Today

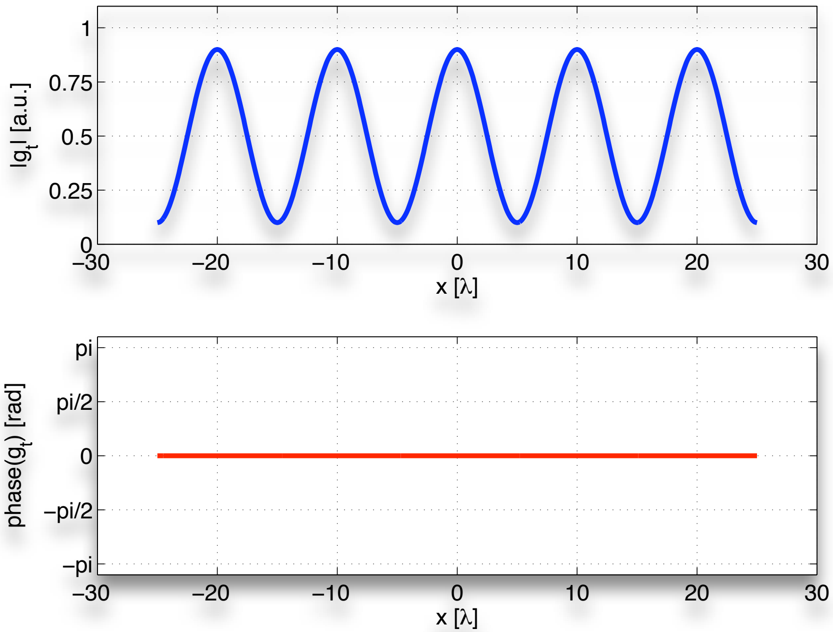
- Gratings:
  - Sinusoidal amplitude grating
  - Sinusoidal phase grating
  - in general: spatially periodic thin transparency

Wednesday

- Fraunhofer diffraction
- Fraunhofer patterns of typical apertures
- Spatial frequencies and Fourier transforms

Gratings

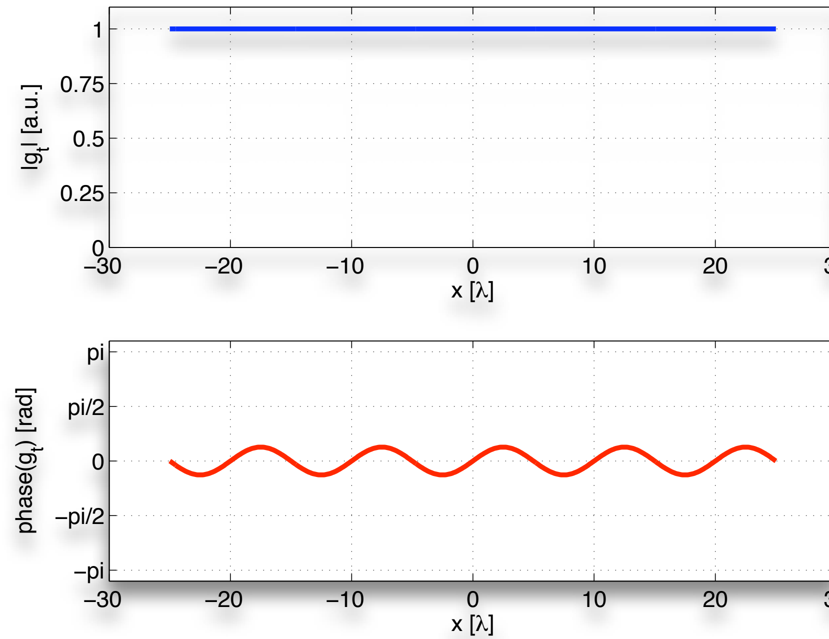
Amplitude grating



$$g_t(x) = \frac{1}{2}\left[1 + m\cos\left(2\pi\frac{x}{\Lambda} + \phi\right)\right]$$
$$|g_t(x)| = \frac{1}{2}\left[1 + m\cos\left(2\pi\frac{x}{\Lambda} + \phi\right)\right]$$
$$\angle g_t(x) = 0.$$

