Wigner Distribution Function and Integral Imaging

Michael McCanna
Michelle Lydia Kam Ye-Sien
Lei Tian
Shalin Mehta

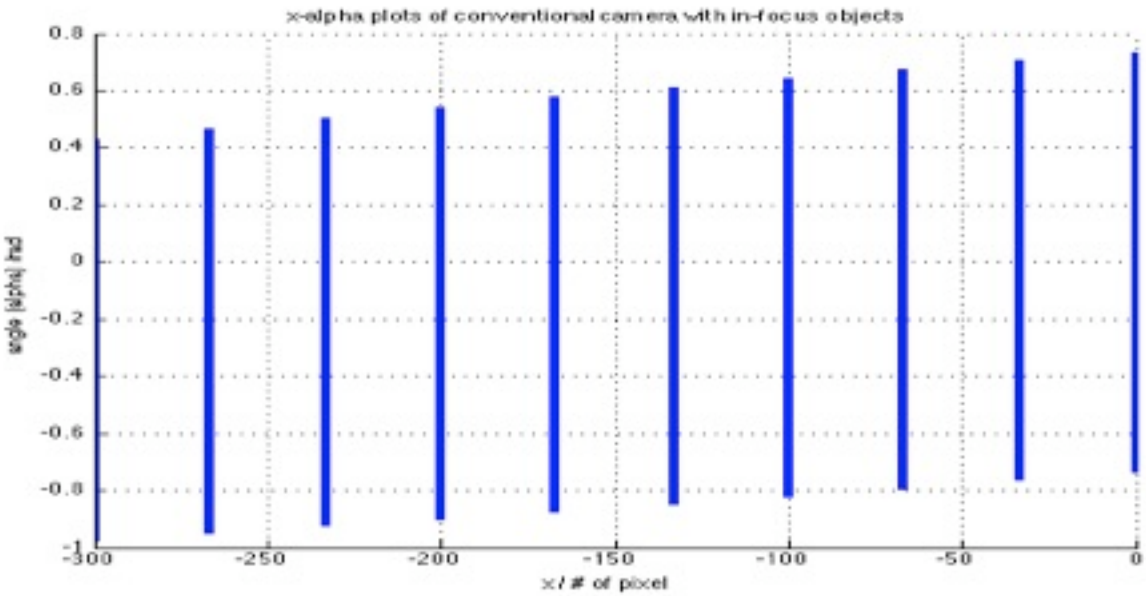
Motivation and Underlying Principles

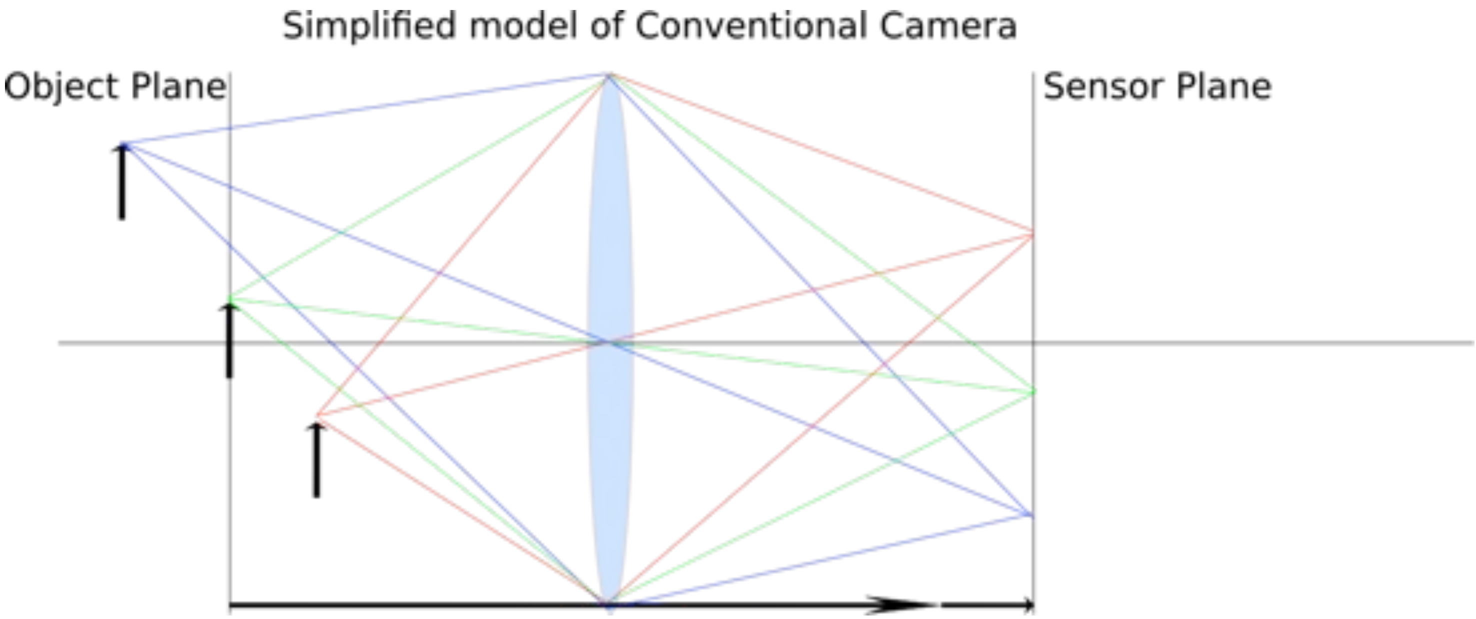
Conventional Cameras produce one view of a scene; they do a good job of imaging objects which fall within a small range of distance from the camera.

