ANALYTICAL SOLUTIONS OF THE WAVE–BODY PROBLEM FORMULATED ABOVE ARE RARE. THE FEW EXCEPTIONS WHICH FIND FREQUENT USE IN PRACTICE ARE :

- WAVEMAKER THEORY (STUDIED)
- DIFFRACTION BY A VERTICAL CIRCULAR CYLINDER (STUDIED BELOW)
- LONG-WAVELENGTH APPROXIMATIONS (STUDIED NEXT).

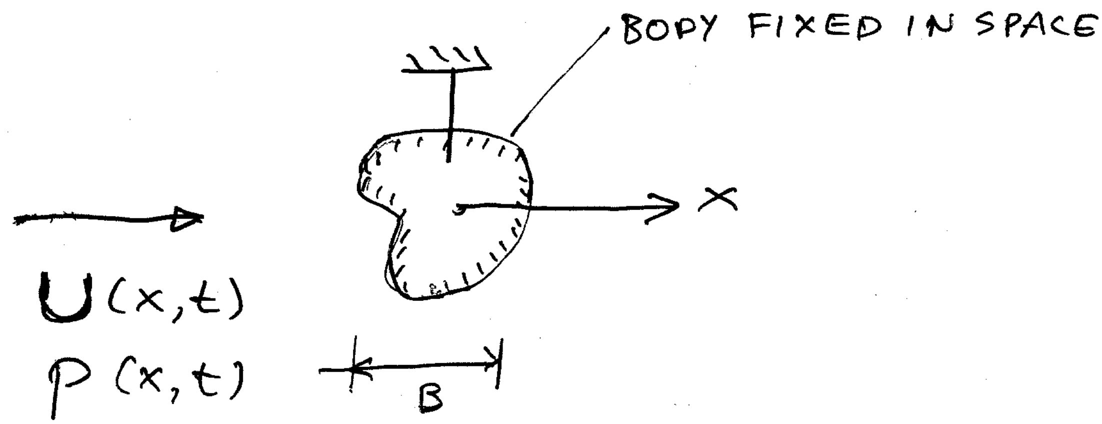
LONG-WAVELENGTH APPROXIMATIONS

VERY FREQUENTLY THE LENGTH OF AMBIENT WAVES λ IS LARGE COMPARED TO THE DIMENSION OF FLOATING BODIES.

FOR EXAMPLE THE LENGTH OF A WAVE WITH PERIOD $T = 10$ sec IS $\lambda \simeq T^2 + \frac{T^2}{2} \simeq 150\,m$

THE BEAM OF A SHIP WITH LENGTH $L = 100\,m$ CAN BE $20\,m$ AS IS THE CASE FOR THE DIAMETER OF THE LEG OF AN OFFSHORE PLATFORM.

GI TAYLOR'S FORMULA



$U(x,t)$: VELOCITY OF AMBIENT UNIDIRECTIONAL FLOW

$P(x,t)$: PRESSURE CORRESPONDING TO $U(x,t)$

- $$\lambda \sim \frac{|U|}{|\nabla U|} \gg B$$ = BODY CHARACTERISTIC DIMENSION

- IN THE ABSENCE OF VISCOUS EFFECTS AND TO LEADING ORDER FOR $\lambda \gg B$:

$$F_x = -\left(\forall + \frac{A_{11}}{\rho} \right) \frac{\partial P}{\partial x} \bigg|_{x=0}$$

$$\left.\begin{array}{l} \bullet\ F_x : \text{FORCE IN X-DIRECTION} \\ \bullet\ \forall : \text{BODY DISPLACEMENT} \\ \bullet\ A_{11} : \text{SURGE ADDED MASS} \end{array}\right\} \text{PROOF IN MH.}$$

AN ALTERNATIVE FORM OF GI TAYLOR'S FORMULA FOR A FIXED BODY FOLLOWS FROM EULER'S EQUATIONS:

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} \simeq -\frac{1}{\rho} \frac{\partial P}{\partial x}$$

THUS:

$$F_x = (\rho \forall + A_{11}) \left(\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} \right)_{x=0}$$