Λ: period; *m*: contrast; *φ*: phase shift

Phase grating

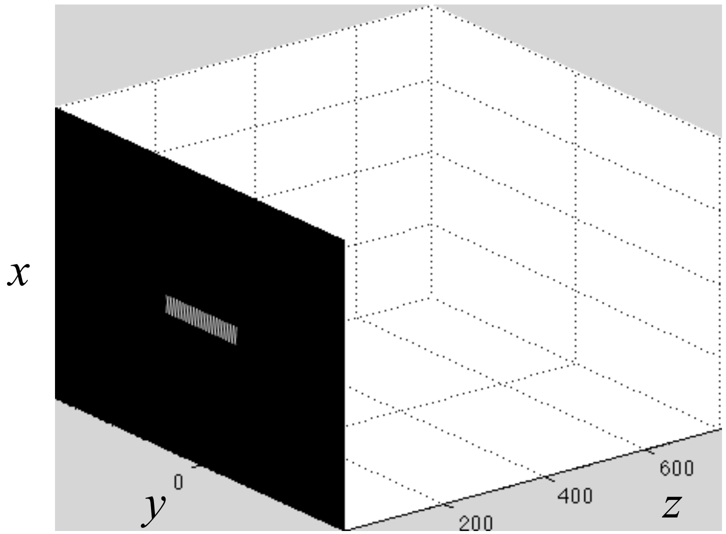


$$g_t(x) = \exp\left[i\frac{m}{2}\sin\left(2\pi\frac{x}{\Lambda} + \phi\right)\right]$$
$$|g_t(x)| = 1$$
$$\angle g_t(x) = \frac{m}{2}\sin\left(2\pi\frac{x}{\Lambda} + \phi\right).$$

Λ: period; *m*: phase contrast; *φ*: phase shift

Gratings

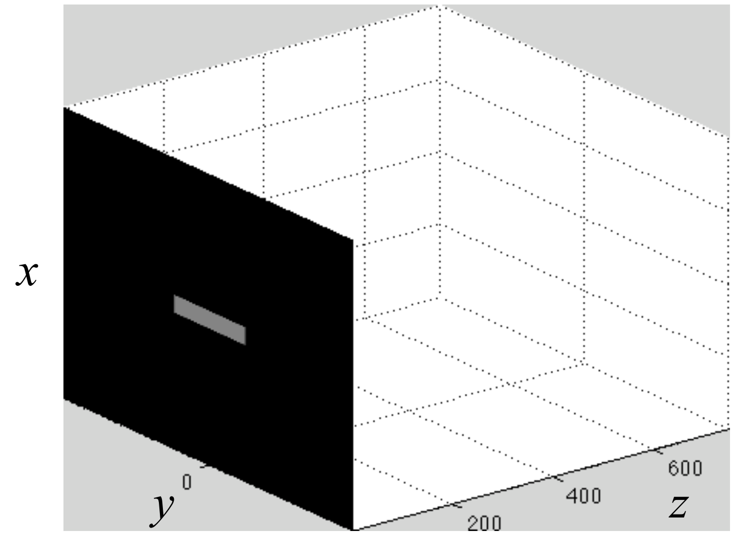
Amplitude grating



$$g_t(x) = \frac{1}{2}\left[1 + m\cos\left(2\pi\frac{x}{\Lambda} + \phi\right)\right]$$

$$\begin{aligned} \Lambda &= 2\lambda \\ m &= 1.0 \\ \phi &= 0. \end{aligned}$$

Phase grating



$$g_t(x) = \exp\left[i\frac{m}{2}\sin\left(2\pi\frac{x}{\Lambda} + \phi\right)\right]$$

