Massachusetts Institute of Technology
Department of Mechanical Engineering

2.12 Introduction to Robotics
Mid-Term Examination
November 2, 2005
2:30 pm – 4:30 pm

Close-Book. Two sheets of notes are allowed.
Show how you arrived at your answer.
Do not leave multiple answers. Indicate which one is your correct answer.

Problem 1 (50 points + 5 points extra)

The MIT Mechanical Engineering is developing a dinosaur robot for a local science museum. Figure 1 shows a schematic of the robot, having three degrees of freedom. The joint angles θ_1, θ_2 and θ_3 are all measured from the positive *x* axis fixed to the floor, and the link lengths are $\overline{OA} = \overline{AB} = 1m$. The head of the dinosaur is at Point E $(x_e, y_e)^T$, which is *3* meters from Joint 3 (Point B), and its orientation φ_e is measured from the positive *x* axis to line EH, the head centerline. Note that line EH bends down $\pi/3$ from line BE, as shown in the figure. Answer the following questions.

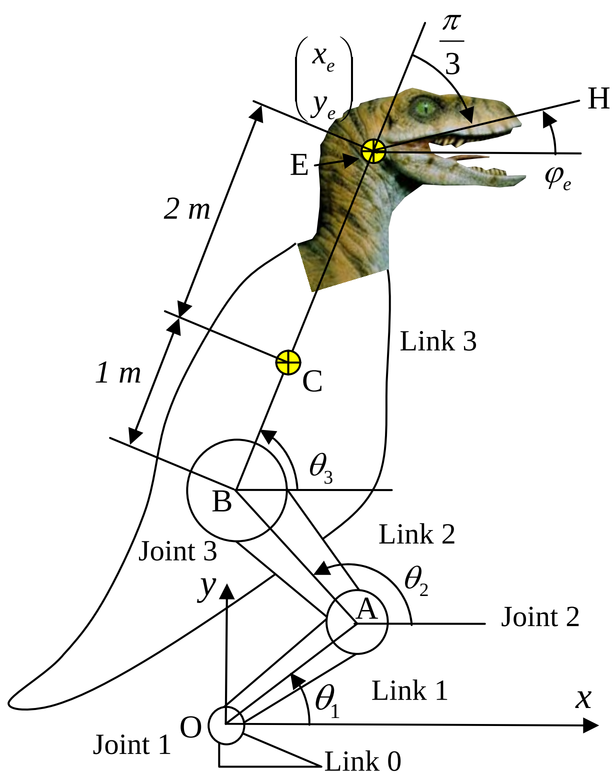

Figure 1 Overall structure of dinosaur robot

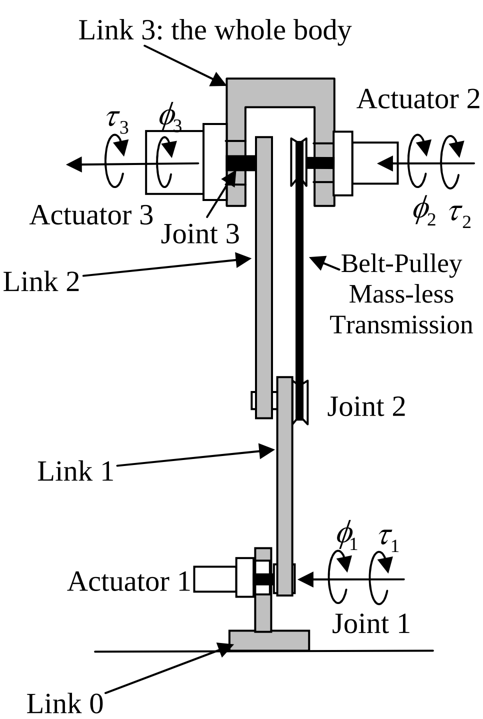

Figure 2 Actuators and leg mechanism

a). Obtain the position and orientation of the dinosaur head, $\mathbf{p} = \begin{pmatrix} x_e & y_e & \varphi_e \end{pmatrix}^T$, as functions of joint angles.

b). Obtain the Jacobian matrix relating the head velocity $\dot{\mathbf{p}} = \begin{pmatrix} \dot{x}_e & \dot{y}_e & \dot{\varphi}_e \end{pmatrix}^T$ to joint velocities: $\dot{\mathbf{q}} = \begin{pmatrix} \dot{\theta}_1 & \dot{\theta}_2 & \dot{\theta}_3 \end{pmatrix}^T$.

c). Obtain singular configurations using the above Jacobian in part b). First, mathematically solve the singularity condition, interpret the result, and sketch the singular configurations.

For the rest of the questions, refer to Figure 2 along with Figure 1, depicting the actuators and transmission mechanisms. Actuator 1 generates torque τ_1 between link 0 and link 1. Note that the body of Actuator 1 is fixed to link 0, while its output shaft is connected to link 1. Actuator 2 is fixed to Link 3, and its output torque τ_2 is transmitted to Joint 2, i.e. the knee joint, through the mass-less belt-pulley system with a gear ratio of 1:1. Actuator 3 is fixed to Link 3, while its output shaft is connected to Link 2. All actuator torques τ_1, τ_2, and τ_3 are measured in a right hand sense, as shown by the arrows in the figure. Displacements of the individual actuators are denoted ϕ_1, ϕ_2, and ϕ_3 are measured in the same direction as the torques.

d). The body of the dinosaur weighs 2,000 kg. The center of mass is at Point C, which is 1 meter from point B on line BE, as shown in Figure 1. How much actuator torques τ_1, τ_2, and τ_3 are needed to bear the weight of the body at the configuration of $\theta_1 = \pi/4$, $\theta_2 = 3\pi/4$, and $\theta_3 = \pi/3$. Ignore friction and the mass of the leg.

e). If one uses the Jacobian matrix relating the head velocity to actuator velocities for examining the singular configuration, are the singular configurations different from the ones obtained in part c)? Explain why.