IF THE BODY IS ALSO TRANSLATING IN THE X-DIRECTION WITH DISPLACEMENT $X_1(t)$ THEN THE TOTAL FORCE BECOMES

$$F_x = (\rho \forall + A_{11}) \left(\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} \right) - A_{11} \frac{d^2 X_1(t)}{dt^2}$$

OFTEN, WHEN THE AMBIENT VELOCITY U IS ARISING FROM PLANE PROGRESSIVE WAVES, $\left| U \frac{\partial U}{\partial x} \right| = O(A^2)$ AND IS OMITTED. NOTE THAT U DOES NOT INCLUDE DISTURBANCE EFFECTS DUE TO THE BODY. —

- APPLICATIONS OF G I TAYLOR'S FORMULA IN WAVE-BODY INTERACTIONS

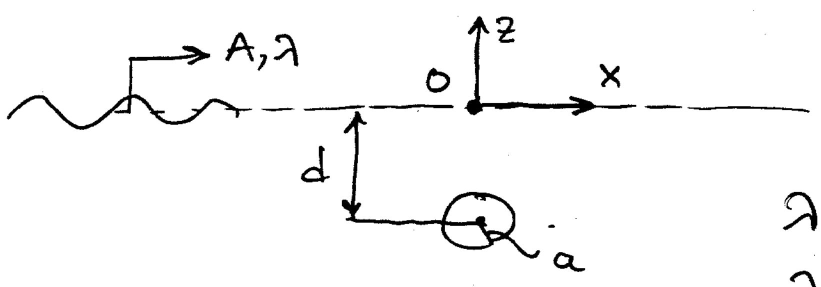
A) <u>ARCHIMEDEAN HYDROSTATICS</u>

$$p = -\rho g z \quad , \quad \frac{\partial p}{\partial z} = -\rho g$$

$$F_z = -\left(\forall + \underset{\substack{\uparrow \\ \text{NO ADDED MASS} \\ \text{SINCE THERE IS} \\ \text{NO FLOW}}}{\phi} \right) \frac{\partial p}{\partial z} = \rho g \forall$$

- SO ARCHIMEDES' FORMULA IS A SPECIAL CASE OF GI TAYLOR WHEN THERE IS NO FLOW. THIS OFFERS AN INTUITIVE MEANING TO THE TERM THAT INCLUDES THE BODY DISPLACEMENT.

B) REGULAR WAVES OVER A CIRCLE FIXED UNDER THE FREE SURFACE



$\lambda >> a$

$\lambda \sim d$

$$\phi_I = \mathbb{R}e\left\{ \frac{igA}{\omega} e^{kz-ikx+i\omega t} \right\} \quad , k = \omega^2/g$$

$$u = \frac{\partial \phi_I}{\partial x} = \mathbb{R}e\left\{ \frac{igA}{\omega}(-ik) e^{kz-ikx+i\omega t} \right\}$$

$$= \mathbb{R}e\left\{ \omega A\, e^{-kd+i\omega t} \right\}_{\substack{x=0 \\ z=-d}}$$

$$V = \frac{\partial \phi_I}{\partial z} = \mathbb{R}e\left\{ \frac{igA}{\omega} k\, e^{kz-ikx+i\omega t} \right\}$$

$$= \mathbb{R}e\left\{ i\omega A\, e^{-kd+i\omega t} \right\}_{\substack{x=0 \\ z=-d}}$$

SO THE HORIZONTAL FORCE ON THE CIRCLE IS:

$$F_x = \left(\forall + \frac{a_{11}}{\rho} \right) \frac{\partial u}{\partial t} + O(A^2)$$

