MIT 2.852
Manufacturing Systems Analysis
Lectures 15–16: Assembly/Disassembly Systems

Stanley B. Gershwin

http://web.mit.edu/manuf-sys

Massachusetts Institute of Technology

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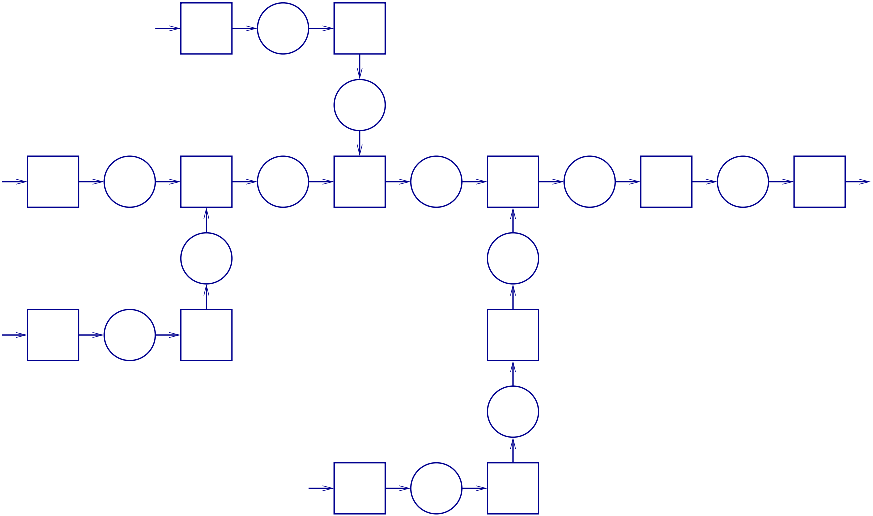
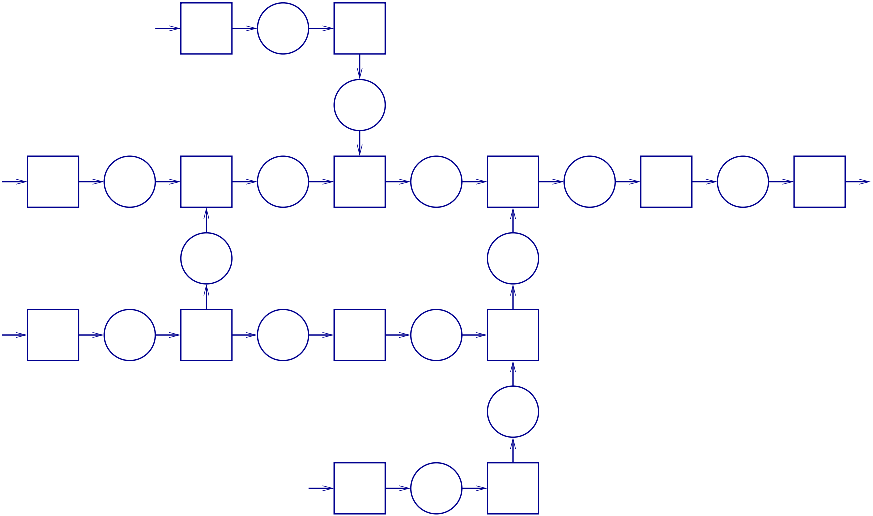
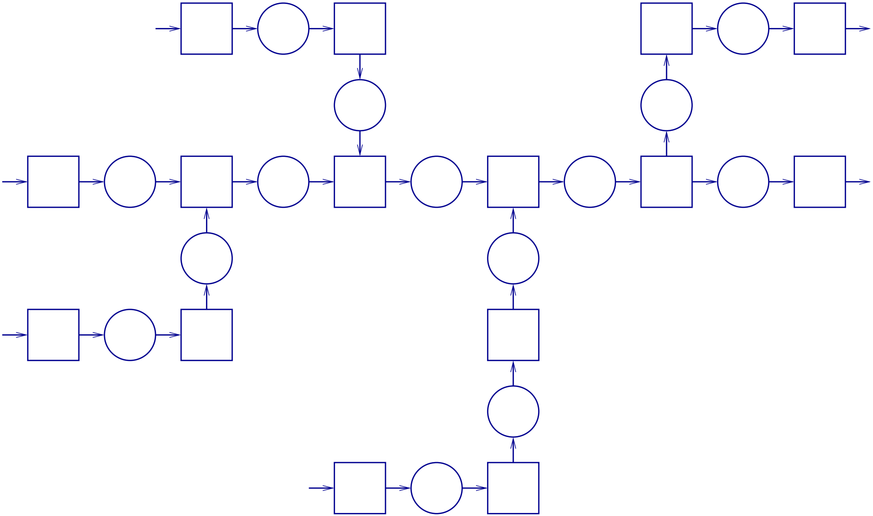


