13.42 Homework #4

Spring 2005

Out: Thursday, February 24, 2005

Due: Thursday, March 3, 2005

Problem 1: Given the probability distribution function,

$$f_X(x) = \begin{cases} \alpha x^2 & \text{if } 0 \le x \le 10 \\ 0 & \text{otherwise} \end{cases}$$

a) Find α.
b) Find $P(X > 5)$.
c) Find the mean, μ_X.
d) Find the variance and standard deviation, σ_X^2 and σ_X respectively.

Problem 2: Gaussian Distribution.

Let random variable X be the number of days it rains in vicinity of a certain weather buoy. Suppose, from historical record, that X is normally distributed with a mean of 60 days and a standard deviation of 15 days.

a) Find the probability distribution function, $f_X(x)$.
b) Find the cumulative distribution function, $F_X(x) = P(X \le x)$.
c) What is the probability that next year the number of rainy days will be between 40 and 70?
d) What is the probability that there will be more than 30 rainy days next year?

[Hint: Use either a computer program such as Mathcad or Excel (Analysis ToolPak Add-In) to calculate the erf or use standard normal distribution tables.]

Problem 3: Poisson Distribution.

Based on historical weather records, there have been an average of 5 rainstorms per year over the last 30 years in vicinity of a particular weather buoy. Assuming that the occurrence of rainstorms is a Poisson process:

a) What is the probability that there will be no rainstorms next year?
b) What is the probability that there will be exactly 5 rainstorms next year?
c) What is the probability that there will be 3 or more rainstorms next year?

Problem 4: Consider wave elevation as a random process:

$$\eta(x,t) = A\sin(\omega_0 t + k_0 x)$$

where ω_0 is a constant and $\theta(x) = k_0 x$. $\theta(x)$ is a random variable which is uniformly distributed from $-\pi$ to π.

a) Find the ensemble average, variance and correlation.
b) Find the temporal average, variance and correlation.
c) Determine whether this is an ergodic process.

Problem 5: Consider an LTI system where the input *u(t)* is stationary and ergodic, and *H(ω)* is the transfer function in the frequency domain. The output is *y(t)*.

a) Is *y(t)* stationary and ergodic?
b) What does it mean to be stationary and what does it mean to be ergodic?
c) Can you have a stationary, non-ergodic process? How about a non-stationary, ergodic process?
d) If the spectrum of *u(t)* is $S_u(\omega)$ and the transfer function is *H(ω)*, what is $S_y(\omega)$?
e) Given $S_u(\omega)$ and $S_y(\omega)$, can you find *H(ω)*? What do you know about its magnitude and phase?

Problem 6: An offshore platform with a deck height of 3 m is subjected to waves with average wave period 12.4 seconds and a standard deviation of 1.25 meters. Assume there is no wave diffraction at the platform.

a) Find the average frequency of water on deck.
b) How high would the platform have to be for the deck to flood once an hour?
c) How high would the platform have to be for the deck to flood once a day?