We can create optical systems which allow the image sensor to capture multiple views of a scene, thereby giving us additional imaging capabilities.

How we actually produce multiple views of the same scene is with a pinhole or microlens array which produces a grid of tiny images.

Conventional Camera





In geometric terms: Each pixel on the sensor "integrates" the variously angled rays coming from each point in space

We lose all sense of angular resolution, and we have no sense of how far a given part of the scene is from the camera.

Aperature limits angular input for a given distance. Smaller angular range means larger distance.

Possible Use Cases

Imaging:
We have systems which trade some spacial resolution for angular resolution.

The extra dimensions of recorded information allow for reconstruction of the original trajectories of rays.

--digital refocusing
--3-d imaging
--imaging of scenes from behind obstructions, such as foliage or murky water

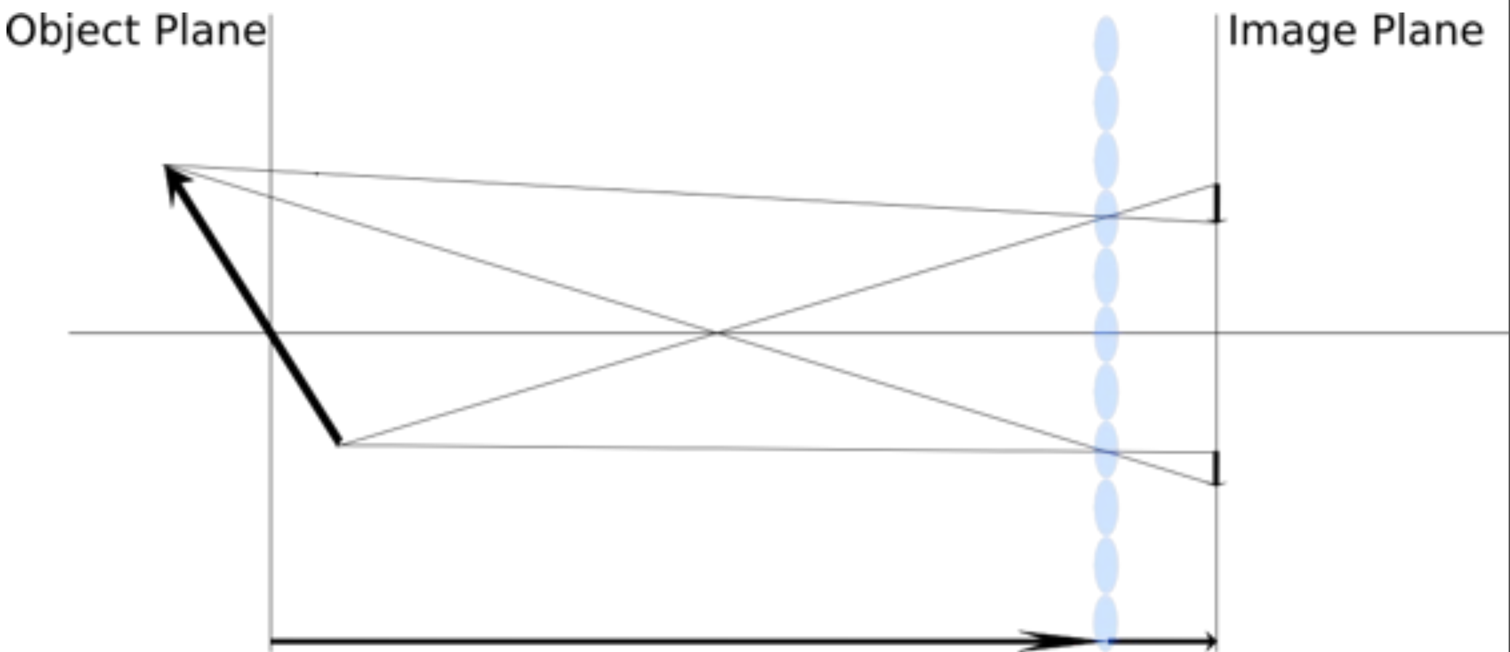
Projection:
A system which allows for selecting various location and angle of rays by selecting just location on the projector could allow for 3-d projection.