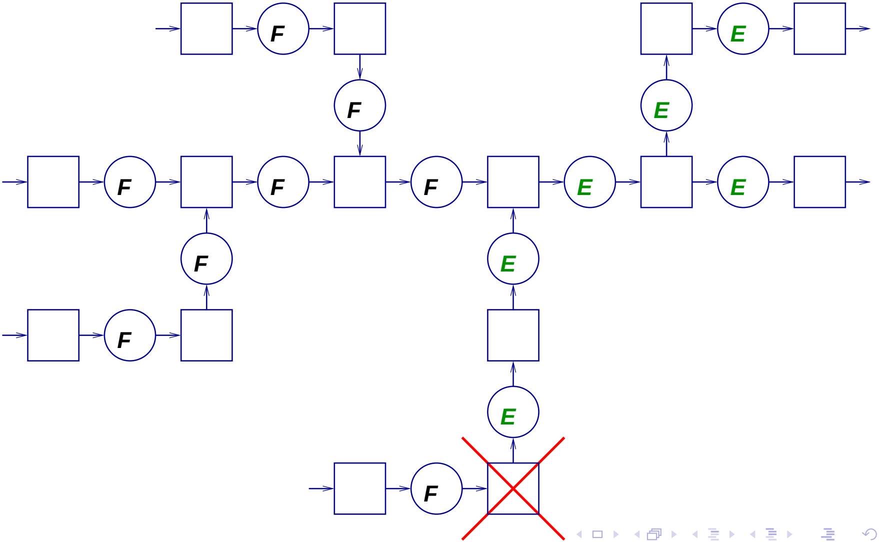
Assembly-Disassembly System with a Loop



Assembly-Disassembly Systems
A-D System without Loops



Assembly-Disassembly Systems
Disruption Propagation in an A-D System without Loops



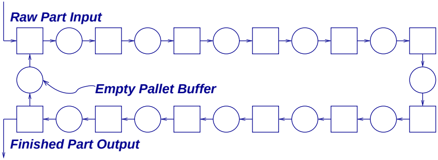
An assembly/disassembly system is a generalization of a transfer line:

- Each machine may have 0, 1, or more than one buffer upstream.
- Each machine may have 0, 1, or more than one buffer downstream.
- Each buffer has *exactly* one machine upstream and one machine downstream.
- *Discrete material systems:* when a machine does an operation, it removes one part from **each** upstream buffer and inserts one part into **each** downstream buffer.
- *Continuous material systems:* when machine M_i operates during $[t, t + \delta t]$, it removes $\mu_i \delta t$ from **each** upstream buffer and inserts $\mu_i \delta t$ into **each** downstream buffer.
- A machine is starved if *any* of its upstream buffers is empty. It is blocked if *any* of its downstream buffers is full.

Assembly-Disassembly Systems
Models and Analysis

- A/D systems can be modeled similarly to lines:
  - discrete material, discrete time, deterministic processing time, geometric repair and failure times;
  - discrete material, continuous time, exponential processing, repair, and failure times;
  - continuous continuous time, deterministic processing rate, exponential repair and failure times;
  - other models not yet discussed in class.
- A/D systems *without loops* can be analyzed similarly to lines by decomposition.
- A/D systems *with loops* can be analyzed by decomposition, but there are additional complexities.

- Systems with loops are *not* ergodic. That is, the steady-state distribution is a function of the initial conditions.
- Example: if the system below has K pallets at time 0, it will have K pallets for all $t \geq 0$. Therefore, the probability distribution is a function of K.



- This applies to more general systems with loops, such the example on Slide 3.

