

13.024 Problem Set 6:

Due: April 15, 2003

Panel Methods

1. For this problem you will develop a 2-Dimensional panel method from scratch.

You are provided with a MATLAB m-function that evaluates the velocity potential at any point (x,y) due to a distribution of sources on a straight segment with vertices at $(-dl/2, 0)$ and $(dl/2, 0)$. The same function also evaluates the potential at (x,y) due to a distribution of normal dipoles on the same segment. The direction of the dipoles is taken parallel to the positive y-axis, and the source strength and dipole moment are taken as 2π . Use Green's Theorem with $G = -\ln r$.

You may also use any method for the solution of a system of linear equations. For example, even though it will be somewhat slow, you can use the Gaussian elimination subroutine, or you can use the LU decomposition method, or you can use the Matrix backwards division function built into MATLAB. For simplicity, the latter is, if you have a matrix equation (set of linear equations $\mathbf{Ax}=\mathbf{b}$, where \mathbf{A} is a well conditioned known matrix and \mathbf{b} is a known column vector, and \mathbf{x} is an unknown column vector, MATLAB will give the solution as $\mathbf{x} = \mathbf{b}\backslash\mathbf{A}$. You may find it helpful to take an approach along the following lines:

1. Write an m-function that reads and processes the input geometry. This might include reading the body geometry (vertices as a series of points in the x-y plane) and gives as output the number of vertices, the (x,y) locations for the vertices and control points, the lengths of the panels and the x and y components of the normal vector to each panel. Another m-function could accept as input the number of panels, the panel vertex and control point locations and the panel lengths; and gives as output values of G and dG/dn integrated over panels. It will use the m.function you will be given, called rank2d.m which calculates the integrals over the panels for you. However, you need to do some coordinate transformations because rank2d.m uses a coordinate system with the panel along the x axis, y perpendicular to x going positive to the right when you go from right to left along the panel and with the origin at the midpoint of the panel length. A third m-function can provide $d\phi/dn$. The next m-function could give the matrices on the left hand and right hand side of the matrices that represent the set of linear equations to be solved. Finally you need to write an m-script to put everything together. Write another module for post-processing the data once the solution has been found. This might include calculating the tangential velocity and the pressure on each panel, as well as a subroutine for pressure integration to get the hydrodynamic force acting on the body. Calculate the pressure coefficient distribution on a cylinder and compare it to the analytical solution you did earlier in the term.
2. Also, calculate the pressure distribution on an ellipse given by

$$\frac{x^2}{2} + \frac{y^2}{1} = 1$$

and plot P vs x on surface.