Common Setups:
Integral Imaging Camera

Integral Imaging Setup (pinhole/ microlens array)

Each pinhole produces an image of the full scene, but from varying viewpoints.

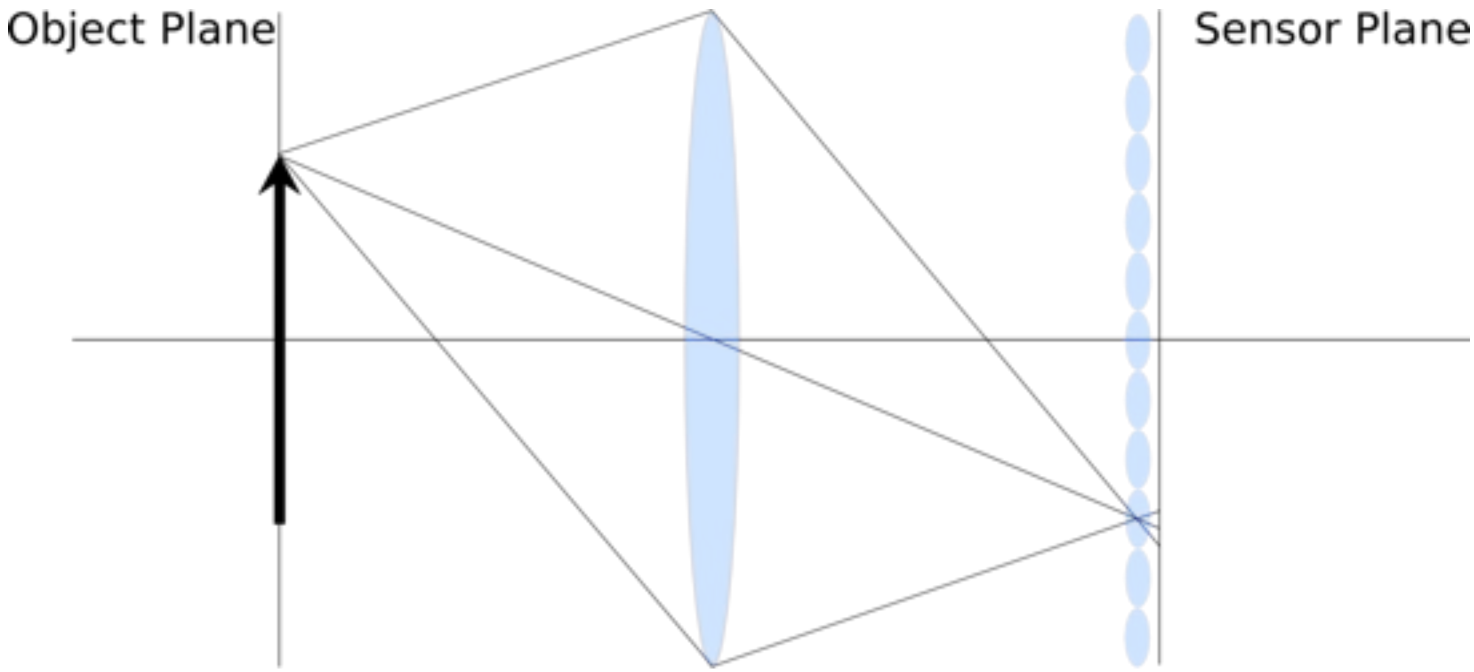
(think of each pinhole as an “eye”)



Common Setups:
Light-Field Camera

Light-Field Setup (large lens + microlens array):

Each microlens provides a picture of a small part of the scene, with many different viewpoints—each pixel in the miniature image is another “view”



Wigner Distribution Function

- Describes a signal in space and spatial frequency simultaneously

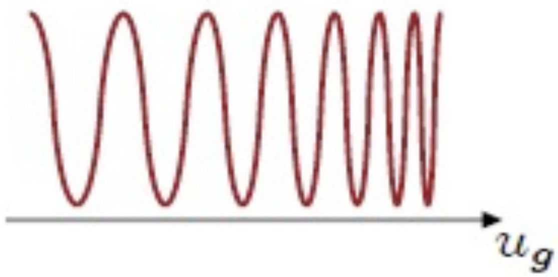
$$W_f(x,\nu) = \int_{-\infty}^{+\infty} f\left(x + \frac{x'}{2}\right) f^*\left(x - \frac{x'}{2}\right) \exp[-i2\pi x'\nu]dx'$$

