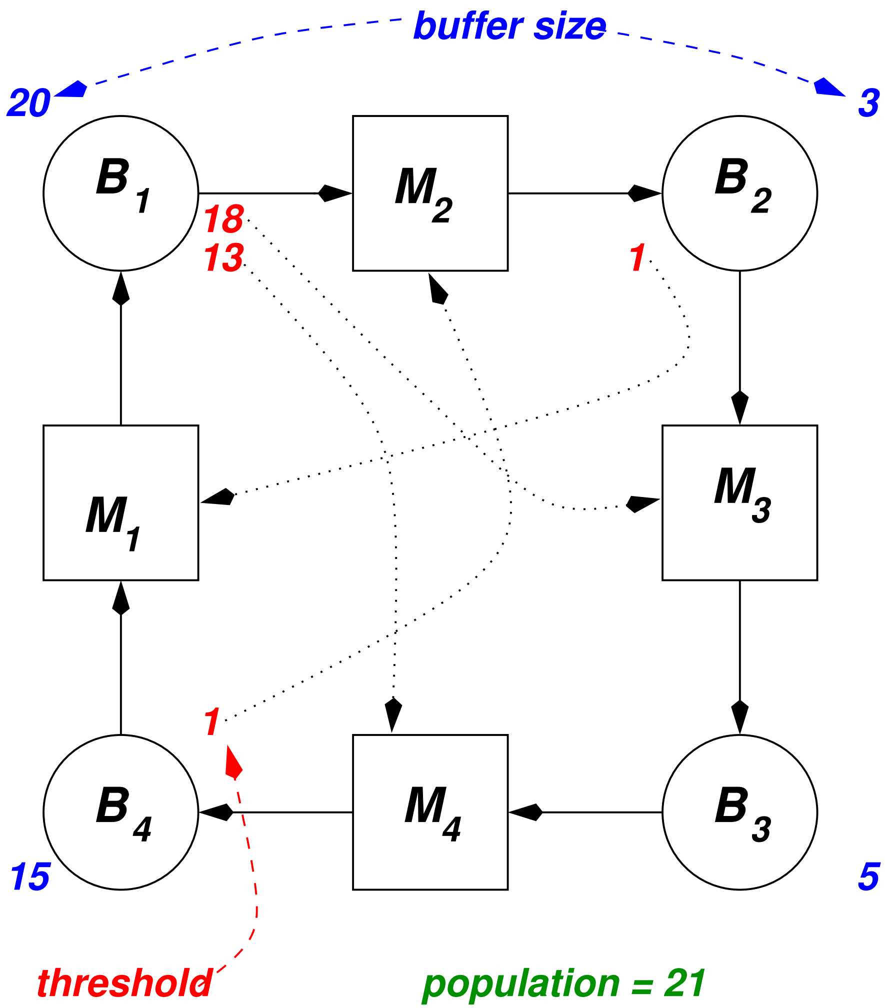
- *Purpose:* to avoid the complexities caused by thresholds.
- *Idea:* Wherever there is a threshold in a buffer, break up the buffer into smaller buffers separated by perfectly reliable machines.