- In general,

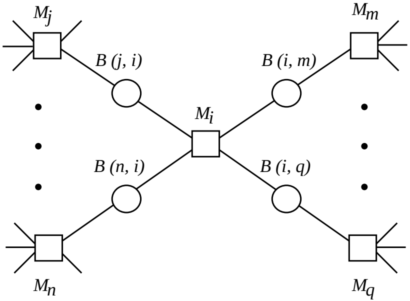
$$\mathbf{p}(s|s(0)) = \lim_{t\to\infty} \text{ prob } \{ \text{ state of the system at time } t = s | \text{ state of the system at time } 0 = s(0)\}.$$

- Consequently, the performance measures depend on the initial state of the system:
  - The production rate of Machine M_i, in parts per time unit, is

$$E_i(s(0)) = \text{ prob } \left[\alpha_i = 1 \text{ and } (n_b > 0 \ \forall \ b \in U(i)) \text{ and } (n_b < N_b \ \forall \ b \in D(i)) \middle| s(0)\right].$$

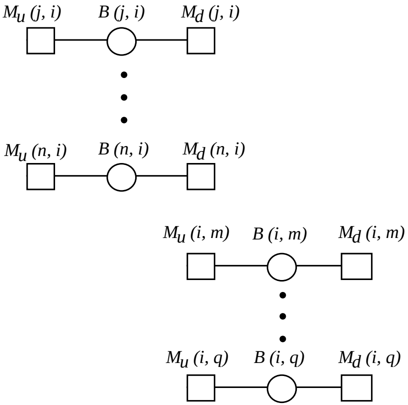
  - The average level of Buffer b is

$$\bar{n}_b(s(0)) = \sum_s n_b \text{ prob } (s|s(0)).$$



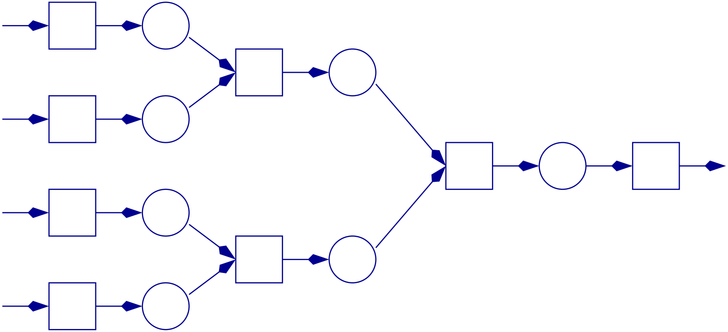
Part of Original Network

Assembly-Disassembly Systems
Decomposition



Part of Decomposition

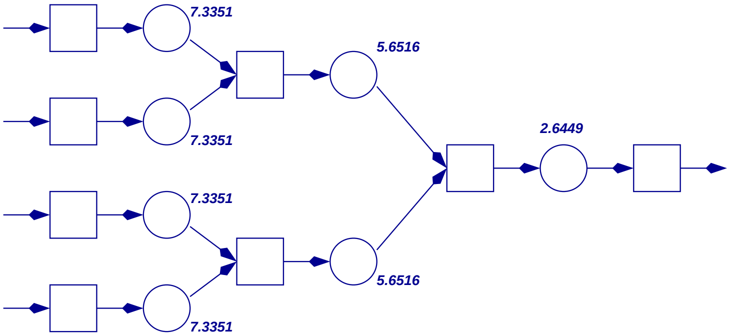
Numerical examples
Eight-Machine Systems



Deterministic processing time model

Numerical examples

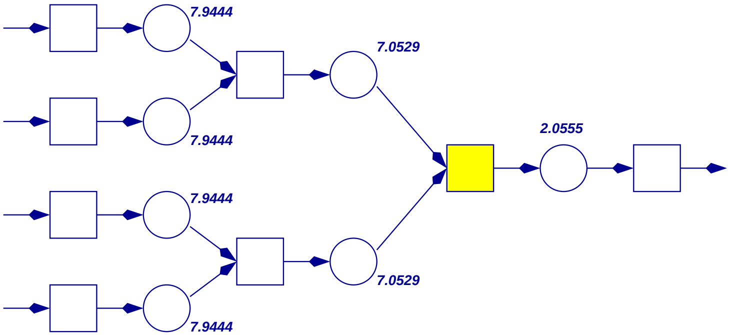
Eight-Machine Systems



Case 1:

$r_i = .1, p_i = .1, i = 1, ..., 8;$

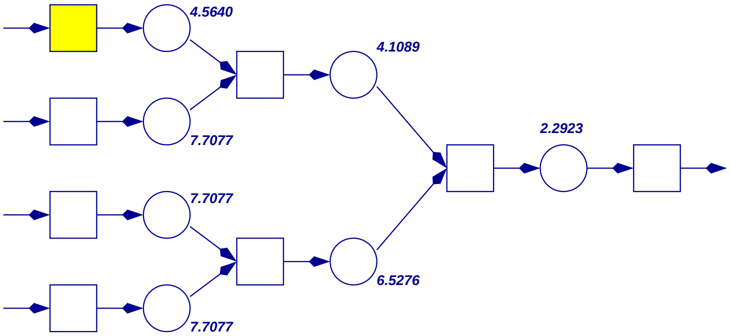
$N_i = 10, i = 1, ..., 7.$



Case 2:

Same as Case 1 except $p_7 = .2$

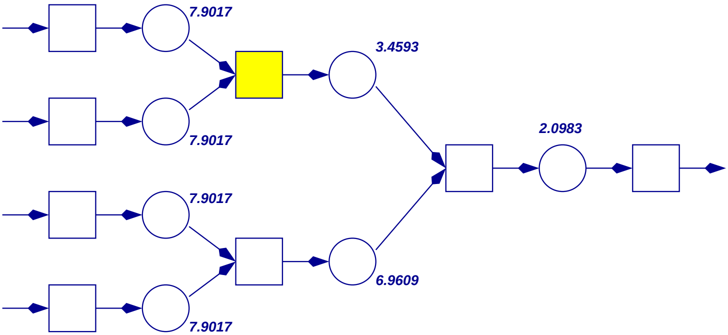
Numerical examples
Eight-Machine Systems



Case 3:

Same as Case 1 except $p_1 = .2$

Numerical examples
Eight-Machine Systems



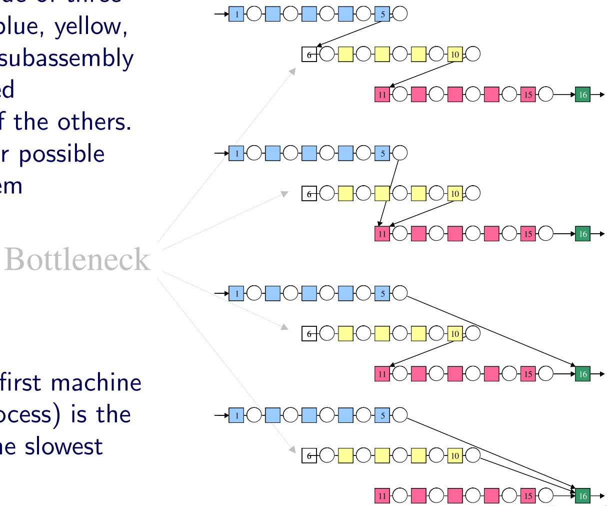
Case 4:

Same as Case 1 except $p_3 = .2$

Numerical Examples

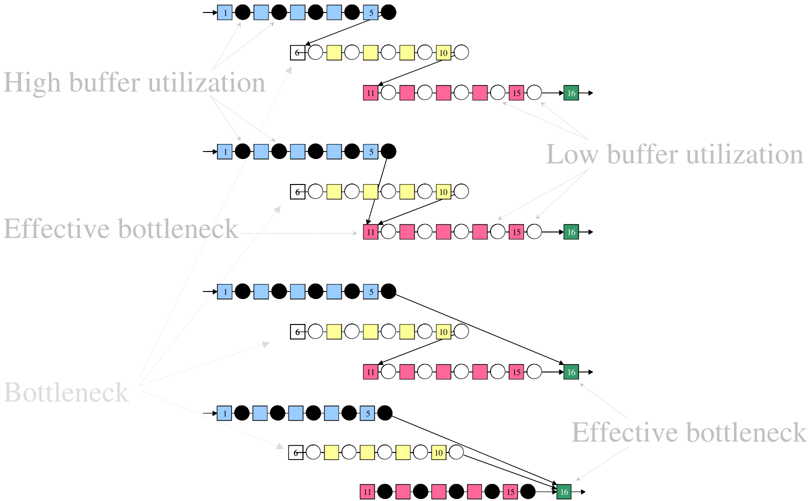
Alternate Assembly Line Designs

A product is made of three subassemblies (blue, yellow, and red). Each subassembly can be assembled independently of the others. We consider four possible production system structures.