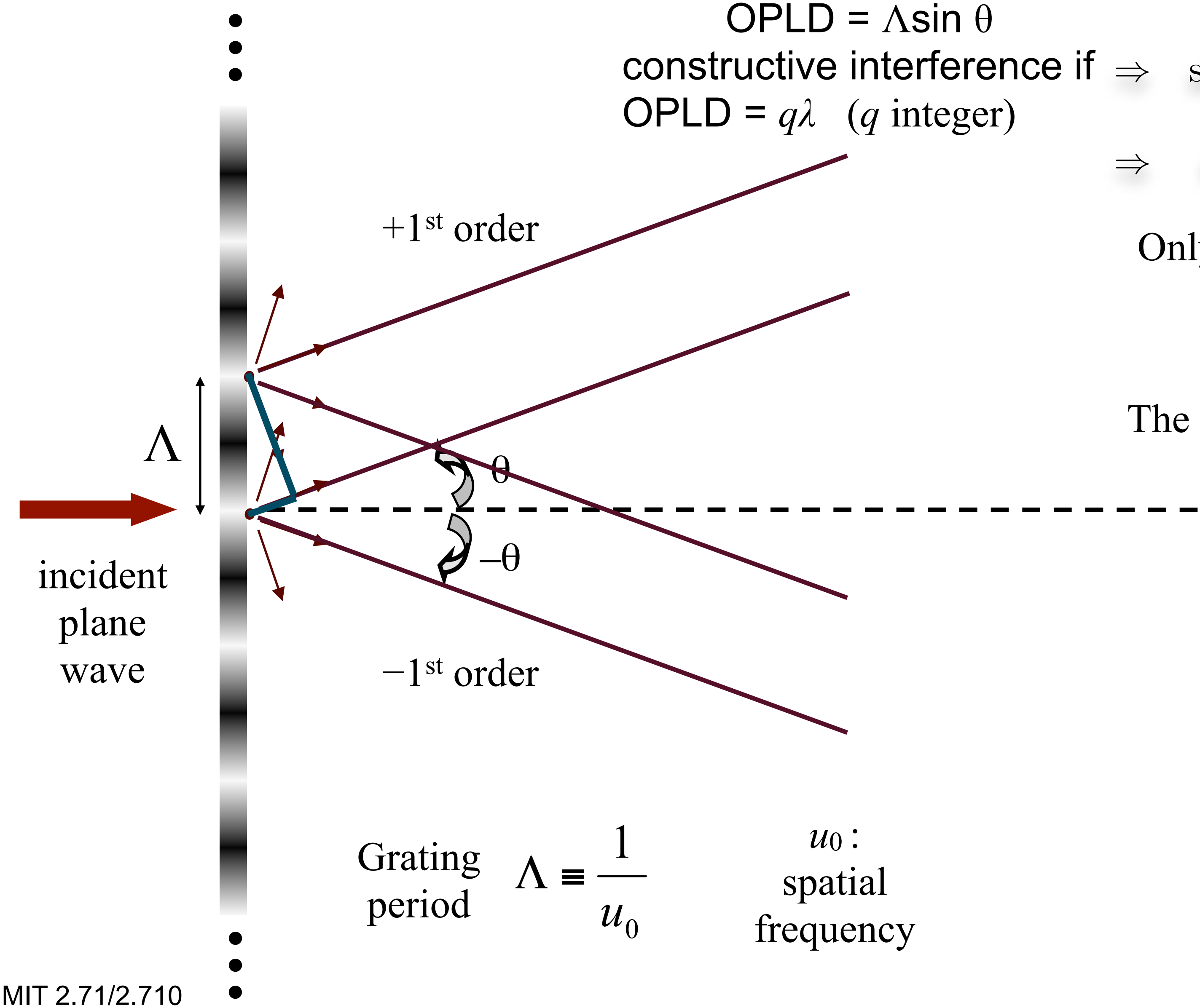
$$\begin{aligned} \Lambda &= 2\lambda \\ m &= 8.44 \text{ rad} \\ \phi &= 0. \end{aligned}$$

Sinusoidal amplitude grating

OPLD = $\Lambda \sin\theta$
constructive interference if
OPLD = $q\lambda$ (q integer)

$$\Rightarrow \quad \sin\theta_q = \frac{\text{OPLD}}{\Lambda} = \frac{q\lambda}{\Lambda}$$