Transformation



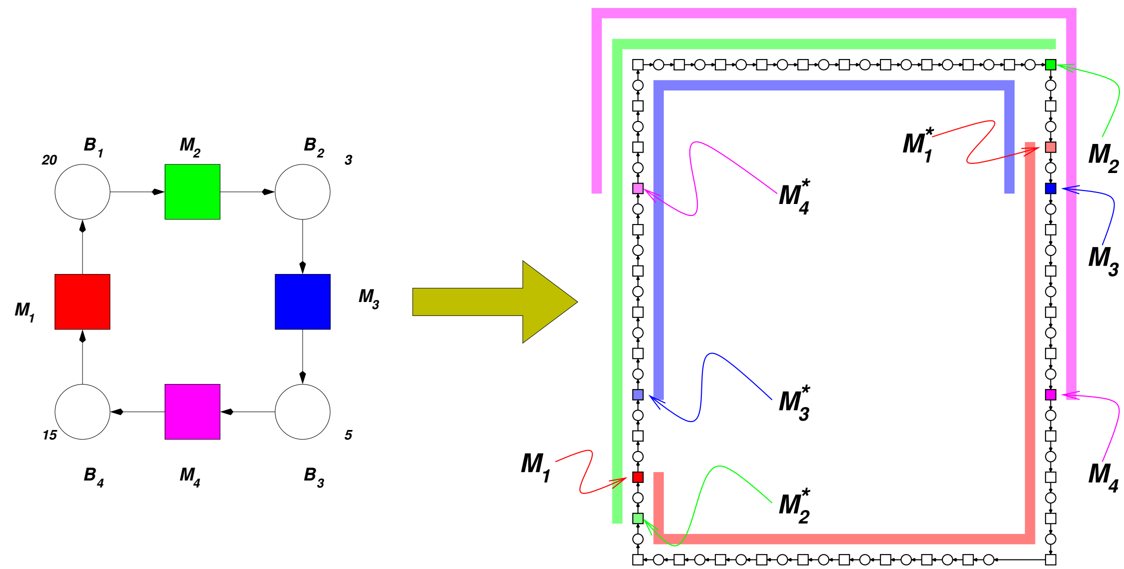
- When M_1 fails for a long time, B_4 and B_3 fill up, and there is one part in B_2. Therefore there is a threshold of 1 in B_2.
- When M_2 fails for a long time, B_1 fills up, and there is one part in B_4. Therefore there is a threshold of 1 in B_4.
- When M_3 fails for a long time, B_2 fills up, and there are 18 parts in B_1. Therefore there is a threshold of 18 in B_1.

Transformation



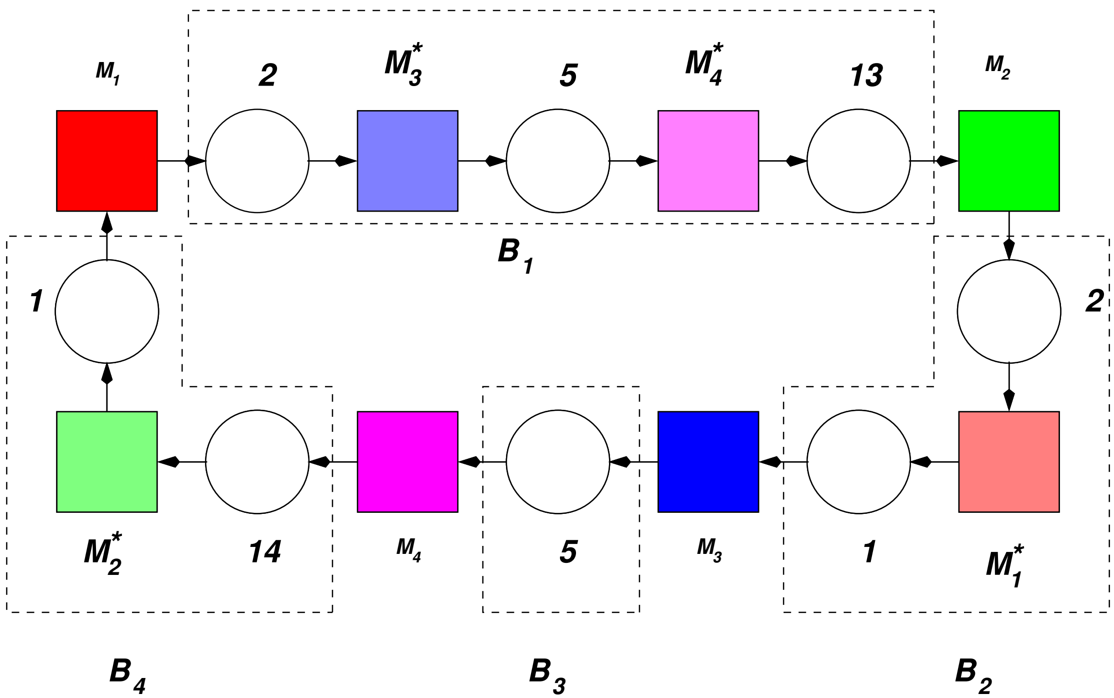
- When M_4 fails for a long time, B_3 and B_2 fill up, and there are 13 parts in B_1. Therefore there is a threshold of 13 in B_1.
- *Note:* B_1 has two thresholds and B_3 has none.
- *Note:* The number of thresholds equals the number of machines.

