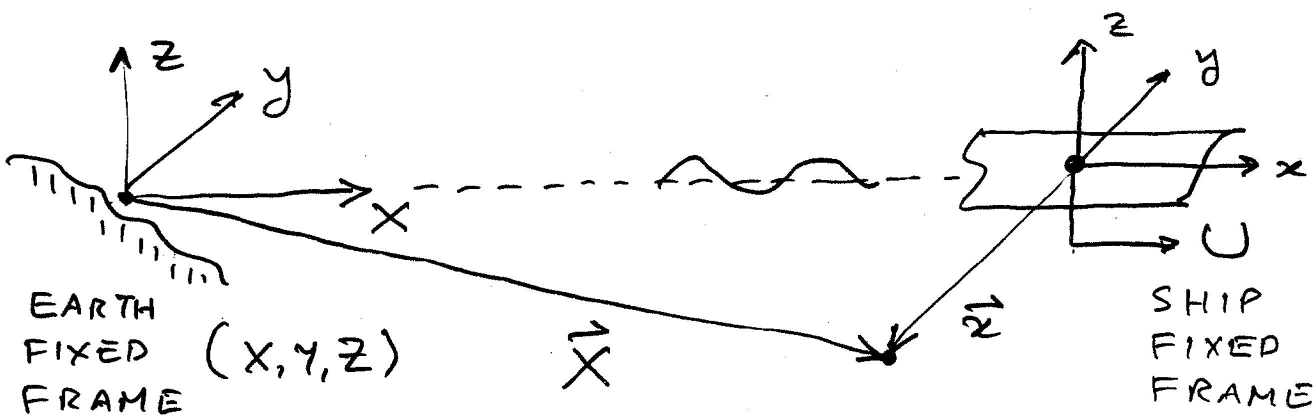
TRANSLATING COORDINATE SYSTEMS



$\vec{X}$ **:** EULERIAN COORDINATE FIXED AND TIME INDEPENDENT RELATIVE TO EARTH FRAME

$\vec{x}$: CARTESIAN COORDINATE OF EULERIAN POINT RELATIVE TO SHIP FRAME. MUST BE TIME-DEPENDENT IF $\vec{X}$ IS FIXED. ITS TIME DEPENDENCE MUST BE ACCOUNTED FOR IN TAKING TIME DERIVATIVES IN FORMULATION OF GOVERNING LAWS AND FREE SURFACE CONDITIONS

$$\begin{cases} x = X - Ut \\ y = Y \\ z = Z \end{cases}$$

- LET $\Phi(\vec{X},t)$ BE THE VELOCITY POTENTIAL DESCRIBING THE POTENTIAL FLOW GENERATED BY THE SHIP RELATIVE TO THE EARTH FRAME

- THE SAME POTENTIAL EXPRESSED RELATIVE TO THE SHIP FRAME IS $\phi(\vec{x},t)$.

THE RELATION BETWEEN THE TWO POTENTIALS IS GIVEN BY THE IDENTITY

$$\Phi(X,Y,Z,t) \equiv \phi(x,y,z,t)$$
$$= \phi(X-Ut, Y, Z, t)$$

WHERE THE RELATION BETWEEN THE COORDINATES OF THE TWO COORDINATE SYSTEMS HAS BEEN INTRODUCED.

NOTE THE TIME DEPENDENCE IN THE ARGUMENTS OF THE TWO POTENTIALS. IT OCCURS IN TWO PLACES IN ϕ AND IN ONE PLACE IN Φ. THE GOVERNING EQUATIONS ARE ALWAYS DERIVED RELATIVE TO THE EARTH COORDINATE SYSTEM AND TIME DERIVATIVES ARE INITIALLY TAKEN ON Φ:

$$\frac{d\Phi}{dt} = \frac{d}{dt}\phi(\underbrace{X-Ut}_{x}, y, z, t)$$

$$= \frac{\partial \phi}{\partial x}\underbrace{\frac{dx}{dt}}_{=-U} + \frac{\partial \phi}{\partial t} = \frac{\partial \phi}{\partial t} - U\frac{\partial \phi}{\partial x}$$