$$\Rightarrow \quad \theta_{\pm1} \approx \pm\frac{\lambda}{\Lambda}.$$



Only the n=0th and n=±1st diffraction orders are generated

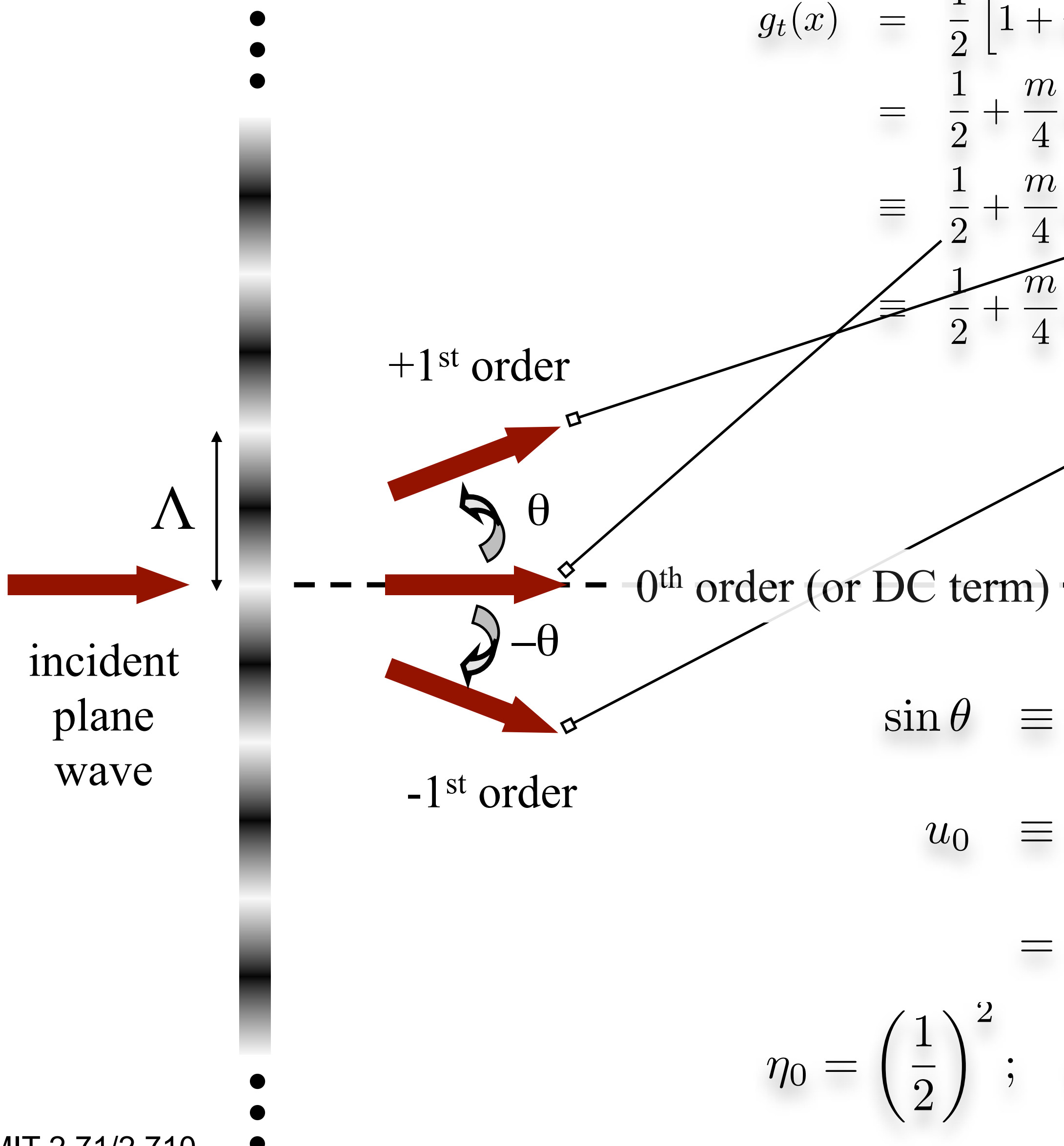
The n=0th diffraction order is also known as the DC term

Grating period $\Lambda \equiv \dfrac{1}{u_0}$

u_0 : spatial frequency

Sinusoidal amplitude grating

$$
\begin{aligned}
g_t(x) &= \frac{1}{2}\left[1 + m\cos\left(2\pi\frac{x}{\Lambda} + \phi\right)\right] \\
&= \frac{1}{2} + \frac{m}{4}\exp\left\{i2\pi\frac{x}{\Lambda}\right\} + \frac{m}{4}\exp\left\{-i2\pi\frac{x}{\Lambda}\right\} \\
&\equiv \frac{1}{2} + \frac{m}{4}\exp\left\{i2\pi\frac{\sin\theta}{\lambda}x\right\} + \frac{m}{4}\exp\left\{-i2\pi\frac{\sin\theta}{\lambda}x\right\} \\
&\equiv \frac{1}{2} + \frac{m}{4}\exp\left\{i2\pi u_0 x\right\} + \frac{m}{4}\exp\left\{-i2\pi u_0 x\right\}.
\end{aligned}
$$

