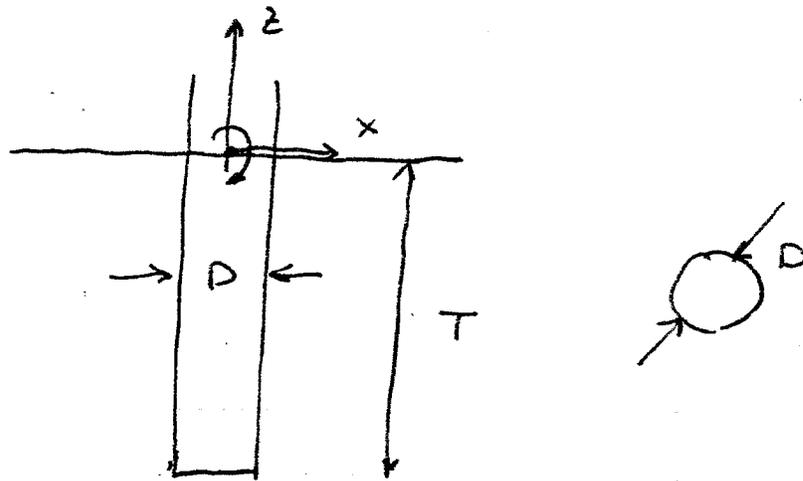13.022
QUIZ # 2
SOLUTIONS

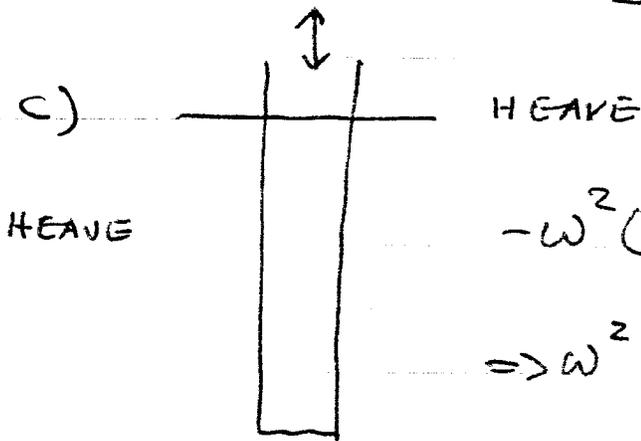— . —

1.

a) $A_{11} = \pi \rho \dfrac{D^2}{4} T$,   IGNORE FS EFFECTS
USE STRIP THEORY

$$A_{55} = \pi \rho \dfrac{D^2}{4} \int_{-T}^{0} z^2 \, dz = \pi \rho \dfrac{D^2}{4} \dfrac{T^3}{3} \quad \bullet —$$

$$A_{33} = \tfrac{1}{2} A_{SPHERE} \; (\omega. \, DIAMETER \; D)$$

$$A_{SPHERE} = \tfrac{1}{2} \rho \, \forall = \tfrac{1}{2} \rho \, \tfrac{4}{3} \pi \left(\dfrac{D}{2}\right)^3 \bullet —$$

b)   HEAVE – UNCOUPLED
SURGE – PITCH – COUPLED
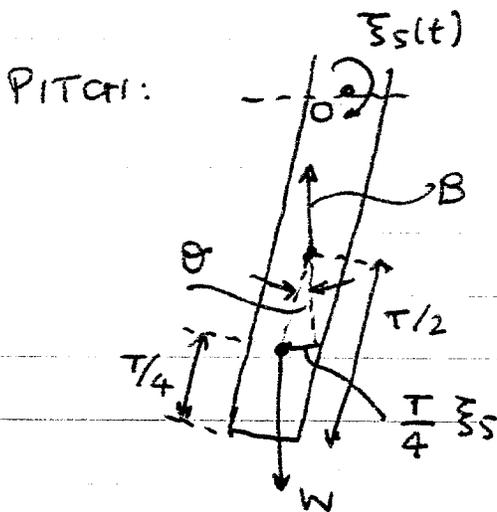SWAY – ROLL – COUPLED
YAW – UNCOUPLED

c)