- Local frequency spectrum of the signal
- Fourier transform gives global frequency spectrum

Wigner Distribution Function

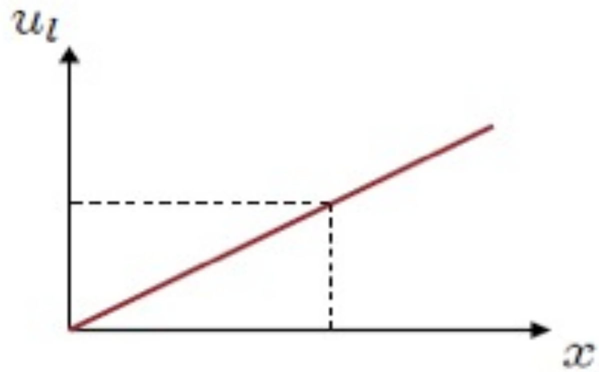
Chirp function: $f(x) = \exp\left\{ i2\pi k x^2 \right\}$

Fourier Transform



$$F(u) = \sqrt{\frac{i}{2}} \exp\left\{ -i\pi \frac{u^2}{2} \right\}$$

Wigner Distribution



$$W_f(x,u) = \delta(u - 2kx)$$

Light Field

- Representation of the light flowing along all rays in free space
- Parameterized by coordinates of two plane

Image removed due to copyright restrictions. Please see Fig. 1 in Levoy, Mark, and Pat Hanrahan."Light Field Rendering." *Proceedings of the 23rd Annual Conference on Computer Graphics and Interactive Techniques* (1996): 31-42.

Plane wave at constant z

$$E(x) = \exp\left\{ i2\pi \frac{\sin\theta}{\lambda} x \right\}$$

Spatial Frequency, u

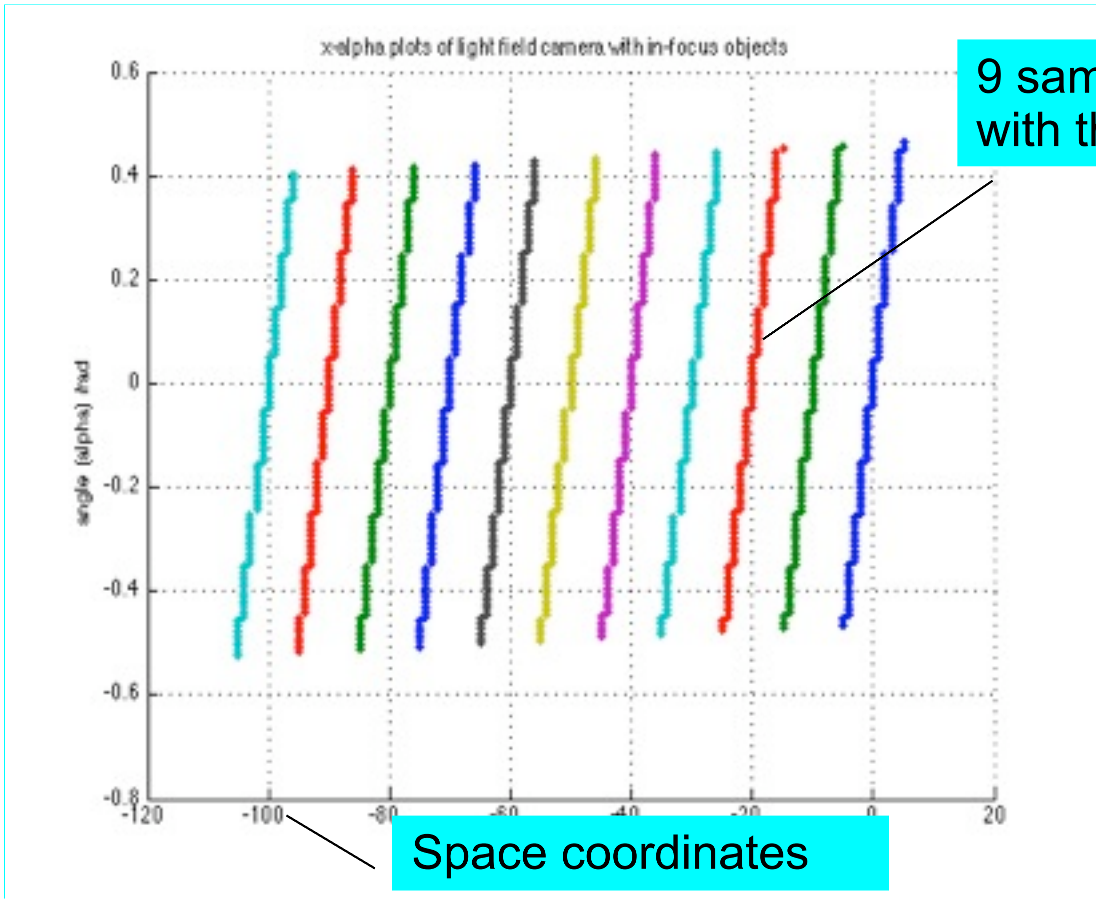
$$u = \frac{\sin\theta}{\lambda} \approx \frac{\theta}{\lambda}$$