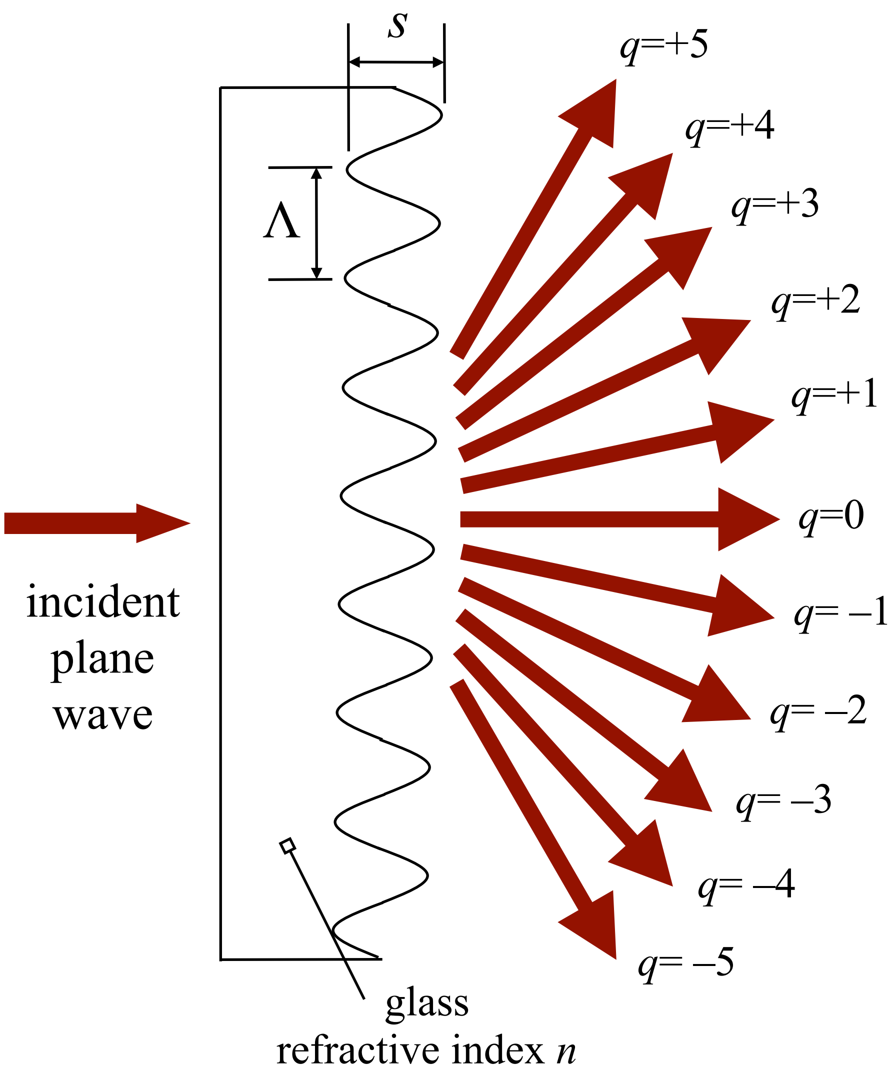


$$\sin\theta \equiv \frac{\lambda}{\Lambda};$$ diffraction angle

$$u_0 \equiv \frac{1}{\Lambda} = \frac{\sin\theta}{\lambda}.$$ spatial frequency

$$\eta_0 = \left(\frac{1}{2}\right)^2; \quad \eta_{\pm 1} = \left(\frac{m}{4}\right)^2$$ diffraction efficiencies

Sinusoidal phase grating



"surface relief" grating

Transmission function

$$g_{\mathrm{t}}(x) = \exp\left\{ i\frac{m}{2}\sin\left(2\pi\frac{x}{\Lambda}\right)\right\}$$