Transformation



- Break up each buffer into a sequence of buffers of size 1 and reliable machines.
- Count backwards from each *real* machine the number of buffers equal to the population.
- Identify the reliable machine that the count ends at.

Transformation



- Collapse all the sequences of unmarked reliable machines and buffers of size 1 into larger buffers.

Transformation

- Ideally, this would be equivalent to the original system.
- However, the reliable machines cause a delay, so transformation is not exact for the discrete/ deterministic case.
- This transformation *is* exact for continuous-material machines.

Transformation

- If the population is smaller than the largest buffer, at least one machine will *never* be blocked.
- However, that violates the assumptions of the two-machine lines.
- We can reduce the sizes of the larger buffers so that no buffer is larger than the population. This does not change performance.

Numerical Results

Accuracy

- Many cases were compared with simulation:
  - ⋆ *Three-machine cases:* all throughput errors under 1%; buffer level errors averaged 3%, but were as high as 10%.
  - ⋆ *Six-machine cases:* mean throughput error 1.1% with a maximum of 2.7%; average buffer level error 5% with a maximum of 21%.
  - ⋆ *Ten-machine cases:* mean throughput error 1.4% with a maximum of 4%; average buffer level error 6% with a maximum of 44%.

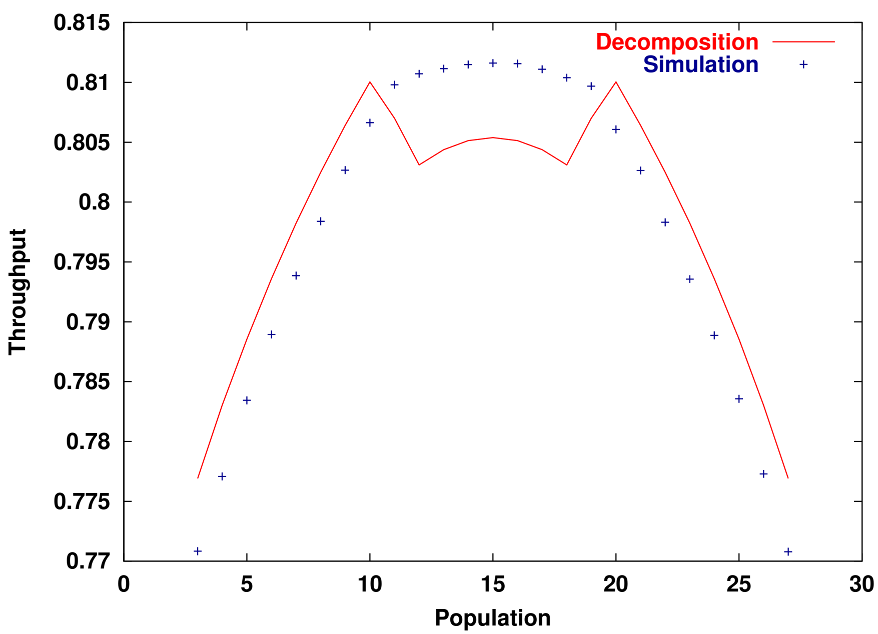
Numerical Results

Other algorithm attributes

- Convergence reliability: almost always.
- Speed: execution time increases rapidly with loop size.
- Maximum size system: 18 machines. Memory requirements grow rapidly also.