THE MAIN RESULT TO FOLLOW IS:

$$\frac{d\Phi}{dt} = \frac{\partial \phi}{\partial t} - U\frac{\partial \phi}{\partial x} \cdot -$$

ALL TIME DERIVATIVES OF THE EARTH FIXED VELOCITY POTENTIAL Φ WHICH APPEAR IN THE FREE SURFACE CONDITION AND THE BERNOULLI EQUATION CAN BE EXPRESSED IN TERMS OF DERIVATIVES OF ϕ USING THE GALILEAN TRANSFORMATION DERIVED ABOVE.

IF THE FLOW IS STEADY RELATIVE TO THE SHIP FIXED COORDINATE SYSTEM:

$$\frac{\partial \phi}{\partial t} = 0$$

BUT:
$$\frac{d\Phi}{dt} = -U\frac{\partial \phi}{\partial x}$$

OR, THE SHIP WAKE IS STATIONARY RELATIVE TO THE SHIP BUT NOT RELATIVE TO AN OBSERVER ON THE BEACH.

FREE SURFACE CONDITION – LINEAR

$$\frac{d^2\Phi}{dt^2} + g\frac{d\Phi}{dz} = 0, \quad z=0$$

- $$\frac{d\Phi}{dt} = \frac{\partial\phi}{\partial t} - U\frac{\partial\phi}{\partial x}$$
- $$\frac{d^2\Phi}{dt^2} = \left(\frac{\partial}{\partial t} - U\frac{\partial\phi}{\partial x}\right)^2 = \frac{\partial^2\phi}{\partial t^2} - 2U\frac{\partial^2\phi}{\partial x\,\partial t} + U^2\frac{\partial^2\phi}{\partial x^2}$$

THIS IS THE FREE SURFACE CONDITION GOVERNING THE FORWARD-SPEED LINEAR SEAKEEPING PROBLEM.

WHEN NO AMBIENT WAVES ARE PRESENT $\partial\phi/\partial t=0$ AND WE OBTAIN THE FREE SURFACE CONDITION FOR THE STEADY KELVIN SHIP WAVE PROBLEM

$$U^2\frac{\partial^2\phi}{\partial x^2} + g\frac{\partial\phi}{\partial z} = 0, \quad z=0$$

THIS IS THE FAMOUS NEUMANN-KELVIN FREE-SURFACE CONDITION GOVERNING THE LINEAR STEADY WAVE PATTERN GENERATED BY A TRANSLATING SHIP.—

BERNOULLI EQUATION

$$p = -\rho\left(\frac{d\Phi}{dt} + \frac{1}{2}\nabla\Phi\cdot\nabla\Phi + gz\right)$$

$$= -\rho\left[\left(\frac{\partial\phi}{\partial t} - U\frac{\partial\phi}{\partial x}\right) + \frac{1}{2}\nabla\phi\cdot\nabla\phi + gz\right]$$

FREE SURFACE ELEVATION

$$\zeta = -\frac{1}{g}\frac{d\Phi}{dt}, \qquad z=0$$

$$= -\frac{1}{g}\left(\frac{\partial\phi}{\partial t} - U\frac{\partial\phi}{\partial x}\right)_{z=0}$$

IN THE KELVIN SHIP WAVE PROBLEM

$$\frac{\partial\phi}{\partial t} = 0$$

HENCE:

$$\zeta = \frac{U}{g}\frac{\partial\phi}{\partial x}, \qquad z=0$$

SO IF THE VELOCITY POTENTIAL $\phi(\vec{x})$ IS AVAILABLE IN SOME FORM RELATIVE TO THE TRANSLATING FRAME, THE WAVE PATTERN FOLLOWS FROM THE ABOVE EXPRESSION.—