$$m \equiv 2\pi\frac{(n-1)s}{\lambda}$$

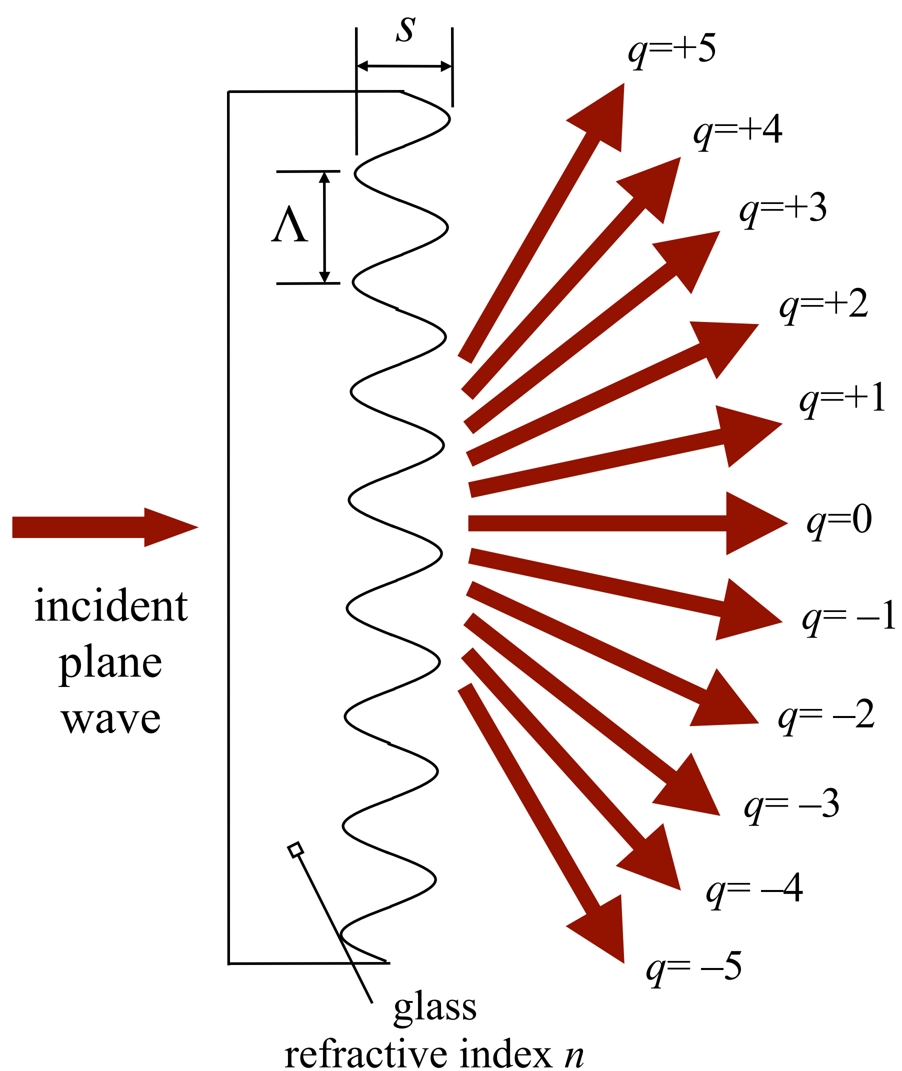
"phase contrast"

Useful math property:

$$\exp\left\{ i\alpha\sin(\theta)\right\} = \sum_{q=-\infty}^{+\infty} \mathrm{J}_q(\alpha)\exp(iq\theta)$$

J_m: Bessel function of 1st kind, *m*-th order

Sinusoidal phase grating



Field after grating is expressed as:

$$g_+(x,z) = \exp\left\{i\,\frac{m}{2}\sin\left(2\pi\frac{x}{\Lambda}\right) + i2\pi\frac{z}{\lambda}\right\}$$

$$= \sum_{q=-\infty}^{+\infty} \mathrm{J}_q\left(\frac{m}{2}\right)\times \quad \text{amplitude}$$

