Homework #4 solution

1) WOT fuel required per cycle $m_f = \eta_{v,i} V_D \left(\frac{p_i}{RT_i}\right)\left(\frac{F}{A}\right)$

where $\eta_{v,i}$ is the volumetric efficiency based on the manifold condition.
At WOT, $\eta_{v,i} \simeq 0.8$ (because of the valve timing etc.)

$$m_f = 0.8 \times (500 \times 10^{-6}) \left(\frac{10^5}{287 \times 300}\right) \frac{1}{14.6} \text{kg} = \underline{\underline{31 \text{ mg}}}$$

The max. time available is to have the injector on continuously.
Thus τ = cycle time = $\frac{60}{6500 \times 1/2}$ = 18.5 ms
for 4-stroke engine

The injector flow rate is then $\dot{m}_f = \frac{31 \text{ mg}}{18.5 \text{ ms}} = \underline{\underline{1.68 \text{ g/s}}}$

At idle, intake pressure is about 0.3 bar, fuel required = 0.3 × 31 mg = 9.3 mg
Injection duration = $\left(\frac{9.3 \times 10^{-3}}{1.68}\right) = \underline{\underline{5.54 \text{ ms}}}$

Note that actual injectors are not sized for continuous injection at max rpm and full load. It is sized for delivery of enough fuel at cold start cranking. So the maximum delivery rate is much higher. The pulse width at idle is then much lower, at about 1 to 2 ms or so.

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2) x - τ model : discrete case

$$M_i - M_{i-1} = x f_i - k M_{i-1}$$
$$m_i = (1-x) f_i + k M_{i-1}$$

(a) At equilibrium with fuel per cycle set at f_0, puddle mass

$$0 = x f_0 - k M_0 \Rightarrow M_0 = \underline{\underline{\frac{x f_0}{k}}}$$

fuel delivered
$$m_0 = (1-x) f_0 + k M_0 = \underline{\underline{f_0}}$$

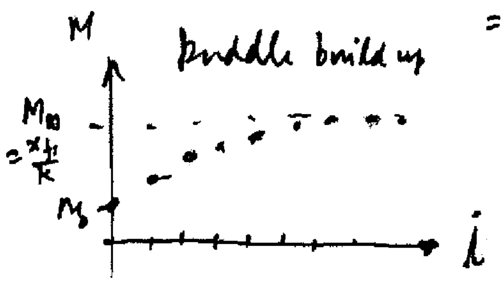
(b) The sequence of port puddle accumulation is

$$M_1 = x f_1 + (1-k) M_0$$
$$M_2 = x f_1 + (1-k)[x f_1 + (1-k) M_0] = x f_1 + (1-k) x f_1 + (1-k)^2 M_0$$
$$\vdots$$
$$M_i = \underbrace{x f_1 + (1-k) x f_1 + (1-k)^2 x f_1 + \cdots}_{i \text{ terms: geometric series}} + (1-k)^i M_0$$

$$= \underline{\underline{x f_1 \left[\frac{1-(1-k)^i}{k}\right] + (1-k)^i M_0}}$$

for $i \to \infty$
$M_\infty = \frac{x f_1}{k}$

puddle build up



Fuel delivered
$$m_i = (1-x) f_1 + k M_{i-1} = f_1 + (k M_{i-1} - x f_1)$$
putting in above expression for M_{i-1}
$$m_i = f_1 + k \left\{ x f_1 \left[\frac{1-(1-k)^{i-1}}{k}\right] + (1-k)^{i-1} M_0 \right\} - x f_1$$
$$= \underline{\underline{f_1 + (1-k)^{i-1} (k M_0 - x f_1)}} \rightarrow \text{for } i \to \infty \quad m_\infty = f_1$$

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