Numerical Results

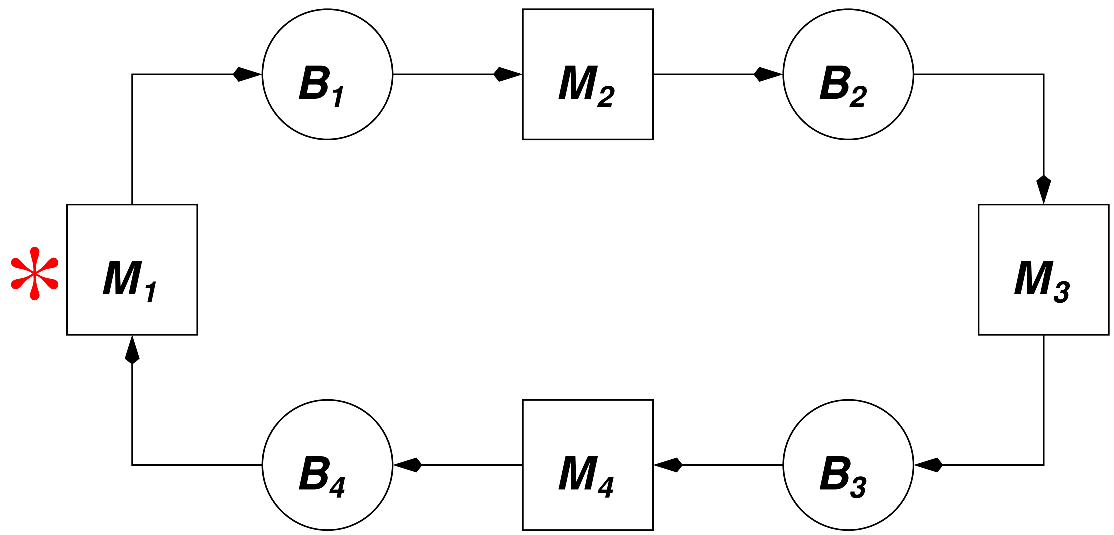
The Batman Effect



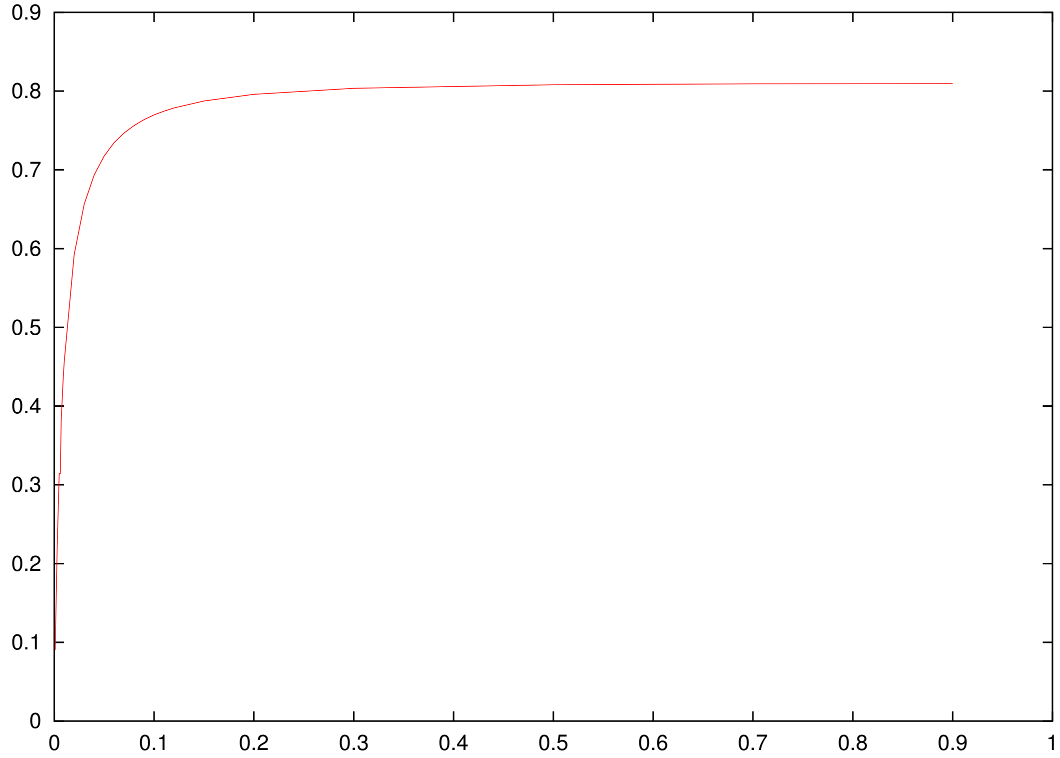
- Error is very small, but there are apparent discontinuities.
- This is because we cannot deal with buffers of size 1, and because we do not need to introduce reliable machines in cases where there would be no thresholds.