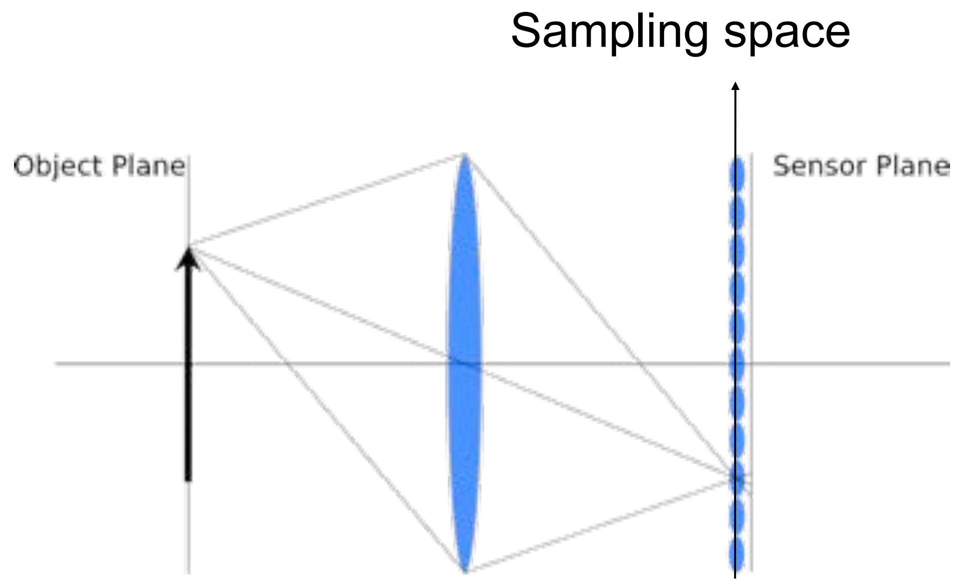
Wigner Distribution Function

- A link between Fourier optics and geometric optics
- Derived in terms of Fourier optics but description of signal closely resembles ray concept in geometrical optics

Sampling in space-frequency domain

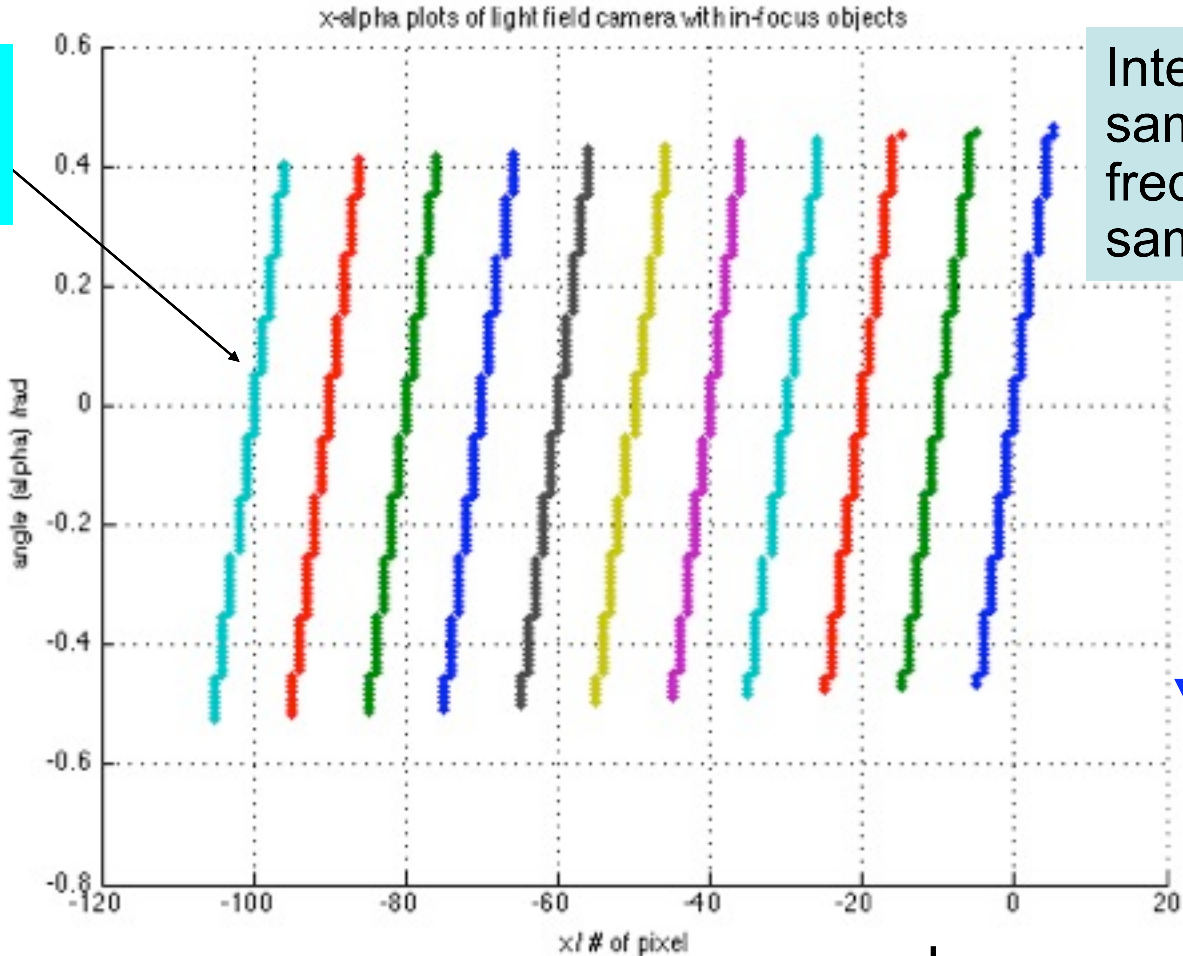
- Integral imaging techniques sample signal in space-frequency domain
- Coordinates of micro-lenses gives samples in spaces
- Sub-images gives samples in frequency domain of corresponding position