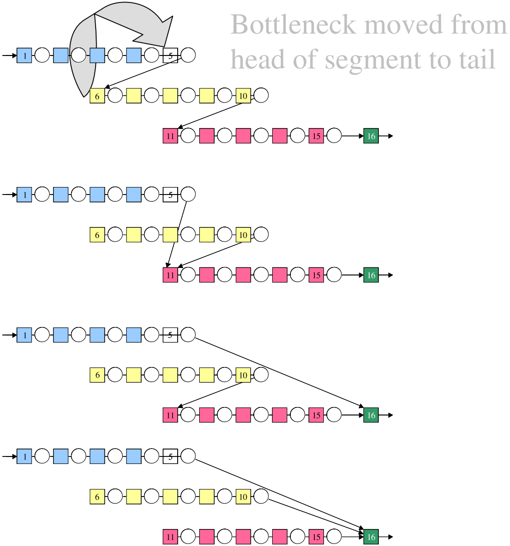
Machine 6 (the first machine of the yellow process) is the bottleneck — the slowest operation of all.

Numerical Examples
Alternate Assembly Line Designs



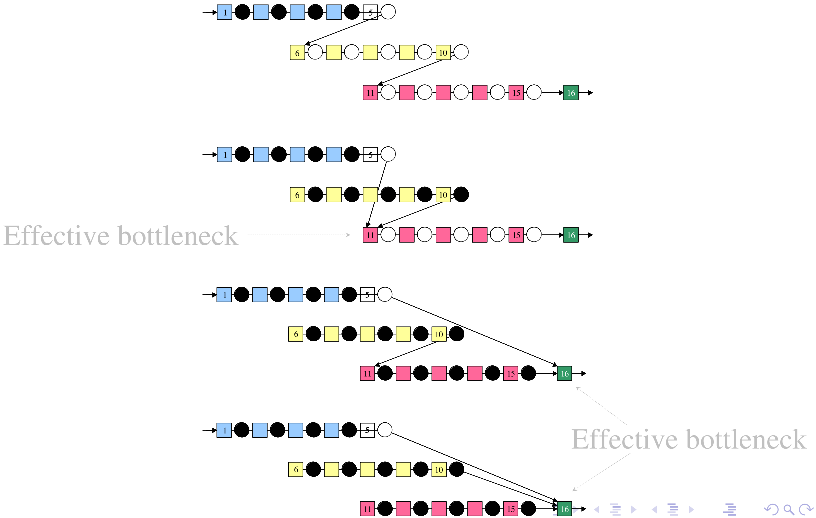
Numerical Examples

Alternate Assembly Line Designs

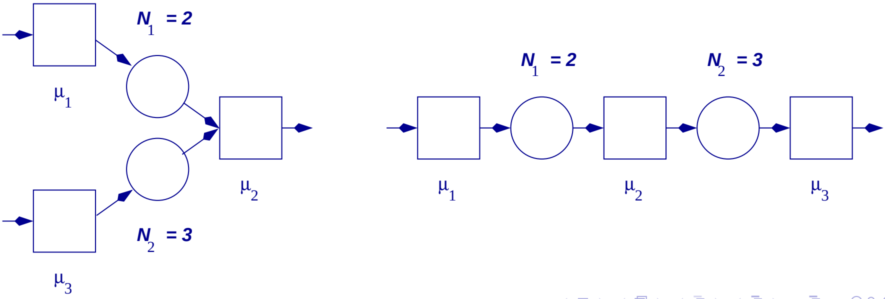


Now the bottleneck is Machine 5, the last operation of the blue process.

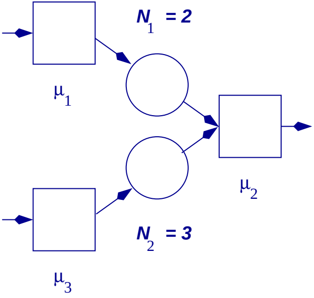
Numerical Examples
Alternate Assembly Line Designs

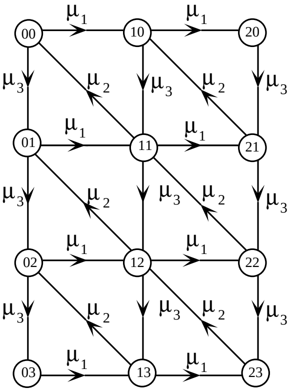


Consider a three-machine transfer line and a three-machine assembly system. Both are perfectly reliable ($p_i = 0$) exponentially processing time systems.