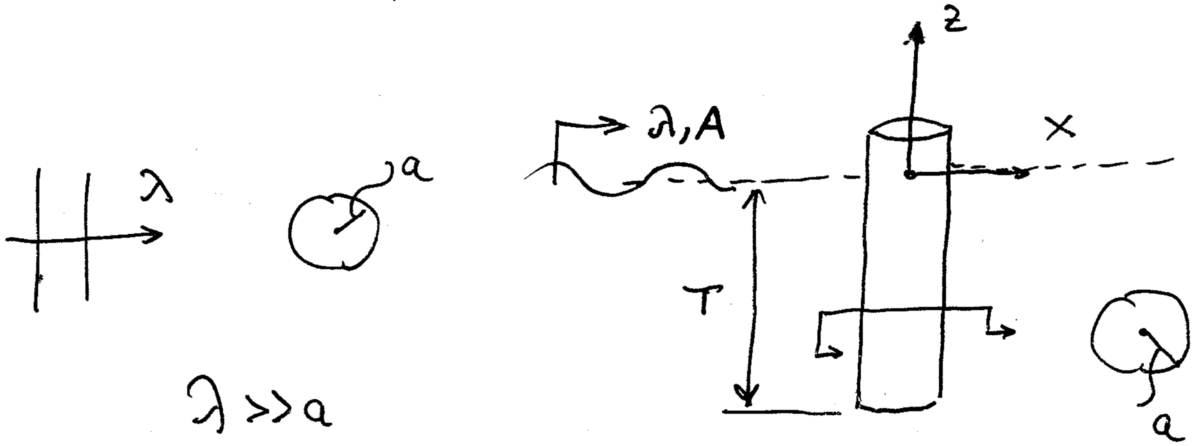
$$\forall = \pi a^2, \quad a_{11} = \pi \rho a^2$$

$$\frac{\partial u}{\partial t} = \mathbb{R}e\left\{ i\omega^2 e^{-kd+i\omega t} \right\}$$

THUS: $$F_x = -2\pi a^2 \omega^2 A\, e^{-kd} \sin \omega t$$

- DERIVE THE VERTICAL FORCE ALONG VERY SIMILAR LINES. IT IS SIMPLY 90° OUT OF PHASE RELATIVE TO F_x WITH THE SAME MODULUS.

C) HORIZONTAL FORCE ON A FIXED CIRCULAR CYLINDER OF DRAFT T:



THIS CASE ARISES FREQUENTLY IN WAVE INTERACTIONS WITH FLOATING OFFSHORE PLATFORMS.

HERE WE WILL EVALUATE $\frac{\partial u}{\partial t}$ ON THE AXIS OF THE PLATFORM AND USE A STRIPWISE INTEGRATION TO EVALUATE THE TOTAL HYDRODYNAMIC FORCE.

$$u = \frac{\partial \phi_I}{\partial x} = \Re\left\{ \frac{igA}{\omega}(-ik)\, e^{kz - ikx + i\omega t} \right\}$$

$$= \Re\left\{ \omega A\, e^{kz + i\omega t} \right\}_{x=0}$$

$$\frac{\partial u}{\partial t}(z) = \Re\left\{ \omega A\, (i\omega)\, e^{kz + i\omega t} \right\}$$

$$= -\omega^2 A\, e^{kz} \sin \omega t.$$

THE DIFFERENTIAL HORIZONTAL FORCE OVER A STRIP dz AT A DEPTH z BECOMES:

$$dF_z = \rho \left(\forall + a_{11} \right) \frac{\partial u}{\partial t} \, dz$$

$$= \rho \left(\pi a^2 + \pi a^2 \right) \frac{\partial u}{\partial t} \, dz$$

$$= 2\pi \rho a^2 \left(-\omega^2 A e^{kz} \right) \sin \omega t \, dz$$

THE TOTAL HORIZONTAL FORCE OVER A TRUNCATED CYLINDER OF DRAFT T BECOMES:

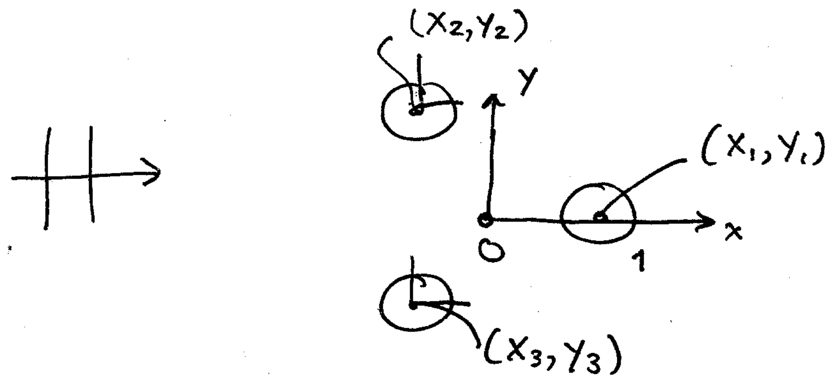
$$F_x = \int_{-T}^{0} dz \, dF = -2\pi \rho a^2 \omega^2 A \sin \omega t \times \int_{-T}^{0} e^{kz} \, dz$$