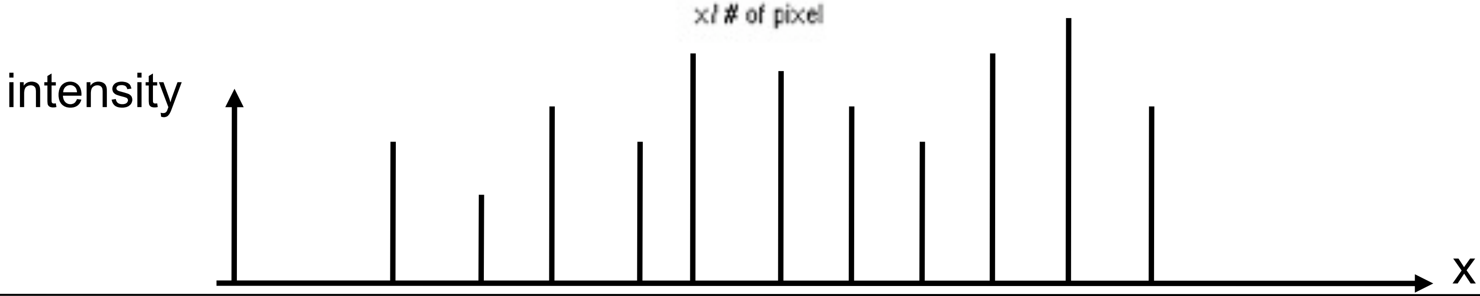


Digital refocusing

Adding all the samples with different angle



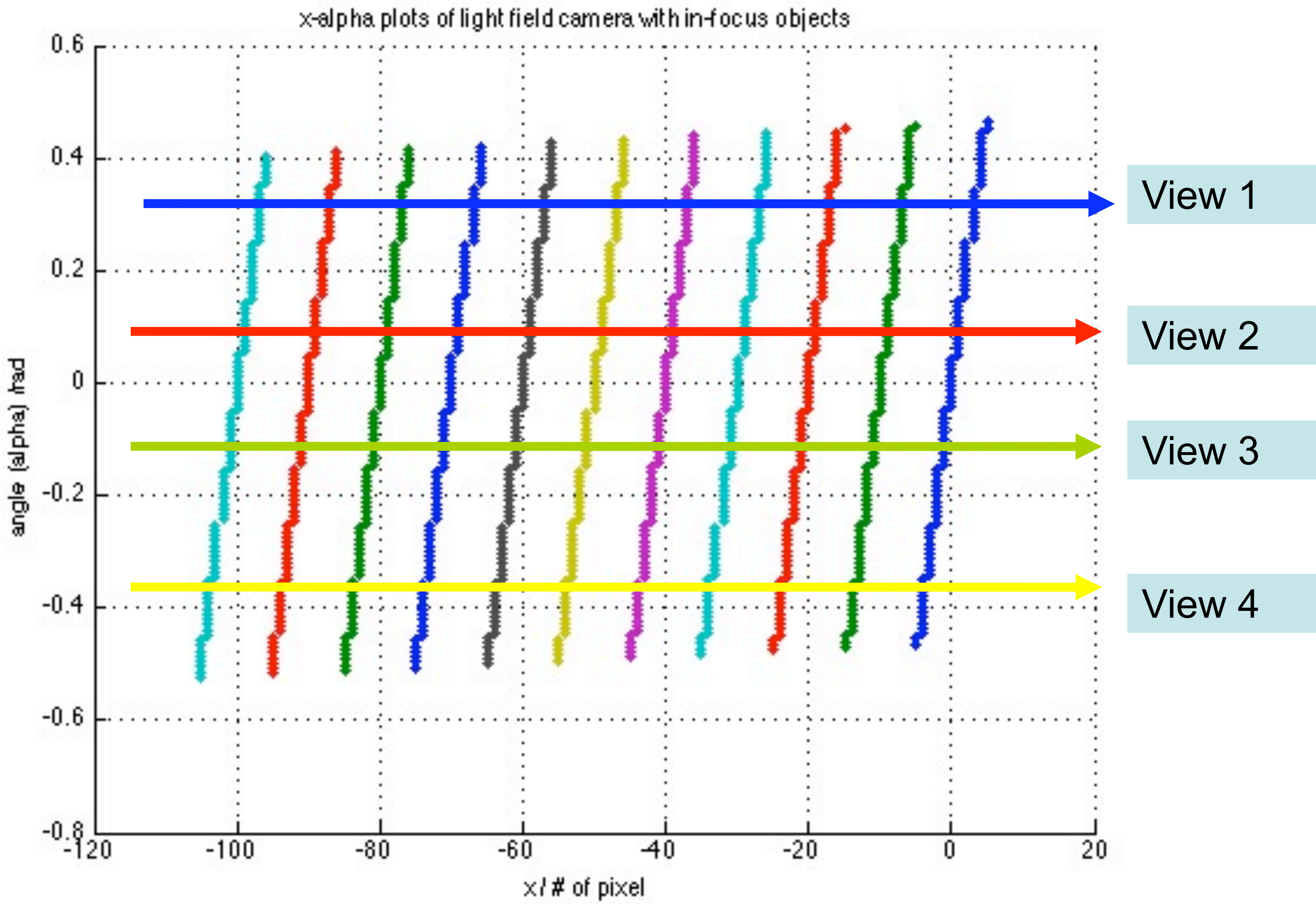
Integrating samples in frequency with the same postion