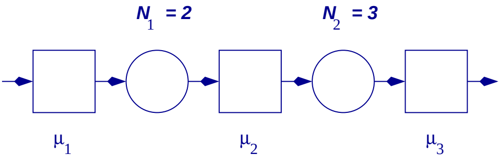


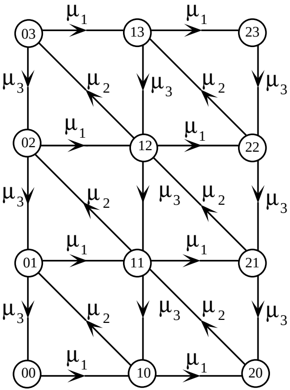
Equivalence
Assembly System State Space





Equivalence
Transfer Line State Space





Equivalence

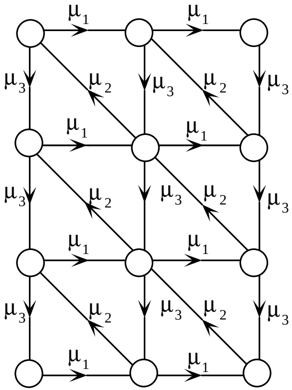
Unlabeled State Space

- The transition graphs of the two systems are the same except for the labels of the states.
- Therefore, the steady-state probability distributions of the two systems are the same, except for the labels of the states.
- The relationship between the labels of the states is:

$$(n_1^A, n_2^A) \Longleftrightarrow (n_1^T, N_2 - n_2^T)$$

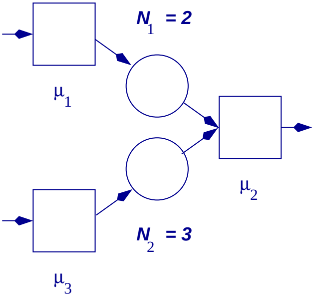
- Therefore, in steady state,

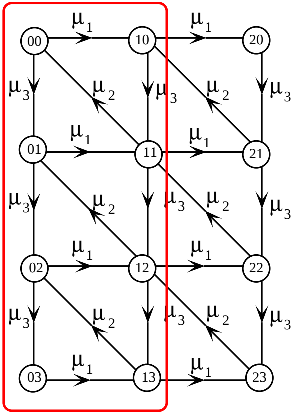
$$\text{prob}(n_1^A, n_2^A) = \text{prob}(n_1^T, N_2 - n_2^T)$$



Production rate = rate of flow of material into M_1

$$= \mu_1 \sum_{n_1=0}^{1} \sum_{n_2=0}^{3} \mathbf{p}(n_1, n_2)$$





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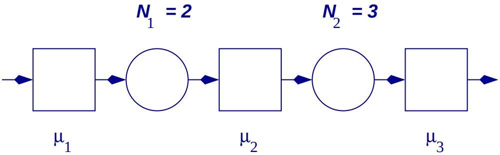
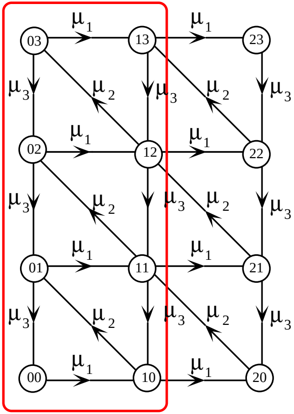




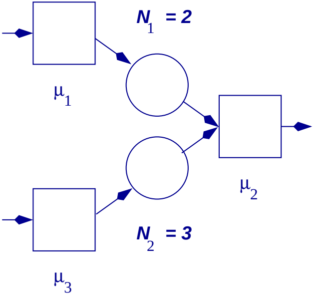
Equivalence
Equal Production Rates

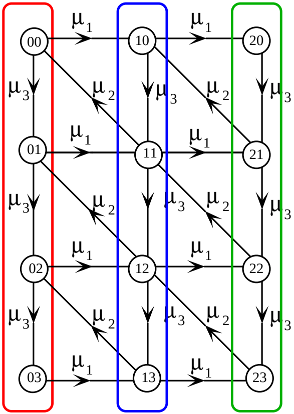
Therefore

$$P^A = P^T$$

Equivalence
Assembly System $\bar{n}_1$

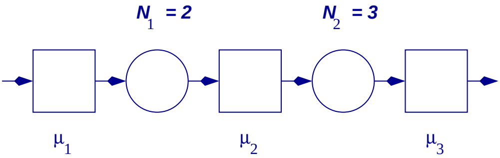
$$\bar{n}_1 = \sum_{n_1=0}^{2} \sum_{n_2=0}^{3} n_1 \mathbf{p}(n_1, n_2) = \sum_{n_1=0}^{2} n_1 \left[\sum_{n_2=0}^{3} \mathbf{p}(n_1, n_2) \right]$$