$$X_1 \equiv F_x = -2\pi \rho a^2 \omega^2 A \sin \omega t \cdot \frac{1 - e^{-kT}}{k} \cdot -$$

- THIS IS A VERY USEFUL AND PRACTICAL RESULT. IT PROVIDES AN ESTIMATE OF THE SURGE EXCITING FORCE ON ONE LEG OF A POSSIBLY MULI-LEG PLATFORM

- AS $T \to \infty$; $\frac{1 - e^{-kT}}{k} \to \frac{1}{k}$

D) HORIZONTAL FORCE ON MULTIPLE VERTICAL CYLINDERS IN ANY ARRANGEMENT:



THE PROOF IS ESSENTIALLY BASED ON A PHASING ARGUMENT. RELATIVE TO THE REFERENCE FRAME:

$$\phi_I = \mathbb{R}e \left\{ \frac{igA}{\omega} e^{kz - ikx + i\omega t} \right\}$$

- EXPRESS THE INCIDENT WAVE RELATIVE TO THE LOCAL FRAMES BY INTRODUCING THE PHASE FACTORS:

$$\mathbb{P}_i = e^{-ikx_i}$$

LET: $\quad x = x_i + \xi_i$

THEN RELATIVE TO THE i-TH LEG:

$$\phi_I^{(i)} = \mathbb{R}e \left\{ \frac{igA}{\omega} e^{kz - ik\xi_i + i\omega t} \, \mathbb{P}_i \right\}$$

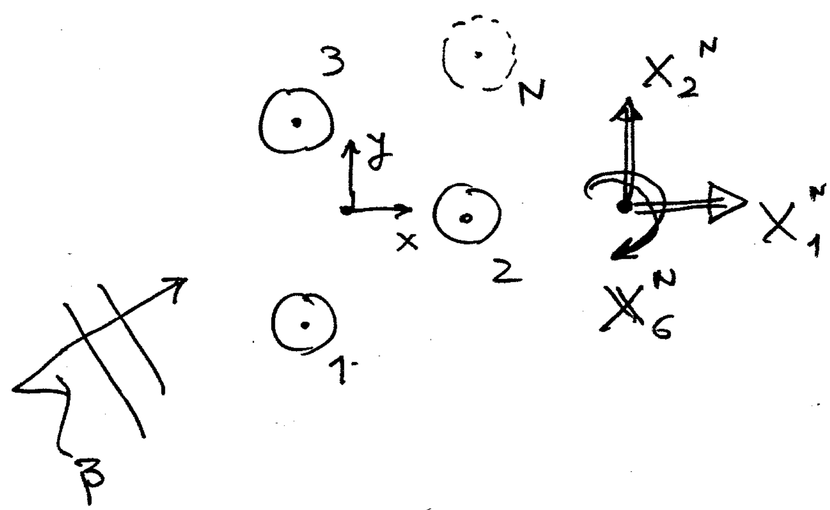
$$i = 1, \dots, N$$

IGNORING INTERACTIONS BETWEEN LEGS, WHICH IS A GOOD APPROXIMATION IN LONG WAVES, THE TOTAL EXCITING FORCE ON AN N-CYLINDER PLATFORM IS:

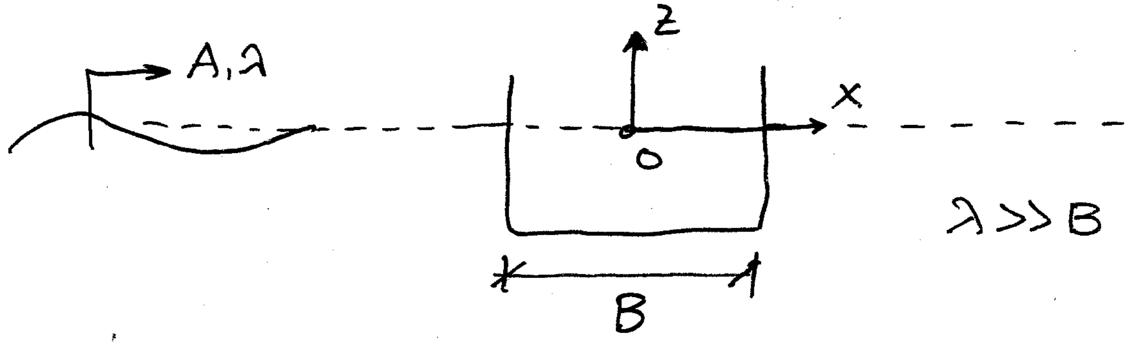
$$\mathbb{X}_1^N = \sum_{i=1}^{N} \mathbb{P}_i \, \mathbb{X}_1$$

THE ABOVE EXPRESSION GIVES THE COMPLEX AMPLITUDE OF THE FORCE WITH $\mathbb{X}_1$ GIVEN IN THE SINGLE CYLINDER CASE.

- THE ABOVE TECHNIQUE MAY BE EASILY EXTENDED TO ESTIMATE THE SWAY FORCE AND YAW MOMENT ON N-CYLINDERS WITH LITTLE EXTRA EFFORT (LEFT AS AN EXERCISE)



E) SURGE EXCITING FORCE ON A 2D SECTION



$$\phi_I = \mathrm{Re}\left\{ \frac{igA}{\omega} e^{kz-ikx+i\omega t} \right\}$$

$$u = \mathrm{Re}\left\{ \frac{igA}{\omega}(-ik)\, e^{kz-ikx+i\omega t} \right\}$$

$$\frac{\partial u}{\partial t} = \mathrm{Re}\left\{ \frac{igA}{\omega}\left(-i\frac{\omega^2}{g}\right)(i\omega)\, e^{i\omega t} \right\}_{\substack{x=0 \\ z=0}}$$

$$= \mathrm{Re}\left\{ i\omega^2 A\, e^{i\omega t} \right\} = -\omega^2 A \sin\omega t$$

$$\mathbb{X}_1 = (\rho\forall + A_{11})\frac{\partial u}{\partial t} = -\omega^2 A \sin\omega t\,(\rho\forall + A_{11})$$

$\rho\forall$: FROUDE KRYLOV

A_{11}: DIFFRACTION

- IF THE BODY SECTION IS A CIRCLE WITH RADIUS a:

$$\rho\forall = A_{11} = \pi\rho a^2/2$$

SO IN LONG WAVES THE SURGE EXCITING FORCE IS EQUALLY DIVIDED BETWEEN THE FROUDE-KRYLOV AND THE DIFFRACTION COMPONENTS. THIS IS NOT THE CASE FOR HEAVE!

F) HEAVE EXCITING FORCE ON A SURFACE PIERCING SECTION

IN LONG WAVES, THE LEADING ORDER EFFECT IN THE EXCITING FORCE IS THE HYDROSTATIC CONTRIBUTION:

$$\mathbb{X}_3 \sim \rho g A_W A$$

WHERE A_W IS THE BODY WATERPLANE AREA IN 2D OR 3D. A IS THE WAVE AMPLITUDE. THIS CAN BE SHOWN TO BE THE LEADING ORDER CONTRIBUTION FROM THE FROUDE-KRYLOV FORCE

$$\mathbb{X}_3^{FK} = \rho g A \iint_{S_B} e^{kz - ikx} n_3 \, ds$$

USING THE TAYLOR SERIES EXPANSION:

$$e^{kz-ikx} = 1 + (kz - ikx) + O(kB)^2$$

IT IS EASY TO VERIFY THAT: $\mathbb{X}_3 \rightarrow \rho g A A_W$.

THE SCATTERING CONTRIBUTION IS OF ORDER KB. FOR SUBMERGED BODIES: $\mathbb{X}_3^{FK} = O(kB)$.