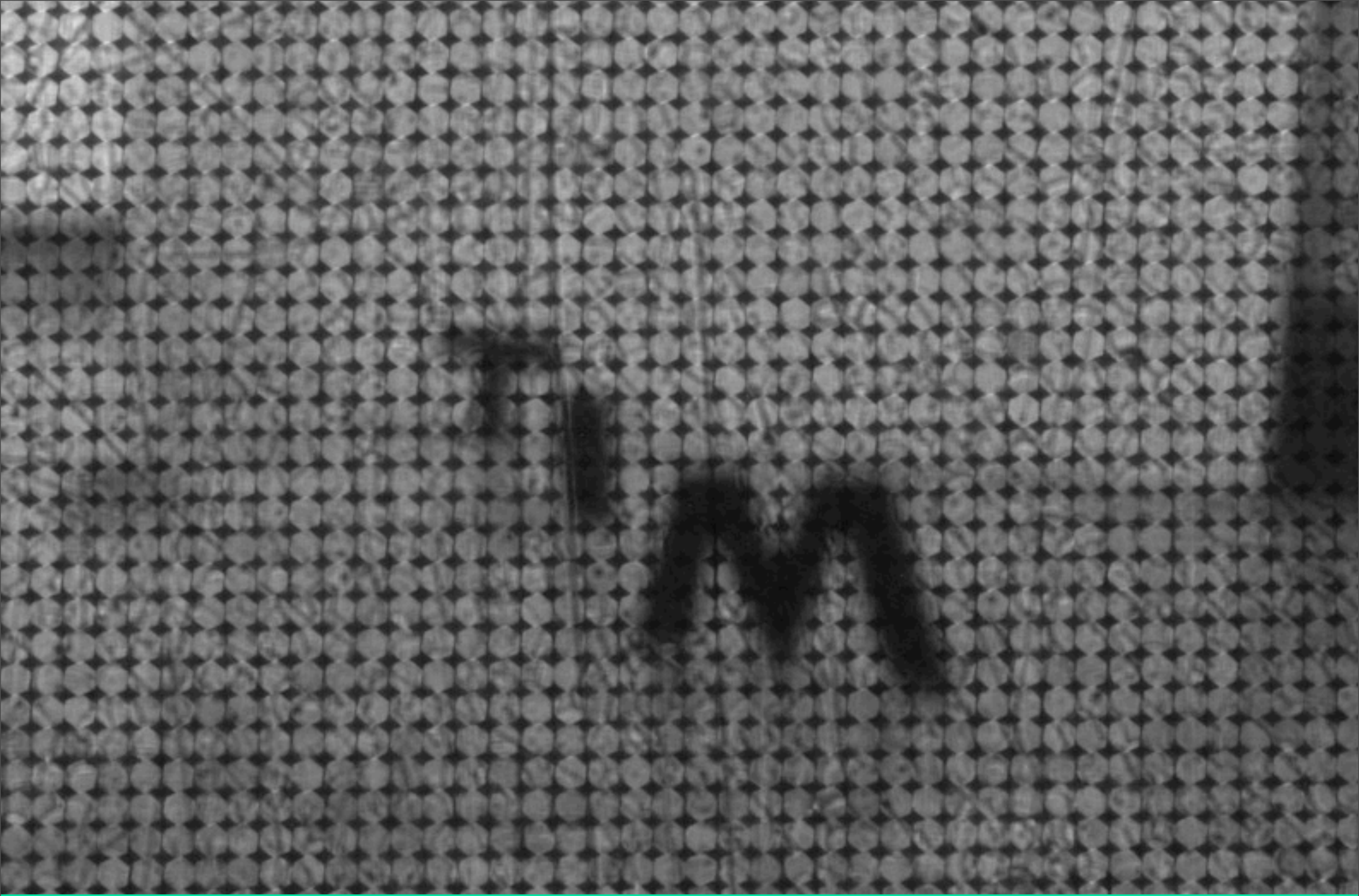
Digital refocusing

- Digitally refocus image from plane 1 to plane 2
  - Shearing in x-direction without changing u-direction
  - Integrating along u-direction

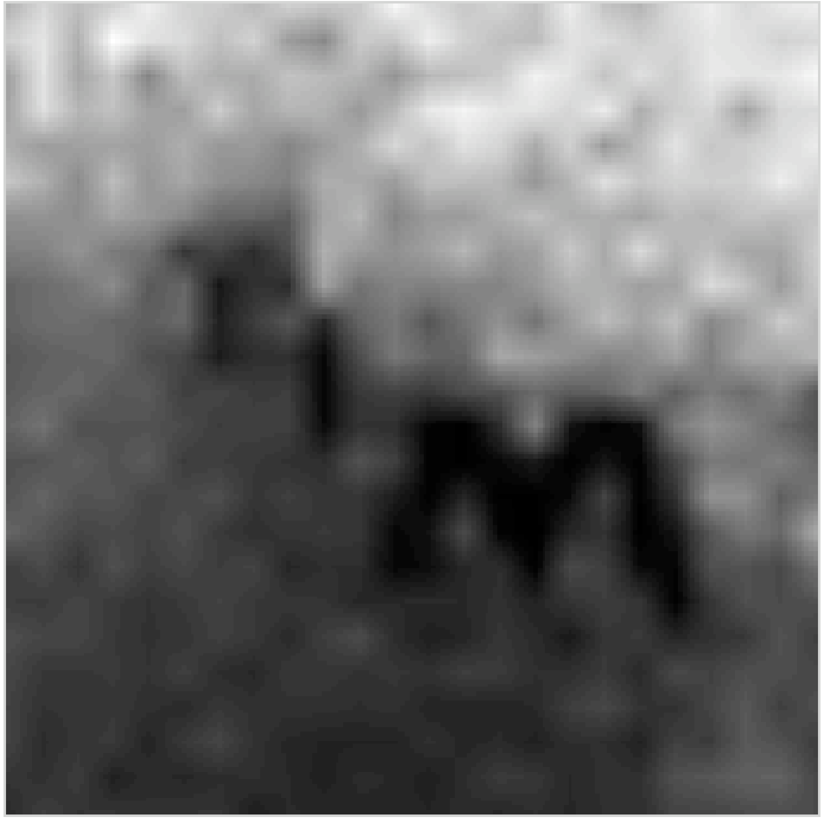
3-D imaging



x-alpha plots of light field camera with in-focus objects
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-120
-100
-80
-60
-40
-20
0
20
angle (alpha) /rad
x / # of pixel
View 1
View 2
View 3
View 4



Diameter of micro-lens: 125um; Pixel size: 2.2 um; spacing between micro-lenses: 11um; Number of pixels: 1488 * 1488 (after cropping)



Reconstruction Movie

- Samples in x directions: 24*24
  - Very blur image
- 9 different views along both x and y.

Acknowledgement

We would like to thank Se Baek for mentoring us, and Pepe for helping us with the experiments.