MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Mechanical Engineering

2.71/2.710 Optics
Spring 2012

Quiz 2

Monday, April 30, 2012

PLEASE DO NOT TURN OVER UNTIL EXAM STARTS

DURATION: 80min (9:35–10:55)

TOTAL PAGES: 3

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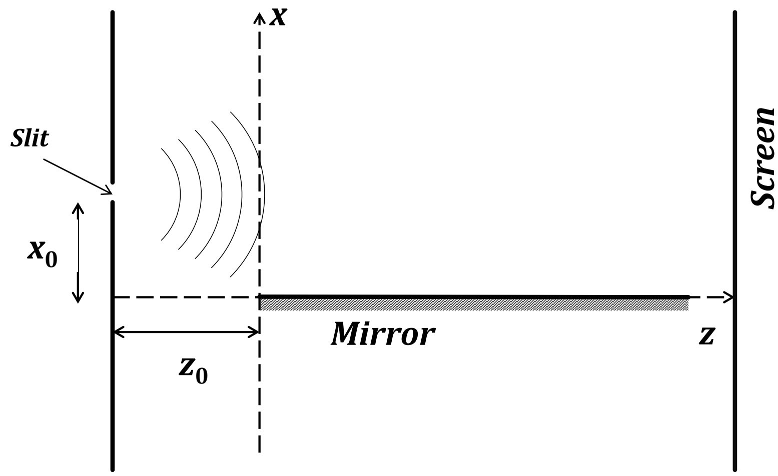
1. **A Lloyd Mirror Interferometer.** The Lloyd mirror is a wavefront-splitting interferometer. It consists of a flat glass mirror that reflects a portion of wavefront that comes from a narrow slit. Another portion of the wavefront proceeds directly to the screen. The interference of the two wavefronts form a set of bright and dark fringes that can be measured on the screen.



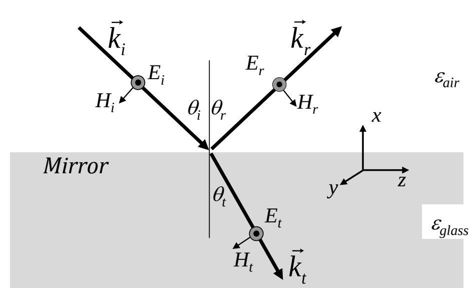
In our problem, let's assume the mirror is placed at the plane $x = 0$ and illuminated by a spherical wave originating from the slit at location (x_0, $-z_0$) (where x_0, $z_0 > 0$). Using the paraxial approximation for a 1D spherical wave (y=0),

$$E(x,z) = E_0 \frac{\exp\left[ik(z+z_0)\right]}{i(z+z_0)} \exp\left[ik\frac{(x-x_0)^2}{2(z+z_0)}\right]$$

a) (10%) The reflected wavefront from the mirror can be considered as spherical wave radiation from a virtual source. Using your knowledge from geometric optics, determine the origin of the virtual source that radiates such spherical wave.

b) (15%) The source illuminating the slit has a wavelength of 500nm in air. If the slit is positioned at x_0=1mm above the flat mirror, and the screen is placed 1 meter away from the slit, please estimate the spacing of the fringes on the screen.

c) (15%) In order to find the relative amplitude and phase of the virtual source with respect to the original spherical wave, we have to consider the Fresnel equations for reflected waves.



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For simplicity, let's assume the E-field of source is polarized in the y-direction (S-polarization) and the slit is placed near the x=0 plane (x_0<<z_0, so $\theta_i=\theta_r$-->90°). Using these assumptions and based on the reflection coefficient for S-polarization at air-glass interface

$$r_S = \frac{E_{0r}}{E_{0i}} = \frac{k_{x,air} - k_{x,glass}}{k_{x,air} + k_{x,glass}}$$

And $k_{x,air} = k\sqrt{n_{air}^2 - sin^2\theta_i}$, $k_{x,glass} = k\sqrt{n_{glass}^2 - sin^2\theta_i}$ (n_{air}=1, n_{glass}=1.5),

Please estimate the relative amplitude (i.e. $|r_S|$)and phase (i.e. arg(r_s)) of the virtual source when θ_i approaches 90°.

***d) (10%)** Using the result you obtained from c), show that the fringe pattern is dark at *x*=0 on the screen.

2. Optical Fourier transforms.

a) (20%) Calculate the Fourier transform of the function:

$$g(x) = \frac{1}{2} exp(-i\pi\alpha^2 x^2)\left[1 + \cos\left(\frac{2\pi x}{\Lambda}\right)\right]$$

b) **(30%)** Assuming α = 10mm^{-1}, Λ = 10μm, design an optical system which at its output plane creates an exact replica of the Fourier transform of the previous question. Additional constraints on your design are:

1. Your system should be designed for spatially and temporally coherent illumination consisting of a plane wave at wavelength λ = 500nm, incident on-axis.

2. You may only use "standard" optical elements such as lenses, gratings, prisms, and free-space propagation.

3. The spatial frequency component u = 50mm^{-1} should be mapped at distance of 3 cm away from the optical axis at the output plane.

GOOD LUCK!

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