13.42 Exam #1 Solutions Spring 2004

Problem 1:

$$f(t) = \underbrace{\tfrac{1}{2}\rho C_d A\, u(t)|u(t)|}_{\substack{\text{non linear} \\ \sim u^2(t)}} + \underbrace{(m + m_a)\frac{\partial u(t)}{\partial t}}_{\text{linear}}$$

Morrison's Equation is not linear in general however thinking back to 13.021 there are instances when the inertial forces dominate & thus the non-linear drag component can be neglected.

Problem 2:

Impulse response $h(t) = h_0 u_0(t - t_0)$

$\int_{-\infty}^{\infty} u(t - t_0)\,dt = 1$ delta fcn.

$$H(\omega) = \text{FourierTransform}(h(t))$$

$$= \int_{-\infty}^{\infty} h(t) e^{-i\omega t}\,dt$$

$$= \underset{\substack{\uparrow \\ \text{constant}}}{h_0} \int_{-\infty}^{\infty} u_0(t - t_0) e^{-i\omega t}\,dt$$

Recall property of delta fcn

$$\int_{-\infty}^{\infty} u(t - t_0) f(t)\,dt = f(t_0)$$

therefore $H(\omega) = h_0 e^{-i\omega t_0}$

b) input $x(t) = x_0 \sin(\omega_0 t + \psi_0)$

output $\rightarrow$ $y(t) = h(t) * x(t)$

-or-

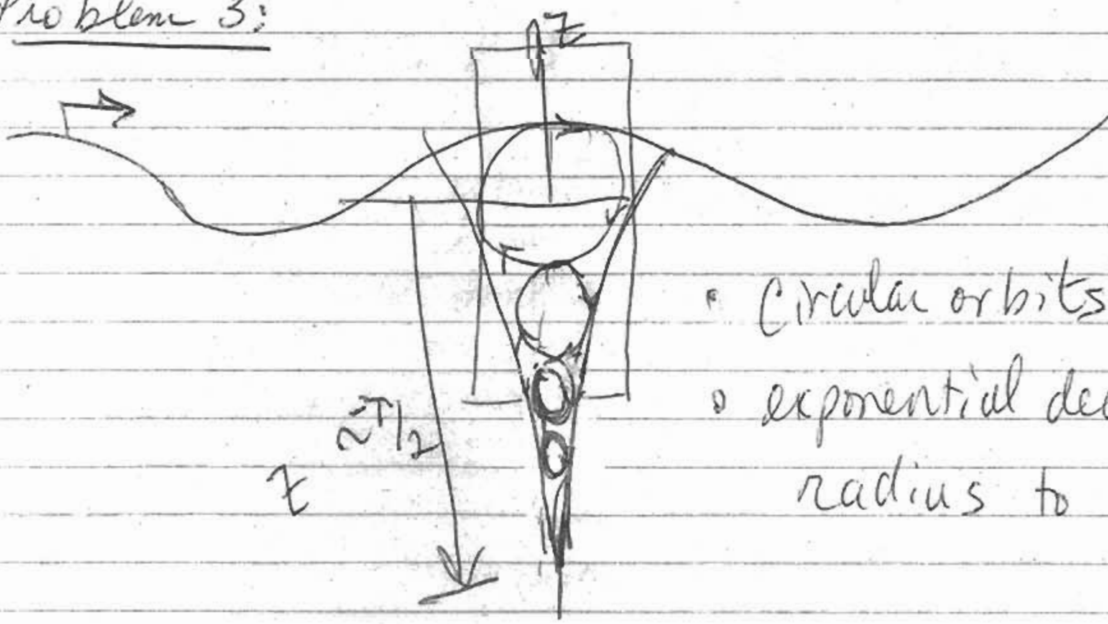
$$y(t) = x_0 |H(\omega_0)| \sin(\omega_0 t + \psi_0 + \measuredangle H(\omega_0))$$

$$H(\omega_0) = \underbrace{h_0}_{|H(\omega_0)|} \, e^{\overbrace{-i\omega_0 t_0}^{\text{arg } H(\omega_0)}} = |H(\omega_0)| \, e^{i \measuredangle H(\omega_0)}$$

$$\Rightarrow y(t) = x_0 h_0 \sin(\omega_0 t + \psi_0 + (-\omega_0 t_0))$$

$$\boxed{\therefore y(t) = x_0 h_0 \sin(\omega_0 (t - t_0) + \psi_0)}$$

Problem 3:



- Circular orbits
- exponential decay in radius to $z = -\lambda/2$

3 b) pressure under waves

Dynamic pressure $= -\rho \frac{\partial \phi}{\partial t}$ ~~c~~ hydroStatic $p = -\rho g z$

$$-p(z,t) = \rho g e^{-kz} \eta(x=0,t) + \rho g z$$

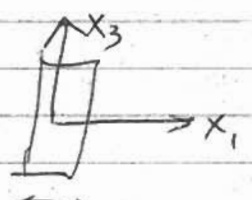
sign ✓

where $\eta = a\cos(kx - \omega t)$

$$\eta(0,t) = a\cos(\omega t) \quad [\cos(-a) = \cos(+a)]$$

pressure is isotropic & acts everywhere normal to surface.

3c) Surge motion (x_1)



$x_1 = x_{1_0}\cos(\omega t)$

equation of motion

$$(m + M_a)\ddot{x}_1 + B_{11}\dot{x}_1 + C_{11}x_1 = f_1(t)$$

mass | added mass from strip theory | estimated linear damping | Tension/length of tether

3d) $$H(\omega) = \frac{1}{-(m+M_a)\omega^2 + i\omega B_{11} + C_{11}}$$

Problem 4

a) - historical wave data
ideally wave measurements over span of at least several decades
- info on wind patterns
- geological info about coastal shorelines & underwater formations that may effect wave development or propagation
- from this we can get significant wave height etc...

b) Uwind limits phase speed

$$U_w \simeq C_p = \omega/k = g/\omega = 4\,m/s$$

deepwater: $\omega^2 = gk \Rightarrow \frac{\omega}{k} = \frac{g}{\omega}$

$$\omega = \frac{g}{U_w} = \frac{10\,m/s^2}{4\,m/s} = 2.5\,rad/s.$$

c) $M_0 = 12\cdot(0.5) + 24(0.5) + 18(0.5) + 6(0.5)$

$\delta(\omega) = 0.5\,rad/s$ $\qquad M_0 = 30\ m^2$ $\quad \left(\frac{m^2}{rad/s}\cdot rad/s\right)$

aside $M_2 = \sum_i \omega_i^2 S^+(\omega_i)\,d\omega_i = 45.75\ m^2\,(rad/s)^2$

$M_4 = \sum_i \omega_i^4 S^+(\omega_i)\,d\omega_i = 105.95\ m^2\,(rad/s)^4$

$(\varepsilon = 0.58)$

d) Sig. wave height $\quad \zeta = 4\sqrt{M_0} = 21.9\,m$!

v. large admittedly

d) $\bar{\eta} = 3/\text{day} = \dfrac{3}{\underset{\text{hrs}}{24}\cdot\underset{\frac{\text{min}}{\text{hr}}}{60}\cdot\underset{\frac{\text{sec}}{\text{min}}}{60}} = \dfrac{3 \text{ times}}{86400 \text{ sec}}$

$\bar{\eta} = 3.47 \times 10^{-5}\ 1/s$

$\bar{\eta} = \frac{1}{2\pi}\sqrt{\frac{M_2}{M_0}}\, e^{-h_0^2/2M_0}$

$= \frac{1}{2\pi}\sqrt{\frac{45.75}{30}}\, e^{-h_0^2/2\cdot 30} = 3.47 \times 10^{-5}$

Solve for h_0 →

$e^{-h_0^2/60} = \frac{3.47\times10^{-5}}{0.1965}$

$\ln\left[e^{-h_0^2/60} = 0.001766\right]$

$-h_0^2/60 = -6.34$

$\therefore h_0^2 = 380.35\ m^2$

$\boxed{\therefore h_0 = 19.5\ m}$

e) • well first off S is quite large probably not realistic ...

• Secondly you want wind mill high enough to be out of wind Boundary layer above ocean waves (regardless of tips getting wet)

~~(w/ 4 m/s wind assuming ~1 km fetch $\delta \approx 5$ m)~~ for smooth surface & turbulent flow] ← aside ...