2.830J / 6.780J / ESD.63J Control of Manufacturing Processes (SMA 6303)
Spring 2008

| **2.830/6.780J — Control of Manufacturing Processes** | Spring '08 |
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# Handout - Kurtosis

MIT OpenCourseWare

I. Moment generating function (raw moments):

$$\begin{aligned} M_x(t) &= \int_{-\infty}^{\infty} e^{tx} f(x)dx = E(e^{tx}) \\ &= \int_{-\infty}^{\infty} (1 + tx + \frac{t^2x^2}{2!} \dots) f(x)dx \end{aligned}$$

II. Normal distribution:

$$\begin{aligned} M_x(t) &= e^{\mu t + \frac{\sigma^2 t^2}{2}} \\ &= (1 + \mu t + \frac{\mu^2 t^2}{2!} + \frac{\mu^3 t^3}{3!} + \frac{\mu^4 t^4}{4!} \dots) + (1 + \frac{\sigma^2 t^2}{2} + \frac{\sigma^4 t^4}{4 \cdot 2!} \dots) \end{aligned}$$

$$\begin{aligned} \mu'_0 &= 1 \\ \mu'_1 &= \mu \\ \mu'_2 &= \text{2nd raw moment} = (\text{coefficient of } t^2) \cdot 2! = \mu^2 + \sigma^2 \\ \mu'_3 &= \text{3rd raw moment} = (\text{coefficient of } t^3) \cdot 3! = \mu^3 + 3\mu\sigma^2 \\ \mu'_4 &= \text{4th raw moment} = (\text{coefficient of } t^4) \cdot 4! = \mu^4 + 6\mu^2\sigma^2 + 3\sigma^4 \end{aligned}$$

III. Kurtosis is defined as

$$\frac{\text{4th central moment}}{\text{2nd central moment}^2} = \frac{\mu_4}{\mu_2^2}$$

Using the definition

$$\mu_n = \sum_{j=0}^{n} \binom{n}{j} (-1)^{n-j} \mu'_j (\mu'_1)^{n-j}$$

,

$$\mu_4 = \binom{4}{0}(-1)^4 \cdot 1 \cdot \mu^4 + \binom{4}{1}(-1)^3 \cdot \mu \cdot \mu^3 + \binom{4}{2}(-1)^2 \cdot (\mu^2 + \sigma^2) \cdot \mu^2 + \binom{4}{3}(-1)^1 \cdot (\mu^3 + 3\mu\sigma^2) \cdot \mu^1 + \binom{4}{4} \cdot (\mu^4 + 6\mu^2\sigma^2 + 3\sigma^4) \cdot 1$$

Then, for the normal distribution,

$$\frac{E(x-\mu)^4}{\sigma^4} = \frac{\mu_4}{\mu_2^2} = \frac{3\sigma^4}{\sigma^4} = 3$$