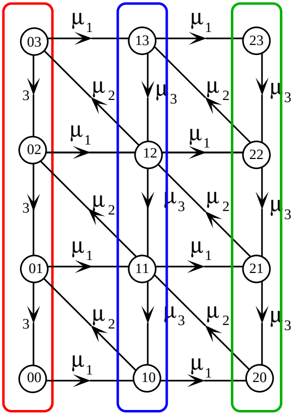




Equivalence
Transfer Line $\bar{n}_1$

$$\bar{n}_1 = \sum_{n_1=0}^{2} \sum_{n_2=0}^{3} n_1 \mathbf{p}(n_1, n_2) = \sum_{n_1=0}^{2} n_1 \left[\sum_{n_2=0}^{3} \mathbf{p}(n_1, n_2) \right]$$





Equivalence
Equal $\bar{n}_1$

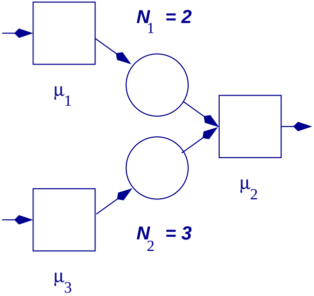
Therefore

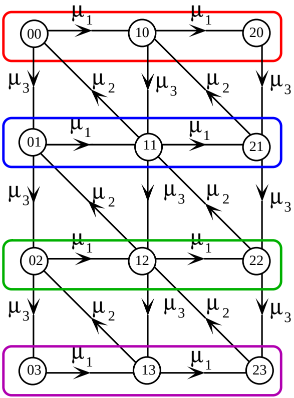
$$\bar{n}_1^A = \bar{n}_1^T$$

Equivalence

Assembly System $\bar{n}_2$

$$\bar{n}_2 = \sum_{n_1=0}^{2} \sum_{n_2=0}^{3} n_2 \mathbf{p}(n_1, n_2) = \sum_{n_2=0}^{3} n_2 \left[\sum_{n_1=0}^{2} \mathbf{p}(n_1, n_2) \right]$$

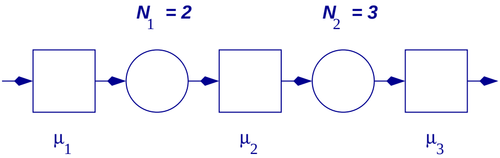


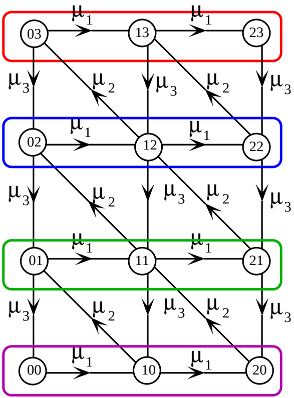


Equivalence

Transfer Line $\bar{n}_2$

$$\bar{n}_2 = \sum_{n_1=0}^{2} \sum_{n_2=0}^{3} n_2 \mathbf{p}(n_1, n_2) = \sum_{n_2=0}^{3} n_2 \left[\sum_{n_1=0}^{2} \mathbf{p}(n_1, n_2) \right]$$





Equivalence

Complementary $\bar{n}_1$

Therefore

$$\bar{n}_2^A = N_2 - \bar{n}_2^T$$

- *Notation:* Let j be a buffer. Then the machine upstream of the buffer is $u(j)$ and the machine downstream of the buffer is $d(j)$.
- *Theorem:*
  - Assume
    - Z and Z' are two exponential A/D networks with the same number of machines and buffers. Corresponding machines and buffers have the same parameters; that is, $\mu'_i = \mu_i, i = 1, ..., k_M$ and $N'_b = N_b, b = 1, ..., k_B$.
    - There is a subset of buffers Ω such that for $j \notin \Omega, u'(j) = u(j)$ and $d'(j) = d(j)$; and for $j \in \Omega, u'(j) = d(j)$ and $d'(j) = u(j)$. That is, there is a set of buffers such that the direction of flow is reversed in the two networks.
  - Then, the transition equations for network Z' are the same as those of Z, except that the buffer levels in Ω are replaced by the amounts of space in those buffers.

