MIT Department of Mechanical Engineering

2.25 Advanced Fluid Mechanics

Vector Calculus Review Problems

- (1) Show that the condition for the vectors $\underline{a}$, $\underline{b}$, and $\underline{c}$ to be coplanar is:
  $\varepsilon_{ijk} a_i b_j c_k = 0$
- (2) Prove the following relationships:
  $\delta_{ij}\delta_{ij} = 3 \;\; \varepsilon_{pqi}\varepsilon_{pqj} = 2\delta_{ij}$
- (3) Use Stokes theorem to prove that $\nabla \times (\nabla\phi) = 0$ for any single-valued twice-differentiable scalar (ϕ) regardless of the coordinate system.
- (4) Problem 3.12 from Panton's Fourth Edition: Write the following formulas in Gibbs's notation using the symbol ∇. Convert the expressions to Cartesian notation and prove that the equations are correct.

$$div(\phi\underline{v}) = \phi\, div\, \underline{v} + \underline{v}\,.\, grad\,\phi$$

$$div(\underline{u} \times \underline{v}) = \underline{v}\,.\, curl\,\underline{u} - \underline{u}\,.\, curl\,\underline{v}$$

$$curl(\underline{u} \times \underline{v}) = \underline{v}\,.\, grad\,\underline{u} - \underline{u}\,.\, grad\,\underline{v} + \underline{u}\, div\,\underline{v} - \underline{v}\, div\,\underline{u}$$