Numerical Results

Behavior



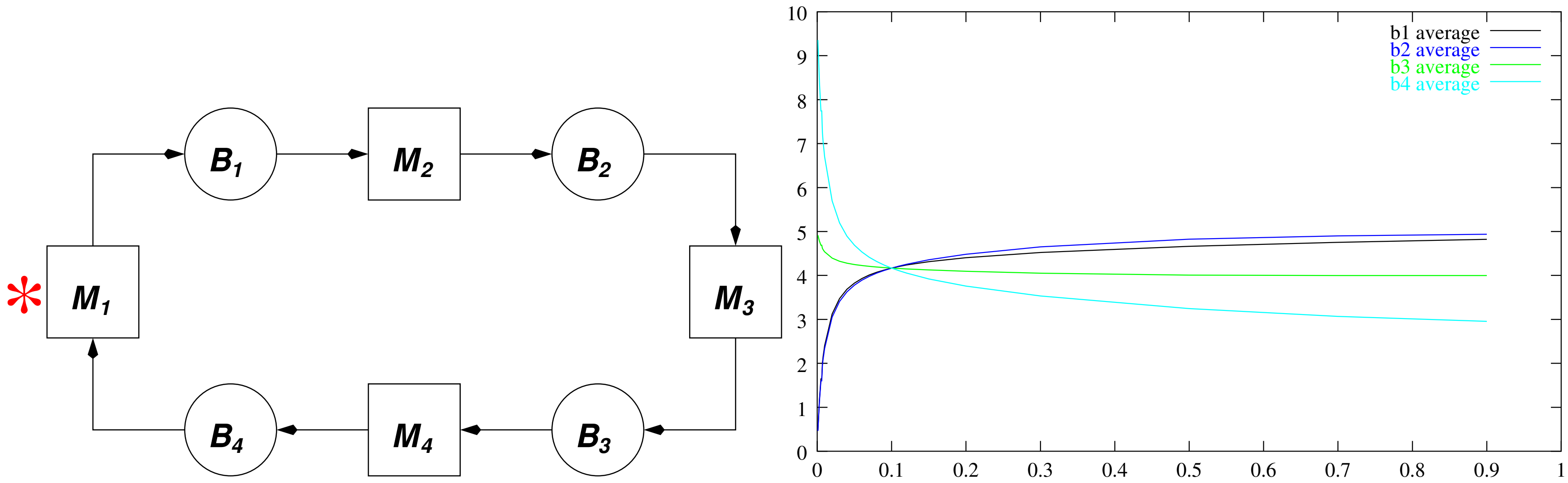
- All buffer sizes 10. Population 15. Identical machines except for M_1.
- Observe average buffer levels and production rate as a function of r_1.



- Production rate vs. r_1.
- Usual saturating graph.

Numerical Results

Behavior



- When r_1 is small, M_1 is a bottleneck, so B_4 holds 10 parts, B_3 holds 5 parts, and the others are empty.
- As r_1 increases, material is more evenly distributed. When $r_1 = 0.1$, the network is totally symmetrical.

Applications

- Design system with pallets/fixtures. The fixtures and the space to hold them in are expensive.
- Design system with tokens/kanbans (CONWIP). By limiting population, we reduce production rate, but we also reduce inventory.

2.852 Manufacturing Systems Analysis
Spring 2010