HEAVE

HEAVE

$$-\omega^2(M + A_{33}) + C_{33} = 0$$

$$\Rightarrow \omega^2 = \frac{C_{33}}{M + A_{33}} =$$

$$C_{33} = \rho g A_w = \rho g \pi \frac{D^2}{4}$$

$$M = \rho T A_w = \rho T \frac{\pi D^2}{4}$$

$$A_{33} = \frac{1}{2} A_{SPHERE} = \frac{\pi \rho D^3}{6}$$

PITCH: $\xi_5(t)$

$$-\omega^2(I_{55} + A_{55}) + C_{55} = 0$$

$$I_{55} \text{ (ABOUT 0)} = A_{55} \text{ (STRIP THEORY)}$$

$$= \pi \rho \frac{D^2}{4} \frac{T^3}{3}$$

$B = W$

$$C_{55} = C_{55}^{HYDROSTATIC} + B\frac{T}{4}$$

$$B = \rho \frac{\pi D^2}{4} T$$

FOR LONG SLENDER BUOY $C_{55} \ll B\frac{T}{4}$

$$\omega^2 = \frac{BT/4}{2 A_{55}}.$$

d)

$$\ddot{\Xi}_3 = \frac{X_3}{i\omega B_{33}},$$

$$B_{33} = \frac{k}{4\rho g v_g} |X_3|^2$$

$$\Rightarrow \ddot{\Xi}_3 = \frac{4\rho g^{3/2}\omega}{\frac{\omega^2}{g} |X_3| i\omega}$$

$$\Rightarrow \ddot{\Xi}_3 = \frac{2\rho g^3}{i\omega^4 |X_3|}$$
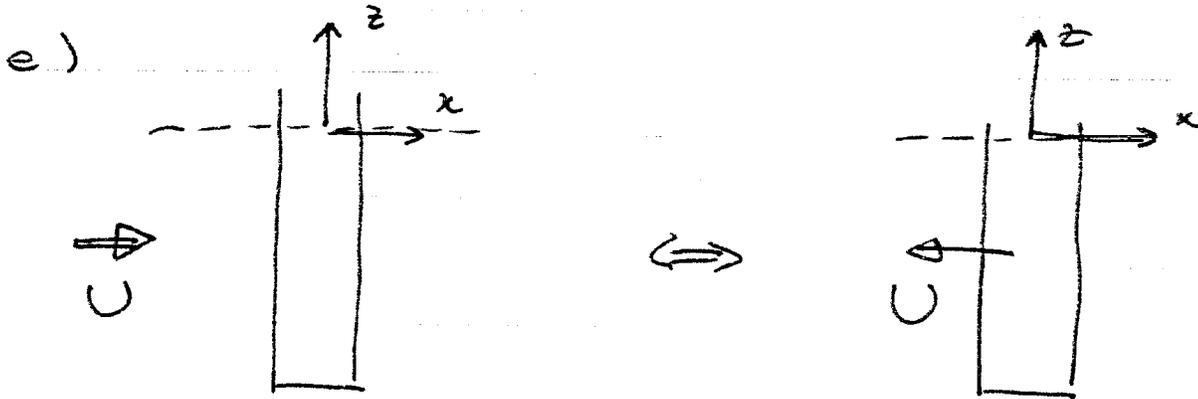
ASSUME WAVELENGTH CORRESPONDING TO RESONANT FREQUENCY IS $\lambda \gg D \Rightarrow$

$$|X_3| = \rho g A_w = \rho g \frac{\pi D^2}{4}$$

$$\Rightarrow \quad \ddot{\Xi}_3 \Big|_{RES} = \frac{2\rho g^3}{i\omega^3 \rho g \frac{\pi D^2}{4}} = \frac{8 g^2}{i\omega^3 \pi D^2}.$$

ADDITIONAL DAMPING ARISES FROM TWO VISCOUS SOURCES WHICH ARE DOMINANT:

- FLOW SEPARATION FROM CYLINDER BOTTOM
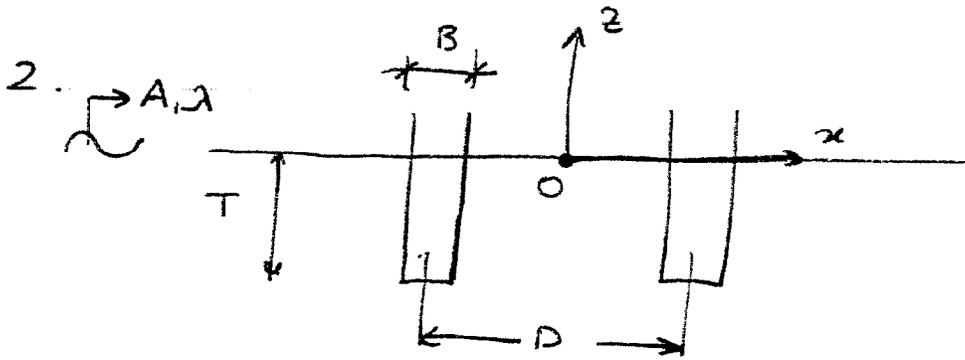- FRICTION DAMPING.

e)

a) SURGE — NO CHANGE DUE TO U. —

HEAVE — NO CHANGE DUE TO U

PITCH — ————— " ————— : NOTE THAT

ANGLE OF ATTACK EFFECT PRESENT
IN PITCH FOR A HORIZONTALLY
SLENDER VESSEL NOT PRESENT HERE

b) NO EFFECT OF U

c) NO EFFECT OF U

d) THE NATURE AND SIGNIFICANCE OF VISCOUS
EFFECTS ARE AFFECTED BY THE CURRENT

2.

a) $\quad \varphi_I = \text{Re}\left\{ \dfrac{igA}{\omega} e^{\nu z - i\nu x + i\omega t} \right\}$

HEAVE EXCITING FORCE VANISHES WHEN VERTICAL
FLUID ACCELERATION AT THE CENTER OF EACH
HULL IS SUCH THAT IN COMBINATION WITH
F-K LEADS TO A TOTAL HEAVE FORCE OUT
OF PHASE. ASSUME $\lambda \gg B$.

$$X_3^{FK} = \rho g B \left[\zeta(x = -D/2) + \zeta(x = +D/2) \right]$$

$$\zeta = A \cos(\kappa x - \omega t)$$

$$X_3^{DIF} = a_{33} \left[\left. \frac{\partial^2 \phi_I}{\partial z \, \partial t} \right|_{\substack{x = -D/2 \\ z = -T}} + \left. \frac{\partial^2 \phi_I}{\partial z \, \partial t} \right|_{\substack{x = -D/2 \\ z = -T}} \right]$$

$a_{33} = \frac{1}{2}$ ADDED MASS OF FLAT PLATE W. BEAM B
$\qquad = \frac{1}{2} \pi \rho \left(\frac{B}{2}\right)^2$

$$\frac{\partial^2 \phi_I}{\partial z \, \partial t} = \text{Re}\left\{ -\omega^2 A \, e^{-\nu T + i\kappa x + i\omega t} \right\}$$

$$= -\omega^2 A \, e^{-\nu T} \cos(\kappa x - \omega t)$$

BOTH F-K AND DIFFRACTION HEAVE FORCES
ARE PROPORTIONAL TO $\cos(kx - \omega t)$, SO
THE TOTAL HEAVE FORCE VANISHES WHEN

$$\cos\left(k\frac{D}{2} - \omega t\right) + \cos\left(-\frac{kD}{2} - \omega t\right) = 0$$

OR $\quad e^{ikD/2} + e^{-i\nu D/2} = 0$

$$\cos\left(\nu D/2\right) = 0 \implies \nu\frac{D}{2} = (2n+1)\frac{\pi}{2}$$

$$n = 0, 1, 2, \ldots$$

$$\implies D = (2n+1)\frac{\pi}{\nu}, \quad n = 0, 1, 2, \ldots$$

b) HEAVE DAMPING VANISHES AT THE SAME
FREQUENCIES WHERE THE HEAVE EXCITING
FORCE VANISHES BY VIRTUE OF THE
RELATION:

$$B_{33} = \frac{|X_3|^2}{2\rho g V_g}$$

$$\implies \frac{\omega^2}{g} = (2n+1)\frac{\pi}{D}$$

$$\implies \omega = \left\{(2n+1)\frac{\pi g}{D}\right\}^{1/2}, \quad n = 0, 1, \ldots, 2$$

c)

$$\omega = \omega_0 + U \frac{\omega_0^2}{g} \quad , \quad \lambda = \frac{2\pi}{\gamma} = \frac{2\pi g}{\omega_0^2}$$

$$\Rightarrow U = \frac{(\omega - \omega_0) g}{\omega_0^2} \cdot \underline{\quad} \quad \Rightarrow \omega_0 = \left(\frac{2\pi g}{\lambda}\right)^{1/2}$$

d)

$$a_1(t) = A_1 \cos(\omega t) = \text{Re}\{A_1 e^{i\omega t}\}$$

$$a_2(t) = A_2 \cos(\omega t + \theta) = \text{Re}\{A_2 e^{i\theta + i\omega t}\}$$

$$a(t) = \text{Re}\{-\omega^2 [\mathbb{H}_3 - x \, \mathbb{H}_5] e^{i\omega t}\}$$

$$a_1(t) = a(t)\big|_{x=0} = \text{Re}\{-\omega^2 \mathbb{H}_3 \, e^{i\omega t}\}$$

$$\Rightarrow +\omega^2 |\mathbb{H}_3| = A_1 \Rightarrow |\mathbb{H}_3| = \frac{A_1}{\omega^2}$$

$$a_2(t) = a(t)\big|_{x=\frac{L}{2}} = \text{Re}\{-\omega^2 [\mathbb{H}_3 - \frac{L}{2}\mathbb{H}_5] e^{i\omega t}\}$$

$$\Delta a = a_2(t) - a_1(t) = \text{Re}\{\omega^2 \frac{L}{2} \mathbb{H}_5 \, e^{i\omega t}\}$$

$$= \text{Re}\{(A_2 e^{i\theta} - A_1) e^{i\omega t}\}$$

$$\Rightarrow$$

$$\omega^2 \frac{L}{2} |\mathbb{H}_5| = |A_2 e^{i\theta} - A_2|$$

$$\Rightarrow |\mathbb{H}_5| = \frac{|A_2 e^{i\theta} - A_2|}{\omega^2 L/2} \cdot \underline{\quad}$$

e) PHASE DIFFERENCE BETWEEN $\mathbb{E}_3$ & $\mathbb{E}_5$

OR $\varphi_3 - \varphi_5$ SAME AS PHASE DIFFERENCE

BETWEEN $-a_1(t)$ AND $a_2(t) - a_1(t)$

OR $\operatorname{Re}\{\underbrace{-A_1}_{z_1} e^{i\omega t}\}$ & $\operatorname{Re}\{\underbrace{(A_2 e^{i\vartheta} - A_1)}_{z_2} e^{i\omega t}\}$

IF z_1 & z_2 ARE TWO COMPLEX NUMBERS THEIR

PHASE DIFFERENCE IS GIVEN BY THE RELATION

$$e^{i(\varphi_1 - \varphi_2)} = \frac{z_1}{z_2}\frac{|z_2|}{|z_1|}$$

OR $\varphi_1 - \varphi_2 = -i \ln\left[\frac{z_1}{z_2}\frac{|z_2|}{|z_1|}\right]$

WHERE

$$z_1 = |z_1| e^{i\varphi_1}$$
$$z_2 = |z_2| e^{i\varphi_2}$$

APPLY THIS RELATION TO $z_1 = -A_1$, $z_2 = A_2 e^{i\vartheta} - A_2$

f) $a(t) = \operatorname{Re}\{-\omega^2 [\mathbb{E}_3 - x \mathbb{E}_5] e^{i\omega t}\}$

$|a(t)| = \omega^2 |\mathbb{E}_3 - x \mathbb{E}_5|$

$= +\omega^2 \left| |\mathbb{E}_3| e^{i\varphi_3} - x |\mathbb{E}_5| e^{i\varphi_5} \right|$

$= \omega^2 \left| |\mathbb{E}_3| - x |\mathbb{E}_5| e^{i(\varphi_5 - \varphi_3)} \right|$.