$$\times\exp\left(i2\pi\, q\frac{x}{\Lambda} + i2\pi\frac{z}{\lambda}\right)$$

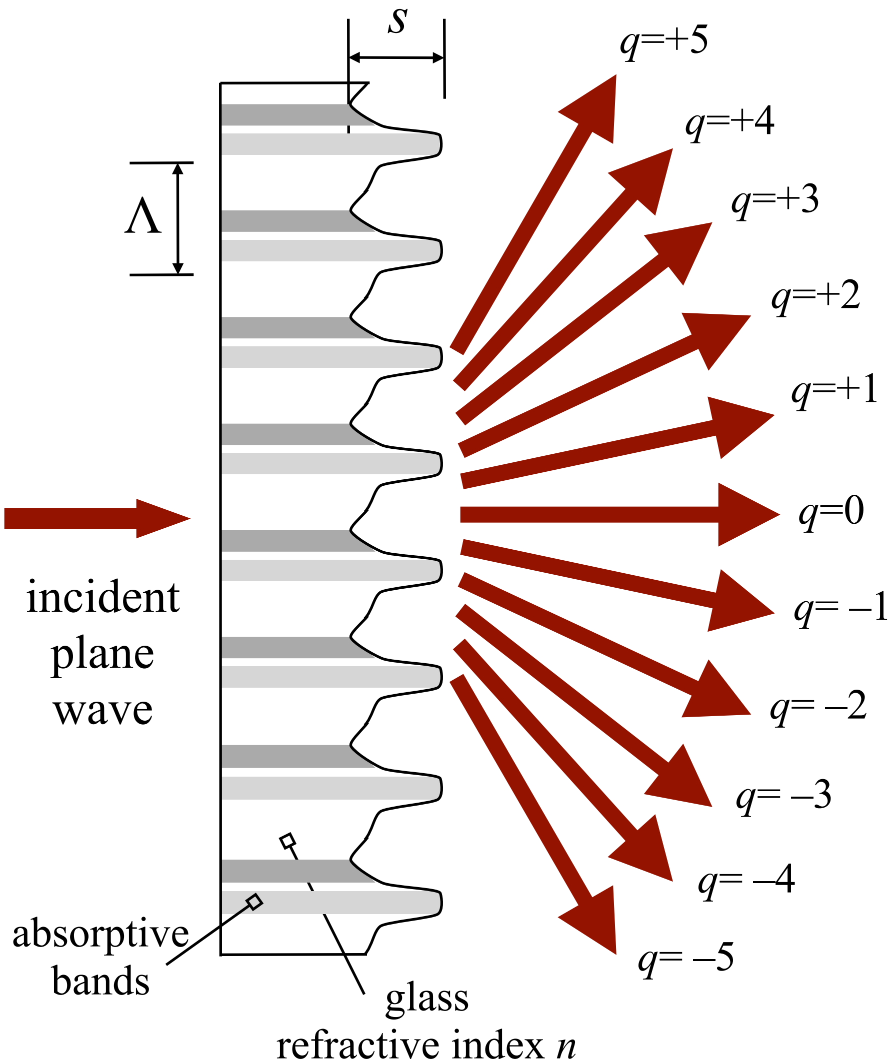
plane waves (diffraction orders),

propagation angle

$$\sin\theta_q \approx \theta_q \equiv q\frac{\lambda}{\Lambda}$$

The *q*-th exponential physically is
a plane wave propagating at angle θ_q
i.e. the *q*-th diffraction order

Fresnel diffraction from a grating as a Fourier series



More generally, a periodic transparency's complex amplitude transmission may be expressed as a Fourier series expansion:

$$g_0(x) = \begin{cases} g_0(x), & 0 \le x < \Lambda \\ 0, & \text{otherwise} \end{cases}$$