Problem 2 (50 points + 10 points extra)

A rescue robot is working at a disaster site moving a heavy object at the tip of the arm. To reduce the load the rescue robot rests on a solid stationary wall at Point A, and slide the second link on the wall, as shown in the figure. The contact between the second link and the wall at Point A is assumed to be friction-less. The coordinates of Point A are x_A, y_A with reference to the base coordinate system. Similar to the 2.12 robot, this rescue robot has a two d.o.f. arm with both motors fixed to the base. (The torque of the second motor is transmitted through a belt-pulley mechanism to the second joint. The two pulley diameters are the same.) Using the parameters and variables shown in the figure, answer the following questions.

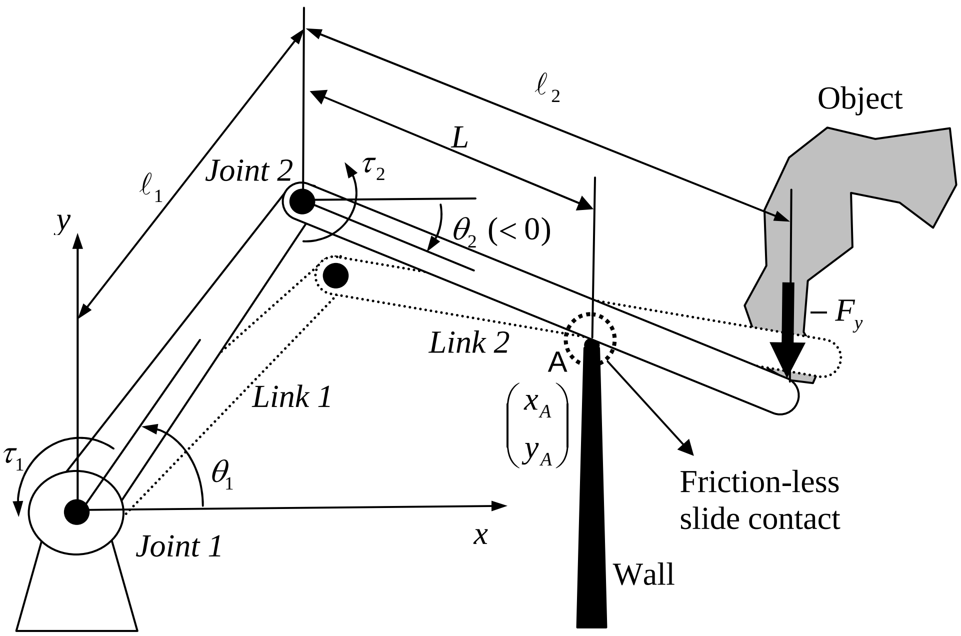

Figure 3 Rescue robot moving a heavy object

a). Under what conditions, can we use the length L between Joint 2 and Point A as a generalized coordinate that locates the system uniquely, assuming that Link 2 is in contact with Point A at all times?

b). When link 2 slides on the wall at Point A, the robot motion has to satisfy some geometric constraint. Obtain constraint equations by writing the coordinates of Point A, x_A, y_A , as functions of the joint angles θ_1, θ_2 and length L, i.e. $x_A = x_A(\theta_1, \theta_2, L)$ and $y_A = y_A(\theta_1, \theta_2, L)$.

c). Differentiating the functions, $x_A = x_A(\theta_1, \theta_2, L)$ and $y_A = y_A(\theta_1, \theta_2, L)$, in part b), find the relationship among virtual displacements $\delta\theta_1, \delta\theta_2$ and δL that satisfy the geometric constraints.

d). Obtain the virtual work done by the motor torques, τ_1, τ_2 , and the vertical endpoint force -F_y .

e). Using the Principle of Virtual Work, show that the joint torques τ_1, τ_2 , and the vertical endpoint force F_y satisfy the following form of condition when the system is in equilibrium:

$$F_y = a\tau_1 + b\tau_2$$

If time permits, obtain *a* and *b* by solving the equilibrium condition derived from the Virtual Work Principle for given joint angles θ_1, θ_2 and length L. You will get additional

10 points. (If you get stuck in computing *a* and *b*, proceed for the next question that can be solved without knowing *a* and *b*.)

f). DC motors are used for driving the two joints of the robot. Let K_{m1} be the motor constant of the first motor, and K_{m2} be that of the second motor. Show that the total power loss at the two motors when generating joint torques τ_1, τ_2 is given by:

$$P_{loss} = \frac{\tau_1^{\ 2}}{r_1^{\ 2} K_{m1}^{\ \ 2}} + \frac{\tau_2^{\ 2}}{r_2^{\ 2} K_{m2}^{\ \ 2}}$$

where r_1, r_2 are gear ratios of the motors.

g). Using parameters *a* and *b* in part e), obtain the optimal values of the joint torques τ_1, τ_2 that minimize the total power loss in both motors, P_{loss}, while bearing the vertical endpoint force $-F_y$.

Class Average	**75.28**	
Standard Deviation	**11.40**	

Point distribution

Problem 1

1-a	10	
1-b	10	
1-c	10	
1-d	10	+ 5 extra
1-e	10	
subtotal	50	

Problem 2

2-a	6	
2-b	6	
2-c	6	
2-d	6	
2-e	10	+ 10 extra
2-f	6	
2-g	10	
subtotal	50	

TOTAL 100 + 15 extra