- That is, the transition (or balance) equations of Z' can be written by transforming those of Z.
- In the Z equations, replace n_j by $N_j - n_j$ for all $j \in \Omega$.

Corollary:

- Assume:
  - The initial states $s(0)$ and $s'(0)$ are related as follows: $n'_j(0) = n_j(0)$ for $j \notin \Omega$, and $n'_j(0) = N_j - n_j(0)$ for $j \in \Omega$.
- Then

$$P'(n'(0)) = P(n(0))$$

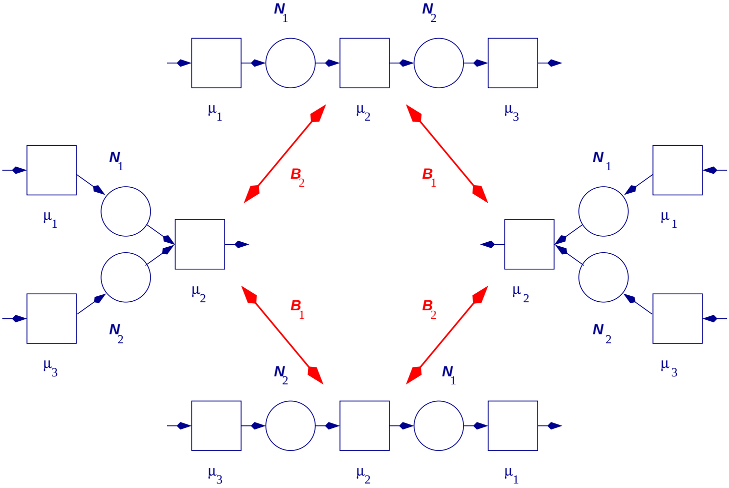
$$\bar{n}'_b(n'(0)) = \bar{n}_b(n(0)), \text{ for } j \notin \Omega$$

$$\bar{n}'_b(n'(0)) = N_b - \bar{n}_b(n(0)), \text{ for } j \in \Omega$$

Corollary: That is,

- the production rates of the two systems are the same,
- the average levels of all the buffers in the systems whose direction of flow has not been changed are the same,
- the average levels of all the buffers in the systems whose direction of flow has been changed are complementary; the average number of parts in one is equal to the average amount of space in the other.

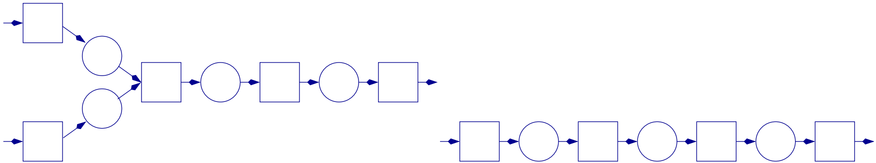
Equivalence
Equivalence class of three-machine systems



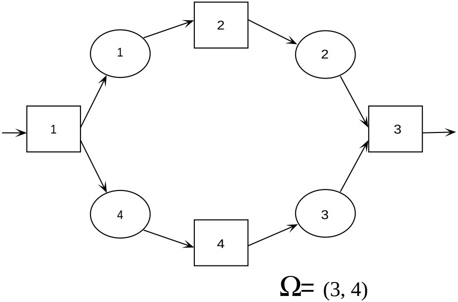
Equivalence

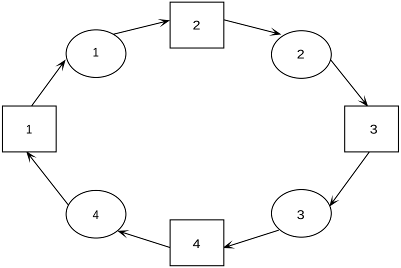
Equivalence classes of four-machine systems

Representative members



Equivalence
Example of equivalent loops





(a) A Fork/ Join Network

(b) A Closed Network

- Loops and invariants
- Two-machine loops
- Instability of A/D systems with infinite buffers

2.852 Manufacturing Systems Analysis
Spring 2010