Wheel

It should not be assumed that linearity may only be achieved for small displacements. Appropriate design can result in devices which are linear over large ranges of displacements. Perhaps the simplest and most ubiquitous example is the wheel. Figure 5.9 depicts a wheel which may rotate about a fixed axis. The rim of the wheel contacts a sliding bar. Due to friction between the rim and the slider, when the wheel rotates, the slider translates. Assuming the wheel and slider are rigid and there is no slip between rim and slider, simple geometry provides a relation between displacements.

$$x = r\,\theta \qquad\qquad (5.22)$$

where r is the radius of the wheel. As before, a relation between flows (speed of the slider, v, and angular speed of the wheel, ω) may be obtained by differentiating with respect to time.

$$v = r\,\omega \qquad\qquad (5.23)$$

The relation between force, F, on the slider to the moment, μ, about the shaft is also linear.

$$\mu = r\,F \qquad\qquad (5.24)$$

These are the equations of a linear transformer (e.g. figure 5.3) relating the translational and rotational domains. Again, the same geometric parameter, r, shows up in both equations.
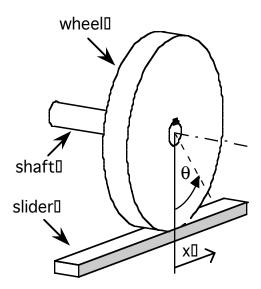

Figure 5.9:   A linear rotation-to-translation transformer composed of a wheel and slider.
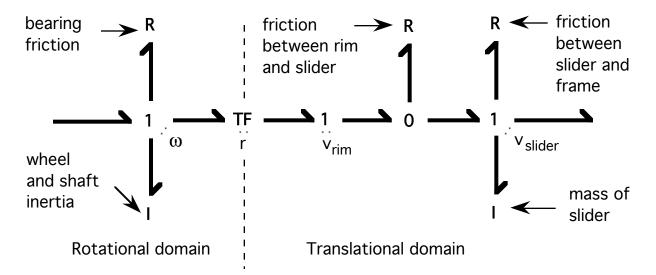
Nevertheless, it is well to keep in mind that in general, a transformer may be modulated, embodying a nonlinear relation between efforts and flows. As any practical machine designer knows, the radius of any real wheel will not be constant. The wheel may be non-circular or may be mounted eccentrically. In either case, its radius will be a function of its angle. If these effects are sufficiently important, they may modeled by describing the device as a modulated transformer as depicted in figure 5.6.
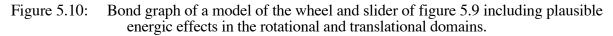
Parasitic Dynamics

As with other elements of a lumped-parameter model, an ideal transformer such as that of figure 5.3 or 5.6 describes a single phenomenon: power transmission. Real power transmission devices such as those sketched in figures 5.7 to 5.9 also store and dissipate energy. Being secondary (and somewhat athwart the primary function of power transmission) these phenomena are often referred to as *parasitic*. A competent model of a real power transmission device may need to include these "parasitic" effects. This may readily be accomplished using the energy storage and dissipation elements defined earlier.

For example, the wheel and slider of figure 5.9 might exhibit substantial energic phenomena in both domains. Likely candidates in the rotational domain are the inertia of the wheel and shaft and the friction due to the bearings which support the shaft (not shown in figure 5.9). All of the rotational phenomena are associated with a single speed, so the inertia, friction and the rotational side of the transformer are connected by a one-junction.

Figure 5.10:   Bond graph of a model of the wheel and slider of figure 5.9 including plausible energic effects in the rotational and translational domains.

Inertia and friction are also plausible in the translational domain. There are two distinct frictional components: One is due to the motion of the slider relative to the frame. That component and the inertia of the slider are associated with the same velocity, v_{slider}, so the two elements are connected to a one junction. The other component is due to the possibility of relative motion between the slider and the wheel rim. The translational velocity of the wheel rim at the point of contact, v_{rim}, is the translation port of the transformer. The frictional element is connected to a zero junction because the force on it is the same as that transmitted to the slider. Including these phenomena, a model of the system might be as shown in figure 5.10.

Of course, it should not be assumed that figure 5.10 is necessarily a better model. For some purposes, to include these phenomena may be to add superfluous detail and an ideal transformer may be the best model.