$$g_t(x) = g_0(x) \times \sum_{q=-\infty}^{+\infty} \delta(x - n\Lambda)$$

$$= \sum_{q=-\infty}^{+\infty} c_q \times$$

$$\times \exp\left(i2\pi\, q\frac{x}{\Lambda} + i2\pi\frac{z}{\lambda}\right)$$

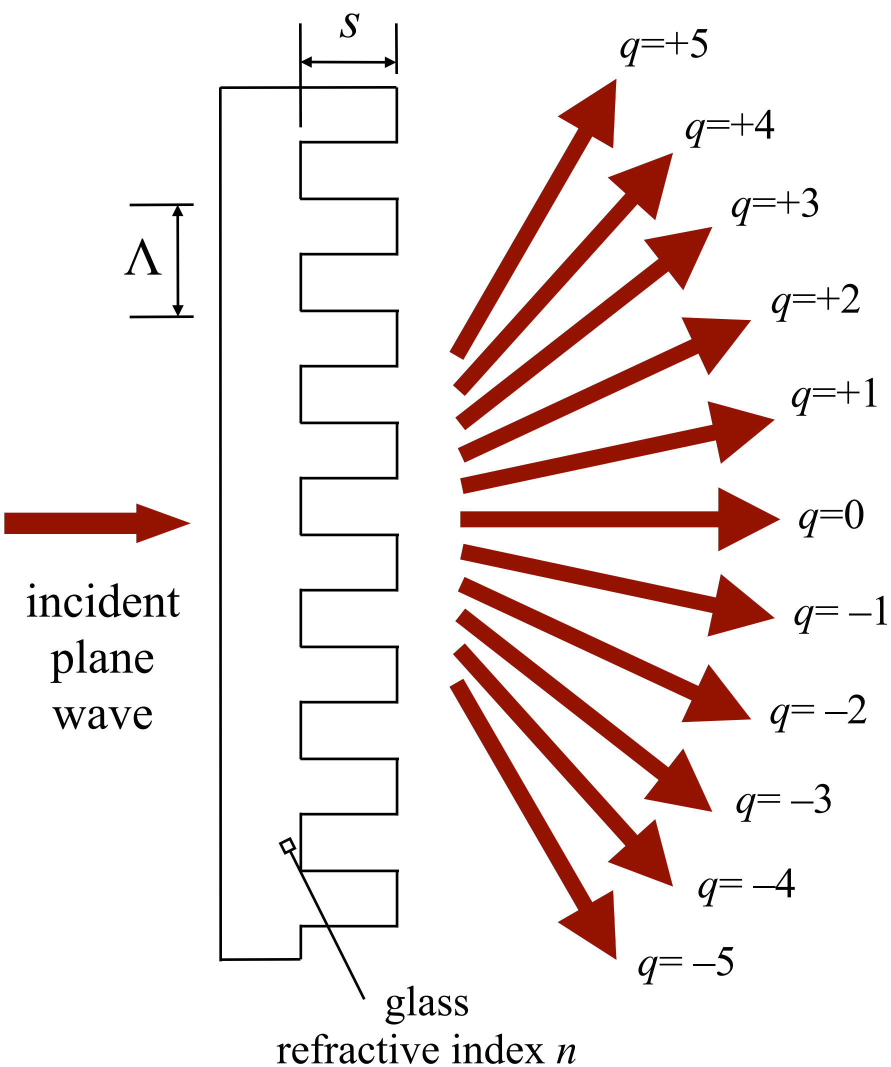
amplitudes

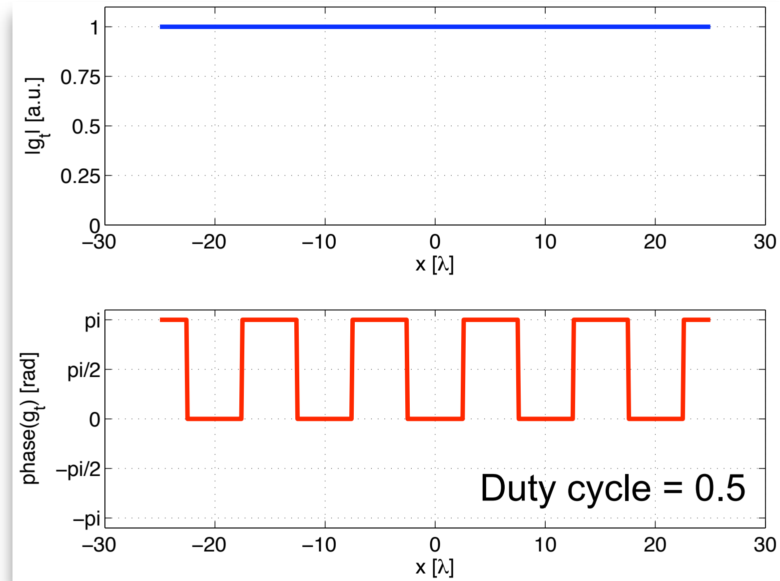
plane waves (diffraction orders), propagation angle

$$\sin\theta_q \approx \theta_q \equiv q\frac{\lambda}{\Lambda}$$

$$\eta_q = \left|c_q\right|^2$$ diffraction efficiencies

Example: binary phase grating





$$g_0(x) = \begin{cases} 1, & 0 \le |x| \le \Lambda/4 \\ -1, & \Lambda/4 < |x| \le \Lambda/2 \end{cases}$$

$$c_q = \frac{1}{\Lambda}\int_{-\Lambda/2}^{\Lambda/2} g_0(x) \exp\left\{i2\pi q \frac{x}{\Lambda}\right\} \mathrm{d}x.$$

$$c_q = \mathrm{sinc}\left(\frac{q}{2}\right) \quad \text{where} \quad \mathrm{sinc}(\xi) \equiv \frac{\sin(\pi\xi)}{(\pi\xi)}.$$

$$\eta_{\pm q} = \left(\frac{2}{\pi q}\right)^2 \quad \text{for} \quad q \text{ odd.}$$

$$\eta_{\pm 1} = \left(\frac{2}{\pi}\right)^2 \approx 40.53\%.$$