13.49 Homework #3

1. Consider a submarine moving forward at speed U, and restricted to small motions in the vertical $(x-z)$ plane. Assume that:

   - The submarine is symmetric in the $x-y$ plane and $y_G = 0$. The submarine is not symmetric in the $x-z$ plane due to the sail, etc., but assume it is *nearly* symmetric, so that you can omit certain hydrodynamic terms.
   - At rest, the submarine is neutrally buoyant and stable with the x and y axes horizontal. What does this tell us about the magnitude of the buoyant force and where it acts with respect to the center of weight?

   (a) Derive the equations of motion for the submarine moving at speed U.

   (b) Derive the hydrostatic, restoring pitch moment for small pitch θ.

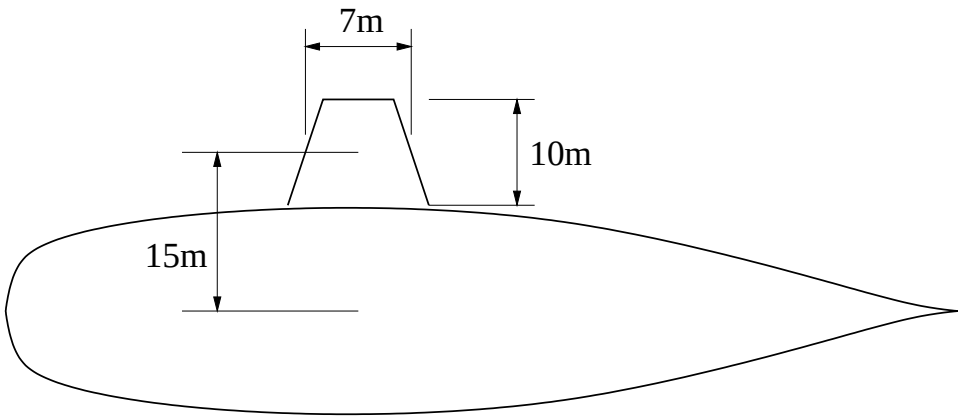
   (c) Linearize the inertial terms in the equations of motion.

   (d) Expand the fluid forces and moments in terms of the motions, as we have done in class for the yaw/sway problem. Omit nonlinear and memory effects, and be sure to include the hydrostatic moment. Explain your choices.

   (e) Write out the complete linearized equations of motion. Does surge decouple from pitch and heave? Write out the coupled equations in matrix form.

   (f) Can the submarine be stable in pitch without feedback control? Can the submarine be stable in depth without feedback control?

2. Using the lifting surface formulas, estimate the coefficient K_v of a submarine sail as shown below, for $U = 10m/s$.



3. Consider a long ellipsoidal body of length l and diameter d.

   (a) With $l/d = 7.0$, approximate the cross-body added mass, using slender-body theory, and compare it with the exact results from the table below (Blevins, 1979).

   (b) Perform the slender body calculation also for a sphere, and compare again to the exact result.

   (c) What is the added mass in the in-line direction?

Sphere added mass:	$2\rho\pi a^3/3$
Ellipsoid cross-body added mass:	$4\alpha\rho\pi ab^2/3$
where $a/b = 0.1$:	$\alpha = 0.075$
0.2:	0.143
0.6:	0.355
1.0:	0.500
2.0:	0.704
5.0:	0.894
7.0:	0.933
10.0:	0.960
∞:	1.000
Ellipsoid longitudinal added mass:	$4\alpha\rho\pi ab^2/3$
where $a/b = 0.1$:	$\alpha = 6.148$
0.2:	3.008
0.6:	0.908
1.0:	0.500
2.0:	0.210
5.0:	0.059
7.0:	0.036
10.0:	0.021
∞:	0.000

$2a$: sphere diameter or ellipsoid length,
$2b$: ellipsoid diameter